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Short-term memories with a stochastic perturbation

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Abstract

We investigate short-term memories in linear and weakly nonlinear coupled map lattices with a periodic external input. We use locally coupled maps to present numerical results about short-term memory formation adding a stochastic perturbation in the maps and in the external input.

1. Introduction

Coupled map lattices are spatially extended dynamical systems, with discrete space and time, while allowing a continuous state variable. They have been extensively used as models for spatio-temporal dynamics, illustrating the interplay between spatial and temporal degrees of freedom [1]. In fact, many phenomena like turbulence, synchronization, collective phenomena, etc. have been investigated in coupled map lattices due to the small computer time required to perform numerical simulations [2].

One of the remarkable collective phenomena displayed by coupled map lattices, due to the self-organization, arising from the competition between nonlinearity and diffusion, is the possibility of short-time memories in lattices with an external input. By short-time memories we mean the ability of the lattice to store an external signal—in our case a periodic kick of constant amplitude applied to all the coupled maps [3]. This information is stored only as long as the input is kept being applied to the system. Once the input is switched off those memories are rapidly lost due to the dissipative nature of the system. This is the reason we are calling them short-time, in opposition to the kind of memories displayed by neural networks [4].

In this paper we consider a coupled map lattice with the purpose of analyzing short-term memory formation in dynamical systems with many degrees of freedom. The use of a coupled map lattice, with a continuous state variable attached to each lattice site, may be advantageous from the point of view of a memory storage, when compared with usual Ising-type models as the Hopfield model [5,6]. As the state variable can assume any real value within a given range, a convenient partitioning of this domain enables us to encode complex information [7,8]. This fact turns to be important in terms of the memory storage capacity of the lattice since we could design, at least in principle, smaller networks with coupled map lattices, while retaining the overall memory capacity [6].

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The possibility of short-term memories was explored to explain the results of a charge density wave experiment in $NbSe_3$, in which the memory encoding manifested itself as a synchronization of the response to a repeated train of driven pulses so that the voltage/current ratio decreases just as each pulse ends. A coupled linear map lattice with periodic inputs was proposed to explain the memory formation [3].

A step forward in this direction was the observation that a coupled map lattice can store multiple short-term memories, such that a certain type of extrinsic noise can stabilize multiple memories, enabling many parameter values to be encode permanently [9]. In a recent paper we presented numerical evidence that weakly nonlinear maps are able to store multiple short-term memories, and we used this fact to encoded symbols in a matrix of pixels, using suitable control laws [7]. We were able to use this technique in lattices of inductively coupled oscillating circuits, in order to encode information which can be used to generate pixel matrices for symbols in the Braille language [8].

In this paper we explore some of the consequences of the coupled map lattice used in Ref. [3] in the modeling of the mentioned CDW experiment. In particular, we analyze the effect of stochastic perturbation in the memory storage. This paper is organized as follows: Section 2 introduces the model and presents the concept of a short-term memory. Section 3 is devoted to numerical analysis of the stochastic perturbation in the map and in the periodic input. The last section contains our conclusions.

2. Short-term memories

We consider a one-dimensional lattice with N sites, each of them characterized by a continuous state variable at time n and site $i = 1, 2, \ldots, N$, denoted by $x_n^{(i)}$. The model to be treated in this work assumes that the coupling of a given site is with its nearest neighbors only. More general couplings are also possible, with somewhat different properties [10]. The coupled map lattice reads

$$x_{n+1}^{(i)} = f(x_n^{(i)}) + \mathrm{int}\{k[f(x_n^{(i-1)}) - 2f(x_n^{(i)}) + f(x_n^{(i+1)})] - (1 + A_n)\}, \tag{1}$$

where k is the coupling strength and int(z) is the largest integer less than or equal to z. A_n represents an external input signal that is periodically applied to lattice sites, and it constitutes the pattern that the network is supposed to memorize. Throughout this work we shall assume the following 2-cycle external input: $A_n = 9(n\ \text{odd}), A_n = 10(n\,\text{even})$.

A coupled map lattice of the form 1, with a linear map $x \mapsto x$ describes the dynamics of an over-damped chain of N particles in a deep periodic potential, with nearest neighbors connected by Hookean springs with elastic constant equal to the unity, and subjected to external force kicks of amplitude $1 + A_n$ [11]. This is related to the dynamics of sliding charge density waves [3]. A drawback of linear maps is their ability to store only one memorized pattern, so we used a weakly nonlinear map of the form $f(x) = x + Rx^2$, where $R \ll 1$.

