Chapter 18
Recurrence Analysis of Turbulent Fluctuations in Magnetically Confined Plasmas

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Abstract Recurrence plots and their quantification became a modern tool in nonlinear data analysis, currently being used in a myriad of scientific disciplines, a diversity that characterizes the Recurrence Plot Workshop since its beginning. In this work we review some applications of recurrence quantification analysis to data analysis in a fusion plasma, namely turbulent fluctuations in the plasma edge of Tokamak Chauffage Alfvén Brésilien tokamak, which is a magnetic confinement plasma machine. A similar analysis was performed in a low-density plasma device called Texas Helimak (University of Texas at Austin). Our results point out that the deterministic content of the fluctuations tends to increase as we approach the plasma edge, indicating a concentration of the recurrences therein. This favours the use of fluid models to describe the physics of the plasma edge turbulence.

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18.1 Introduction

Recurrence plots are an extremely useful tool in nonlinear data analysis, particularly when the data series are short, noisy and even non-stationary, a very typical situation in realistic applications [1–3]. One promising field of study where recurrence plots are potentially advantageous is the analysis of fusion plasmas such those generated by tokamaks [4]. These advantages come from the complex nature of experimental signals from magnetically confined high-temperature plasmas, where we have a mixture of deterministic and stochastic features, and which can be sorted out by recurrence-based methods, unlike linear methods like Fourier analysis. Recurrence plots and recurrence quantification analysis have been applied to a wealth of problems involving technological low-temperature plasmas [5–7], space and geophysical plasmas [8] and fusion high-temperature plasmas [9–12].

Tokamaks are toroidal schemes of magnetic confinement of high-temperature plasmas, and it is thought to be the basis of a future thermonuclear fusion reactor, as in large-scale projects like the ITER (International Thermonuclear Experimental Reactor), currently being built in France by an international consortium [13]. ITER, which is to be completed in the next decade, will be the largest tokamak in the world, expected to generate 500 MW of power, ten times more than the power used to generate and confine the plasma in which the fusion reactions take place [14].

Once a plasma column is generated in a tokamak, one wants to confine it for a sufficiently large time, and for this to happen it is necessary that cross-field transport of particles and energy be controlled [15]. A major goal in the study of magnetically confined fusion plasmas has been to understand the causes and associated rates of anomalously large cross-field transport, which is thought to be caused by plasma turbulence [16]. One experimental signature of plasma turbulence in the plasma edge of a tokamak is the fluctuating behavior of the electrostatic floating potential, which often displays a broad fluctuation spectrum [17].

The deterministic content of the electrostatic fluctuations in the plasma edge has been assigned to physical mechanisms governed by nonlinear systems, like the interaction between drift waves which appear due to the steep density gradients in the plasma edge region of a tokamak [17, 18]. These drift wave interactions are known to depend critically on the radial position such that the electrostatic turbulent fluctuations should also exhibit some radial dependence.

However, the radial variation of the dynamical properties of the plasma turbulence may not be immediately apparent from the experimental data obtained in tokamaks. We used recurrence plots as a tool to quantify the recurrence properties of such series to provide a precise diagnostic of the dynamical properties of the system and other related measures of recurrence quantification analysis and their dependence on the radial position near the plasma edge [4, 19]. It turned out that this radial dependence, formerly elusive in conventional analysis using, e.g. spectral methods, is best observed using recurrence-based techniques.

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We also have used recurrence-based techniques to investigate the effect of high MHD activity in the electrostatic fluctuations in TCABR. The MHD activity increases at different instants of time during the discharge, and reaches high amplitudes with a narrow wave-number spectrum and a well-defined peak on the Mirnov frequency [20].

This chapter is organized as follows: In Sect. 18.2 we present the experimental setting and the kind of floating potential fluctuation data considered in our recurrence-based analyses. Section 18.3 presents results for recurrence-based diagnostics of potential fluctuations in different positions of the plasma column. Section 18.4 considers the effect of bias voltage on the recurrence-based diagnostics. The last section is devoted to our conclusions.