In numerical simulations of the spatio-temporal dynamics generated by Eq. (1) we used initial conditions $x_0^{(i)}, i = 1, 2, \ldots, N$, randomly chosen over a given interval, and mixed boundary conditions: one end $(x_n^{(0)})$ is kept fixed, whereas the other extremity is free: $x_n^{(N+1)} = x_n^{(N)}$. The external input A_n is applied simultaneously to all lattice sites, and can be echoed due to self-organization generated by the spatio-temporal lattice dynamics, forming the short-time memories. They are characterized by a *curvature variable* $c_n^{(i)}$, defined as

$$c_n^{(i)} = k[f(x_n^{(i-1)}) - 2f(x_n^{(i)}) + f(x_n^{(i+1)})], \tag{2}$$

such that memory storage is given by the clustering of curvature variables with the same value:

$$c_n^{(1)} = c_n^{(2)} = \cdots c_n^{(N)}, \tag{3}$$

where the time n is supposed to be large enough for the transients to die out. The resulting memorized pattern is of a short-term type, because the encoded information (the value of the curvature variable) is stored only while the external input is kept active. After the input ceases being applied, the information stored is rapidly lost.

To illustrate short-time memory formation let us consider Fig. 1, where we used a linear ($R = 0$) map lattice with $N = 10$ coupled maps, $k = 0.1$, and the already mentioned 2-cycle external input $A_n = 9(n\,\text{odd})$, $A_n = 10(n\,\text{even})$. There is one curve for each site, such that in the end all sites achieve the same value for the curvature variable. Why does this memory appear can be understood by considering that, firstly, each input causes an increment of same value to all sites, except for that at the fixed (nailed) end $x_n^{(0)} = 0$, so that this site begins to present an increase in the curvature variable. As time goes on, this increase also occurs for the other sites towards the free end of the lattice until a saturation value is reached for the curvature variable, which corresponds to a single permanent memory.

The transient time in Fig. 1 is the time τ it takes for the curvature variable to achieve its stationary value. Considering periodic sequences of external kicks of constant amplitude, we observed that the transient time depends on the

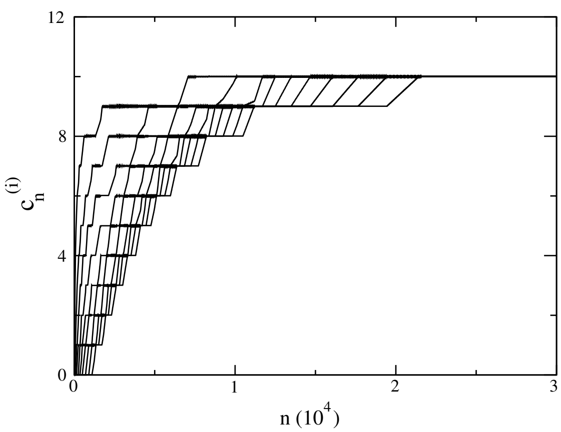

Fig. 1. Time evolution of the curvature variable for a lattice of $N = 10$ linear coupled maps. Each curve corresponds to given site ($i = 1, 2, 3, \ldots, 10$).

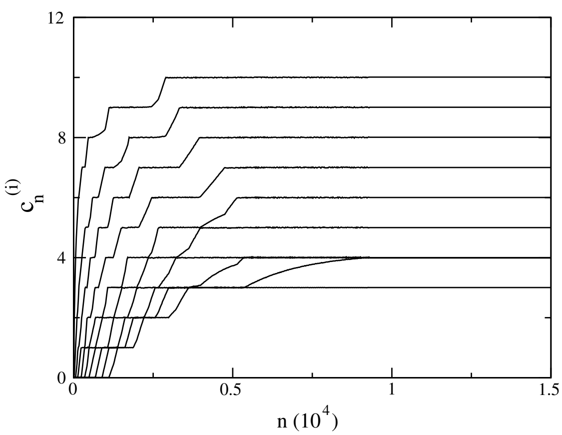

Fig. 2. Time evolution of the curvature variable for a lattice of $N = 10$ weakly nonlinear ($R = 10^{-8}$) coupled maps. Each curve corresponds to given site ($i = 1, 2, 3, \ldots, 10$).

position of the site according to a fourth-order polynomial, with the inverse of the coupling constant k and with the fifth power of the input amplitude A [7].

Chains of coupled weakly nonlinear maps, for which $R \ll 1$, present multiple memories, as depicted in Fig. 2. This fact now enables us to make a suitable partition of the curvature variable interval to encode signals of a graphical matrix [7,8]. We stress that this is a nontrivial phenomenon since the spatio-temporal dynamics, even with a tiny nonlinearity is able to store multiple information from a *single* input. The same kind of effect is produced by extrinsic noise added to the coupled map lattice [9]. In fact, in the experiment involving sliding charge density waves, where there are many sources of noise, there are multiple memories manifested as echoes of the external input represented by electric signals applied to the crystal [3].