18.2 Electrostatic Potential Fluctuations

The experiments were performed in a hydrogen circular plasma in the Brazilian tokamak TCABR operating at the Institute of Physics of University of Sao Paulo (Fig. 18.1a), whose main parameters are listed in Table 18.1 [21]. Langmuir probes

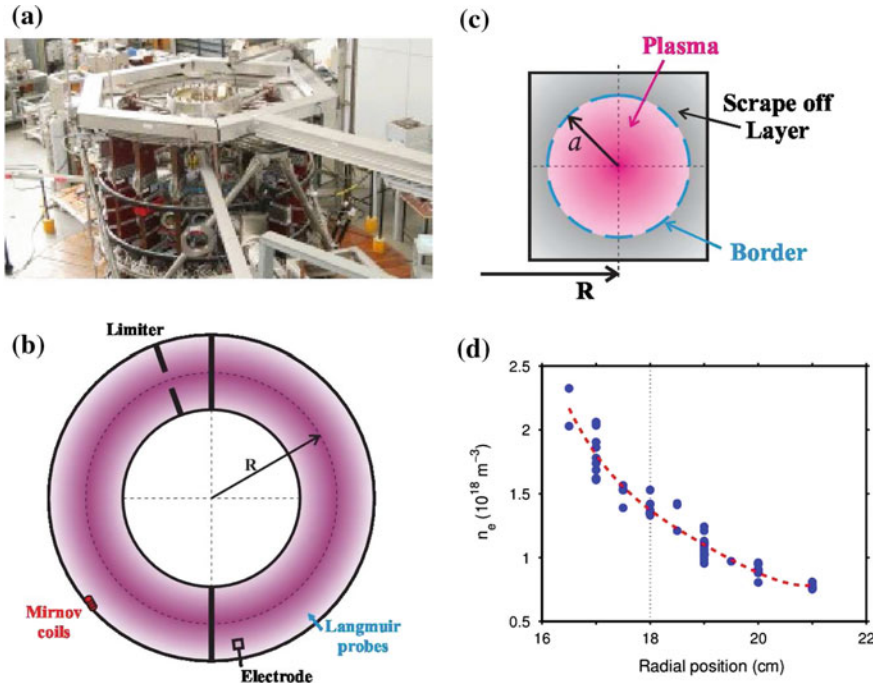


Fig. 18.1 **a** Picture of the TCABR tokamak. **b** Top view of the toroidal chamber and the Langmuir probes used to measure fluctuations of the floating electrostatic potential at the plasma edge. **c** Cross section of the toroidal chamber. **d** Plasma density radial profile

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Table 18.1 Basic parameters of the TCABR tokamak

Major radius (R)	61 cm
Minor radius (a)	18 cm
Maximum plasma current ($(I_p)_{max}$)	100 kA
Plasma duration (τ_p)	100 m
Hydrogen filling pressure (p_0)	3×10^{-4} Pa
Toroidal magnetic field ($B_{\phi 0}$)	1.1 T

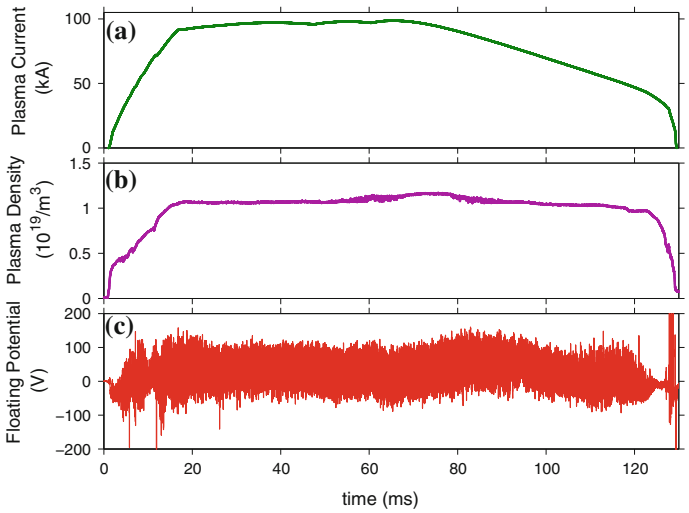


Fig. 18.2 Time evolution of the **a** plasma current, **b** plasma density, and **c** floating potential for a discharge of TCABR tokamak

were used to measure the floating potential whose fluctuations are mainly due to the the fluctuations of the plasma electrostatic potential (Fig. 18.1b) [22, 23]. The probes are mounted on a movable shaft that can be displaced radially with respect to the center of the plasma column, so as to cover both the plasma edge and the so-called scrape-off layer, the latter comprising part of the vacuum layer existent between the plasma column and the vessel wall (Fig. 18.1c, a and R stand for the minor and major radius of the vessel, respectively, see Table 18.1). The probe displacement, however, occurs only for separate discharges [4, 19]. The plasma density in the region surrounding the plasma edge is shown in Fig. 18.1d, showing a significative reduction as we go outwards to the scrape-off layer.

Figure 18.2 shows the time evolution of a typical tokamak plasma discharge in TCABR. The plasma current (Fig. 18.2a) grows rapidly in the first dozen of milliseconds and reaches a plateau where the current stays at a nearly constant level, decaying slowly during the second half until the end of the discharge. The plasma density evolution, indicated by Fig. 18.2b, exhibits a similar evolution. The time

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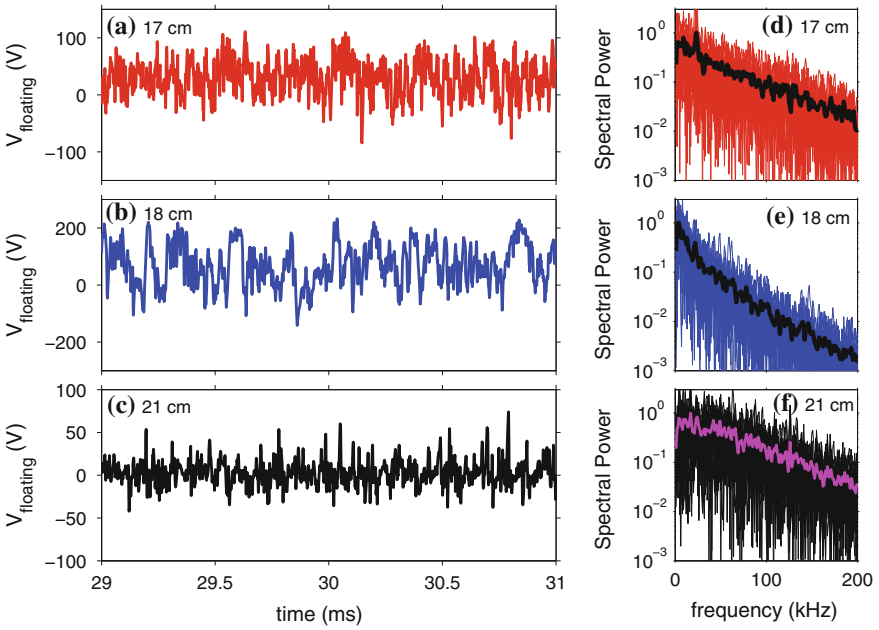


Fig. 18.3 Time evolution of the floating potential measured by Langmuir probes placed at radii **a** 17 cm, **b** 18 cm, and **c** 21 cm. *Light lines* in (**d**), **e** and **f** are the power spectral densities in each 1 m of the discharges corresponding to (**a**), (**b**) and (**c**), respectively. The *bold lines* in **d**–**f** are the average power spectra

signals we are particularly interested to study are for the floating electrostatic potential, a representative example being depicted by Fig. 18.2c, which shows highly irregular fluctuations.

The nature of the electrostatic potential fluctuation is dependent on the radial location where the probe is placed, as shown by Fig. 18.3, where two milliseconds of the time window in the plasma current plateau have been selected. Figure 18.3 also shows the corresponding power spectra in each 1 m (light lines) and the average power spectra in a 10 m window. Within the plasma column (Fig. 18.3a) the potential fluctuations present a −50 to +50 V range. As we move to the edge of the plasma column (Fig. 18.3b) the amplitude of such fluctuations increases by a factor of four, indicating that the fluctuation level augments as we approach the plasma radius. Outside the plasma column (but inside the scrape-off layer of vacuum magnetic field) (Fig. 18.3c) the floating potential amplitude range decreases to an in-between level. Hence the turbulent fluctuations become weaker as we move both outside and inside the plasma edge. This increase of amplitude at the plasma edge has been observed in other Tokamaks [24].