3. Stochastic perturbation

We add a stochastic perturbation in the coupled map lattice to analyze the behavior of the short-time memory formation under realistic conditions, since we are always subjected to environmental and/or parametric noise in experiments. Firstly, we insert an additive stochastic perturbation to each map of the lattice in the form

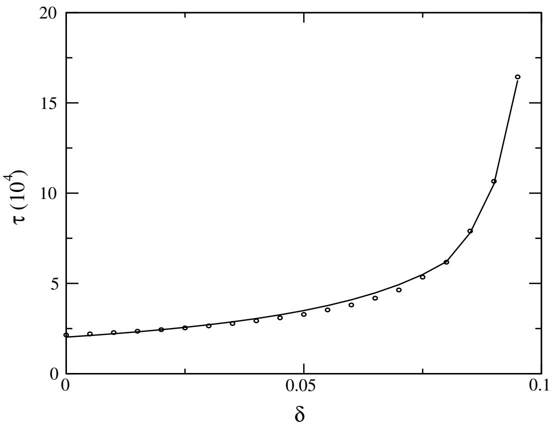

Fig. 3. Transient time for a lattice of $N = 10$ linear coupled maps with $k = 0.01$ and $A_n = 10$.

$$f(x) = x + Rx^2 + \delta r_{n,j}, \tag{4}$$

where $r_{n,j}$ are pseudo-random variables in space j and time n with uniform distribution in the interval [0, 10]; and δ is the level of the stochastic perturbation.

In this case there occurs an increase in the transient time τ it takes for the lattice to achieve permanent memories, and with a nonlinear rate (Fig. 3). The numerical results are fitted by a power-law

$$\tau = \frac{a}{b - c\delta}, \tag{5}$$

where $a = 1.00 \times 10^5$, $b = 4.94$ and $c = 41.56$. In the interval $0 < \delta < 0.95$ there is no noticeable change in the value of the permanent memory, followed by a decrease in both the values of the transient time τ as well as in the value of the curvature variable corresponding to a permanent memory.

We can also consider an additive stochastic perturbation in the periodic external input

$$1 + A_n \rightarrow 1 + A_n + \delta r_n, \tag{6}$$

where r_n are pseudo-random variables in time with uniform distribution in the interval [0, 10], δ being the level of the stochastic perturbation. Fig. 4 displays the temporal evolution of the curvature variable for the site $i = 10$ (in a chain with $N = 10$ maps) *versus* the stochastic perturbation amplitude δ, for a 2-cycle external input $A_1 = 9$ and $A_2 = 10$. The value of the curvature variable for the stationary memory increases with the level of the stochastic perturbation and the transient time shrinks down to zero.

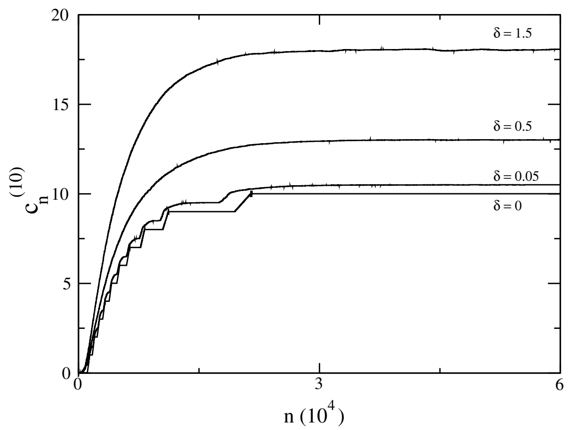

Fig. 4. Stationary value of the curvature variable for the $i = 10$ site of a lattice with $N = 10$ linear coupled maps.

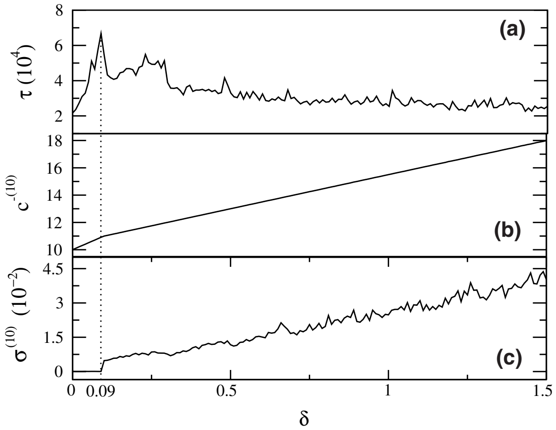

Fig. 5. (a) Transient time, (b) mean curvature variable for the $i = 10$ site, and (c) corresponding standard deviation for a lattice with $N = 10$ coupled linear maps with a 2-cycle external input.