The radial dependence of the electrostatic fluctuations level at the vicinity of the plasma radius is a signature of the role played by radial density gradients in the generation of drift waves, which is the essential cause of turbulence in the plasma

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edge [18]. In fact, the presence of steep density gradients in the plasma edge can give rise to fully developed wave turbulence, which is considered a likely candidate for explaining anomalous transport observed in experiments [25].

However, characterizing turbulence in order to quantify its level and radial dependence is a difficult task, what can be illustrated by Fig. 18.3, where we also show the power spectra of the potential fluctuations measured at three radial positions just analyzed [23, 26]. All of them are broadband, which is already expected from the chaotic behavior related to turbulence but, apart from some unessential rippling, those spectra do not show a distinguish feature which could be used to quantify the fluctuation level and specific different dynamical regimes [27, 28].

18.3 Recurrence-Based Analysis of Turbulent Fluctuations

It turns out that recurrence quantification analysis can help us to characterize the fluctuation level observed in tokamak experiments, especially because it does not impose stationarity nor long series length as necessary conditions, and can also work satisfactorily with moderate noise levels in data acquisition system. Basically the major source of noise is thought to be the due to wave turbulence. Moreover, it also yields reliable estimates of the Shannon entropy and can also indicate the amount of determinism in a given time series, what gives us an idea of the noise level added to the chaotic signal [29].

The recurrence plots for the first five hundred points (corresponding to half a millisecond interval) of the time series previously considered are depicted in Fig. 18.4. We choose the embedding dimension as four and the time delay was selected by considering the first local minimum of the autocorrelation function. We can

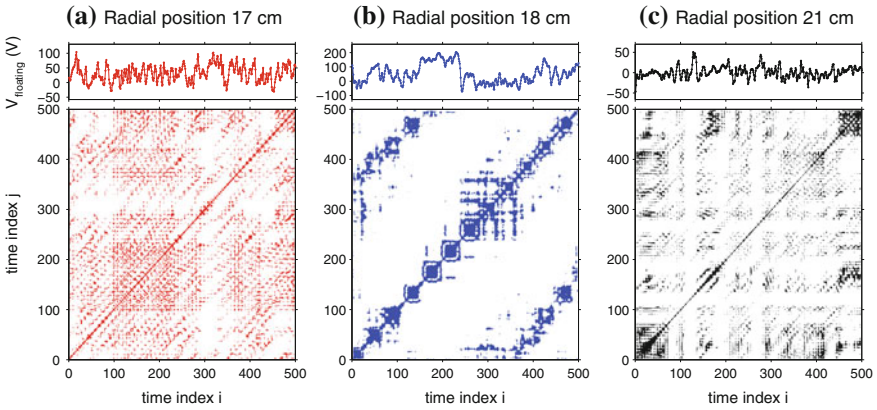


Fig. 18.4 Recurrence plots for the time series of the floating potential for radial positions **a** Radial position 17 cm; **b** Radial position 18 cm; and **c** Radial position 21 cm

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recognize the changes in the turbulent behavior at the plasma radius by comparing the diagonal and vertical structures of the time series at the plasma edge (middle of Fig. 18.4) with the scattered nature of the recurrence plots depicted in left and right of Fig. 18.4, which suggest a pronounced stochastic effect, probably related to noise and/or other mechanisms not accounted for in a deterministic theory.

From the recurrence plots of the time series of the floating potential for different positions we have computed a number of recurrence-based diagnostics. One of them is the determinism (DET) of the signal. By sliding the time window used to compute this quantity in each radial position we obtain a statistical distribution of values of DET. In Fig. 18.5 we plot histograms which stand for numerical approximations of the probability distribution function of the values of DET for different radial positions, from 17 to 21 cm. This range was chosen not just because it is accessible for measurements but also because it is one relevant region for cross flow transport. We clearly see that the average value of DET increases as we approach the edge of the plasma column, but decrease afterwards, with a nearly constant dispersion.

In Fig. 18.6a we plot the radial profile of DET from these average values. The line joining those points is a polynomial fit just to guide the eye, i.e. it is not intended to give a radial profile DET (r) but rather a trend: the degree of determinism increases significantly as we approach the plasma radius, and decreases afterwards. By sliding the time window used to compute this quantity in each radial position we obtain a statistical distribution of values of DET in each 1 m (1000 points, without

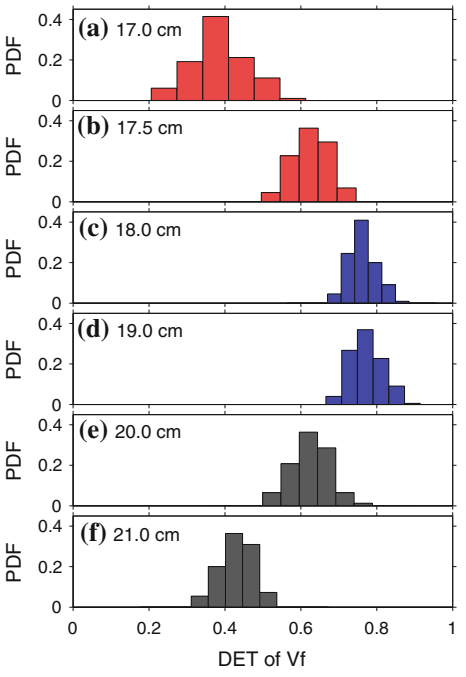


Fig. 18.5 Numerical approximations of the probability distribution function of the determinism for floating potential fluctuations measured at different radial positions

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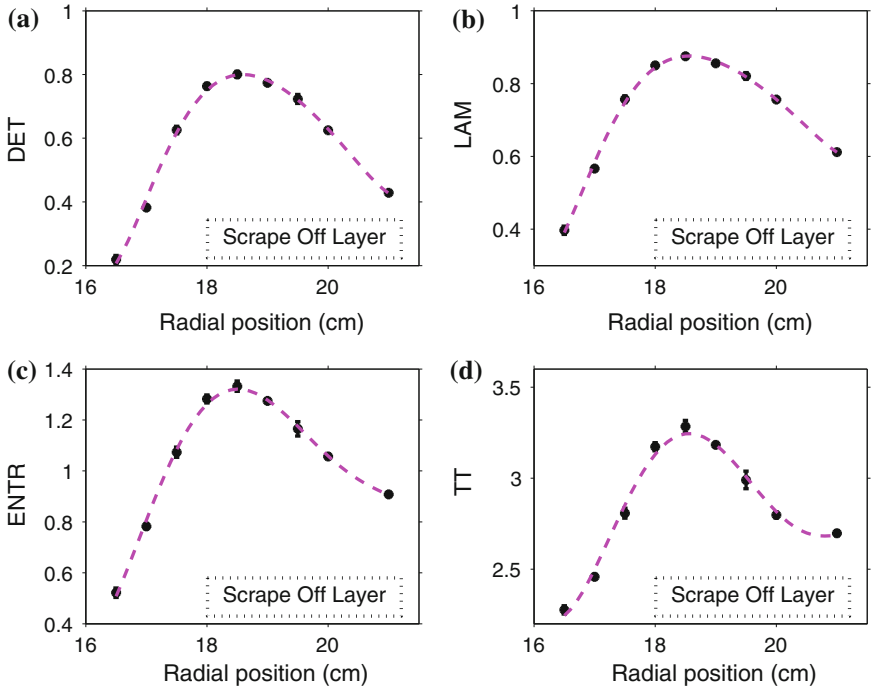


Fig. 18.6 Radial profiles for several recurrence-based diagnostics: **a** determinism; **b** laminarity; **c** entropy; **d** trapping time. The position of the scrape-off layer between the plasma column and the tokamak wall is indicated by a *horizontal bar*

superposition). This suggests a clear modification on the dynamical properties of the plasma floating potential fluctuations in the analyzed region. The same trend is exhibited by other recurrence-based diagnostics like the Laminarity LAM (Fig. 18.6b), entropy of the diagonal line lengths (Fig. 18.6c), and trapping time (Fig. 18.6d). These results reinforce our conclusion that the dynamical properties of the turbulent fluctuations changes substantially in this region, with the simultaneous peak of the DET, LAM and TT indicating that the recurrences are most concentrated around the plasma edge than elsewhere.

18.4 Effects of a Bias Radial Electric Field

The investigation we just reported was the first work about recurrence plots in fusion plasmas and one of the first on plasmas in general [4]. We have, since then, continued our pursuit of applying recurrence-based techniques to characterize a wealth of dynamical phenomena related with turbulent plasma fluctuations. In order to control plasma turbulence, external electric potentials have been applied, in several devices,

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to change the electric field profile. This turbulence reduction decreases the particle loss at the plasma edge and, consequently, improves plasma confinement [14, 26]. In a recent paper [19] we applied recurrence quantification analysis to the electrostatic fluctuations at the edge of the plasma in TCABR tokamak with and without the application of a radial electric bias in order to control the turbulence and improve plasma confinement. The latter is a form of creating internal transport barriers in the plasma so as to decrease the radial transport. Hence the quality of plasma confinement is expected to improve through this procedure.

In order to keep the recurrence plots comparable among signals with very different standard deviations, we fixed the recurrence rate. In other words, we consider two points as being recurrent or not depending on the overall recurrence rate of the series be equal to some given value. Hence, if most points are so close that they would be considered recurrent if a fixed threshold be given, then with a fixed recurrence rate they will be recurrent only if their contribution falls into that percentage. We observed substantial changes in the turbulent behavior at the plasma edge by comparing the diagonal and vertical structures of Fig. 18.7a—obtained without biasing—with the recurrence plot with biasing depicted in Fig. 18.7b. Such comparison suggests a reduction in the deterministic effect, probably related to the existence of a lower number of large-scale structures due to biasing.

In Fig. 18.8 we show the radial profile of the determinism (from potential fluctuations time series) with and without electric bias. The recurrence rate was kept at 1, 2.5 and 10 % in Fig. 18.8a–c, respectively. The overall trends are the same for the three cases here considered, apart from presenting higher values of DET as the recurrence rate is larger, as expected from the higher density of recurrent points. The radial

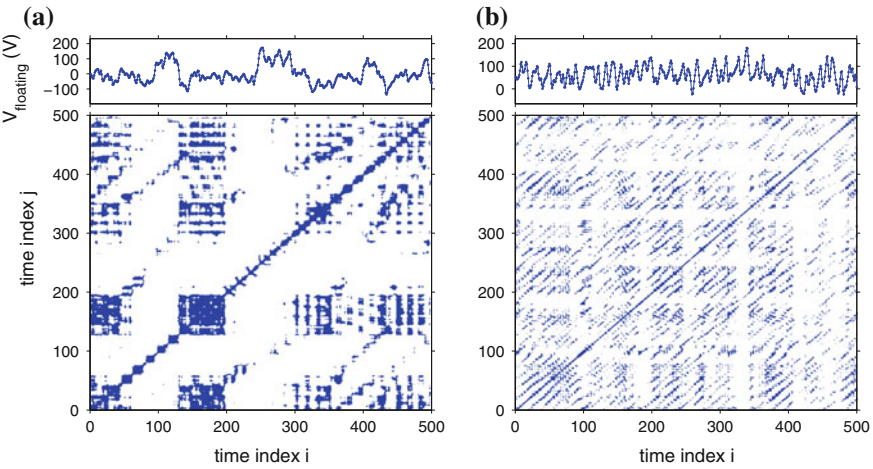


Fig. 18.7 Recurrence plots for the time series of the floating potential for radial positions 19 cm without bias (**a**) and with bias (**b**)

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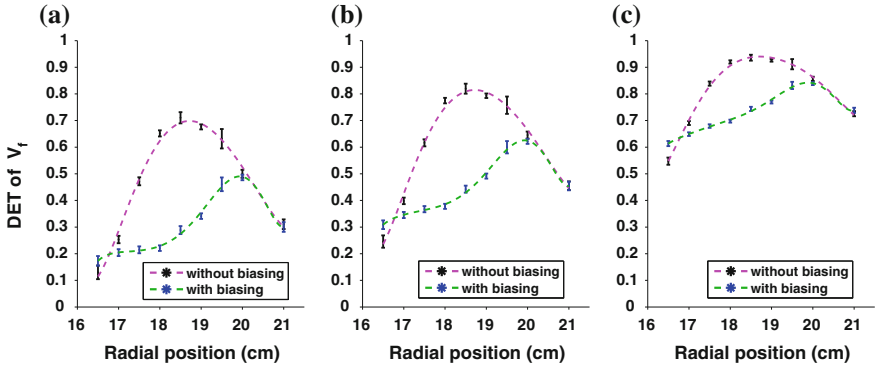


Fig. 18.8 Radial profile of determinism for fluctuations with (*red*) and without (*blue*) radial electric field bias. The recurrence rate was fixed on 1.0, 2.5 and 5.0 %. **a** RR = 1 %, **b** RR = 2.5 %, **c** RR = 10 %

profile of determinism shows an increase as we approach the plasma radius with a maximum value just after that location already, as we shown in the previous Section.

With biasing, however, the radial profile of determinism suffers a radical change, since it decreases the most where it is more intense, namely at the vicinity of plasma edge, an effect possibly caused by the breakup of vertical and horizontal large-scale structures in that region. This structure reduction decreases the particle transport, so improving plasma confinement. Moreover, this alteration in the recurrence properties occurs over a wide radial interval, indicating that a broad region is modified due to the biasing. The results are qualitatively similar as recurrence rate has been varied, what suggests that our results are not artifacts of a too small or too large threshold radius. Hence, as a general trend, the determinism decreases after biasing. We speculate that this is due to the destruction of highly recurrent regions within the plasma that enhance particle and heat transport.

Another experimental investigation of the effects of external biasing on turbulence was performed in the Texas Helimak, a plasma toroidal device located at the University of Texas at Austin [30]. It has a vacuum vessel with rectangular cross section of external radius 1.6 m, internal radius 0.6 m and height 2.0 m (Fig. 18.9a). In this machine the combination between the toroidal and a small vertical field creates a helical magnetic field with curvature and shear, with magnetic field lines with long (*circa* 40 m) connection lengths (in such a way that end effects can be neglected). One of the goals of this machine is to investigate plasma edge turbulence, with a diagnostic system of more than 700 Langmuir probes mounted at four sets of bias plates, where a bias electric field can be applied (Fig. 18.9b) [30]. For negative biasing, turbulence control has been investigated and states of greatly reduced turbulence have been achieved [17]. On the other hand, for positive biasing turbulence shows enhanced broadband spectran and non-gaussian probability distribution functions (PDF) with extreme events associated to bursts [17].

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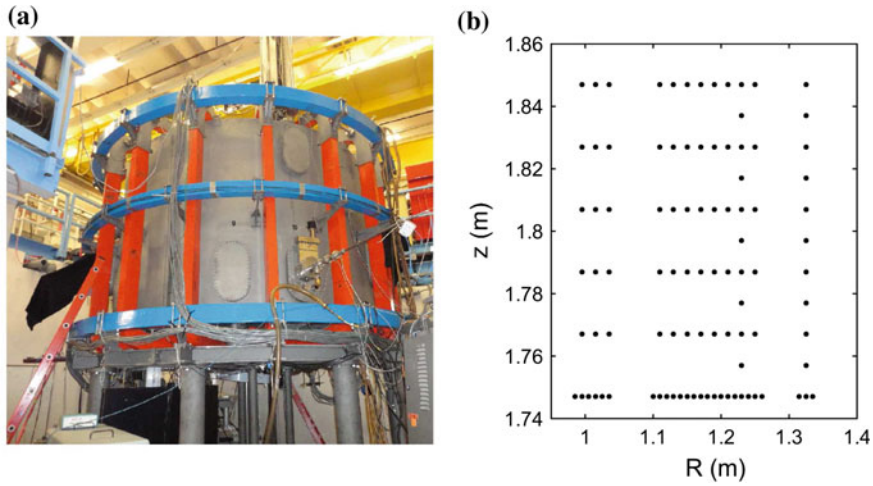


Fig. 18.9 **a** Picture of the Texas Helimak. **b** Langmuir probe distribution on the top plates

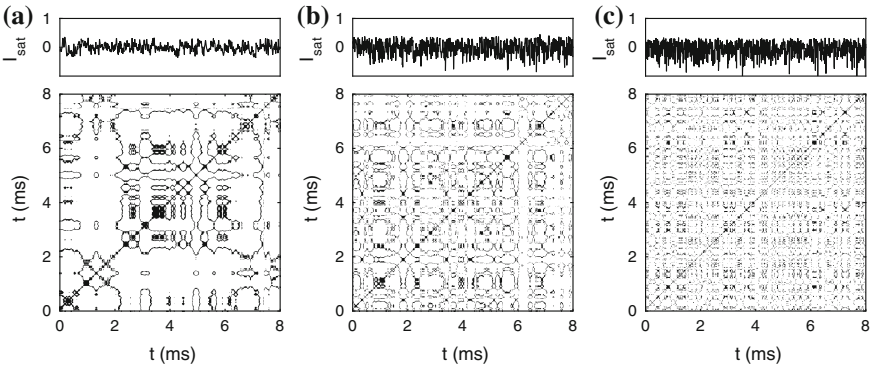


Fig. 18.10 Recurrence plots for plasma turbulence data from Texas Helimak discharges with voltage bias **a** +20 V, **b** zero, and **c** −25 V

In Fig. 18.10, we present the recurrence plots obtained from the measured fluctuation data for short time intervals (of 4 m, corresponding to 2.000 points) with and without bias [31]. We see that for negative bias the size of the recurrence structures in the RPs is bigger than for positive bias. Hence the regularity is lower at positive bias than at negative one, which can be interpreted as an increase of the turbulence. Moreover the determinism of the corresponding time series increases from 0.61 (Fig. 18.10a), for a bias +20–0.85 V (Fig. 18.10c), for a bias −25 V). Hence the value of determinism decreases as we go from negative to positive bias. These results confirm those previously obtained from TCABR discharges in Brazil [19].

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18.5 Conclusions

The dynamical characterization of plasma turbulence is one of the outstanding theoretical and experimental problems in modern fusion plasma physics, specially taking into account the current large-scale projects for fusion tokamaks like ITER. It is thought that at least part of the turbulent phenomena are related to deterministic mechanisms, such as the existence of fluid wave turbulence [32]. On the other hand there must be a variety of other factors that influence turbulent data, mostly from a stochastic nature and hence only amenable to statistical analysis.

We have used recurrence plots as a major tool on analysing turbulent fluctuation data so as to give a precise indication about the dynamical properties of them, in order to understand how long we can advance using theoretical models to characterize plasma edge turbulence in tokamaks. In this article we review past work on this direction using data from the Brazilian tokamak TCABR [4, 19] and the Texas Helimak [31].

Our results show that the recurrence-based diagnostics like determinism, laminarity, entropy and trapping time increase as we approach the plasma edge. This suggests that the concentration of the recurrences is higher at the plasma edge than in its neighbourhood, where the stochastic content is comparatively higher. From the theoretical point of view this means that fluid models like, for example, drift-wave or interchange turbulence, are expected to explain better the features at the plasma edge than elsewhere.

Mier et al. [9] also made a recurrence quantification analysis of fluctuating potential data (of the TJ-II stellarator), with focus on the radial dependence of the determinism and other recurrence-based diagnostics, in a way similar to our analysis. Their results indicate that the deterministic content of the fluctuations increase as we approach the plasma edge, confirming our findings. Moreover, they show an interesting increase of the determinism with the magnetic shear, a feature which we did not consider up to now.

We have performed recurrence-based investigations on the application of a bias electric field in the radial direction. We show that biasing is responsible to the concentration of the recurrences in most of the radial positions considered. Indeed, in the Texas Helimak we found that the determinism actually increases as we go from a positive to a negative bias.

In retrospect we see a wealth of potential applications for recurrence-based diagnostics in plasma physics, since the physical setting involved in plasma phenomena are so complex that it is quite often the case in which the experimental data have both deterministic and stochastic contents.

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