The stochastic perturbation applied to the external input also influences the transient time necessary for achieving a stationary value of the curvature variable. Fig. 5(a) shows that, for $\delta < 0.09$, the transient time increases and, after this value, τ decreases in an oscillating fashion. Fig. 5(b) and (c) shows the temporal mean value of the curvature variable for a given lattice site

$$\overline{c^{(i)}} = \frac{1}{n_2 - n_1} \sum_{n=n_1}^{n_2} c_n^{(i)}, \tag{7}$$

and the corresponding standard deviation,

$$\sigma^{(i)} = \sqrt{\overline{\left(c^{(i)}\right)^2} - \overline{c^{(i)}}^2}, \tag{8}$$

respectively, where n_1 is the transient time for stationary memories and n_2 is the number of further iterations we have used ($\sim 10^6$). We can see that until $\delta \approx 0.09$ the mean curvature variable for the $i = 10$ site increases linearly according to $\langle c^{(10)} \rangle = 10 + 10\delta$ and the standard deviation vanishes. For $\delta > 0.09$ we still observe a linear behavior for the mean curvature variable, but with a slightly different slope: $\langle c^{(10)} \rangle = 10.5 + 5.0\delta$ (Fig. 5(b)). Moreover, the corresponding standard deviation begins to increase with the amplitude of the stochastic perturbation in a quite irregular way (Fig. 5(c)). The mean curve can be fitted by a linear function $\sigma^{(10)} = 0.0005 + 0.0259\delta$.

When we consider weakly nonlinear maps ($R \neq 0$) the coupled map lattice exhibits multiple memories [7]. As we just have done for linear maps, we also introduced an additive stochastic perturbation in the external input and observed a general increase of all the curvature variables. To analyze this effect we calculated the average value of the curvature variable over the entire lattice

$$c_L = \frac{1}{N} \sum_{i=1}^{N} \overline{c_n^{(i)}}, \tag{9}$$

where $n > \tau$ is a time large enough so that the lattice has achieved permanent memories. The corresponding standard deviation is

$$\sigma_L = \sqrt{\overline{\frac{1}{N} \sum_{i=1}^{N} \left(\overline{c_n^{(i)}} - c_L \right)}^2}. \tag{10}$$

The dependence of both quantities with the level of the stochastic perturbation is depicted in Fig. 6, where we considered a lattice with $N = 10$ weakly nonlinear maps ($R = 10^{-8}$), $k = 0.01$, and a two-cycle input. There is no noticeable change in the standard deviation of the curvature variables until $\delta \approx 0.1$. After this value the deviation σ_L, increases linearly. In the interval $0 < \delta < 1.5$ the average curvature variable increases as $c_L = 5.9 + 2.1\delta$, and the corresponding standard deviation as $\sigma_L = 2.2 + 1.5\delta$.

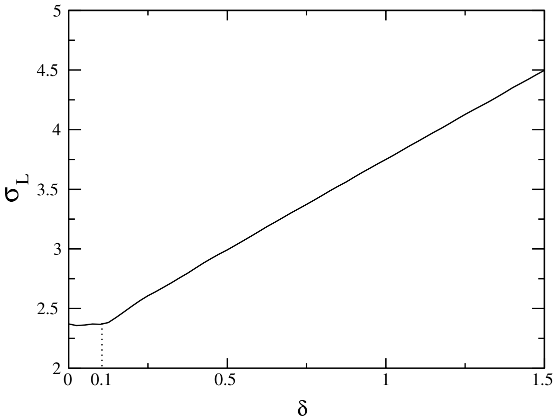

Fig. 6. Standard deviation of the curvature variables for a lattice with $N = 10$ coupled weakly nonlinear maps with a 2-cycle external input.

4. Conclusions

We have investigated the behavior of a coupled maps lattice which has the capacity to encode multiple short-term memories. We inserted an additive stochastic perturbation in each local map and in the external periodic input. The stochastic perturbation in the maps increases the duration of the transient memories and influences the transient time of the permanent memory. When we increase the amplitude of the stochastic perturbation in the external input the transient memories vanish and the average value of the memory increases linearly. The time necessary for the system to obtain the stationary value of these memories increases until a critical value of the stochastic perturbation, decreasing afterwards. There is a limiting value for the amplitude of the stochastic perturbation which causes a linear increase in memory with a vanishing standard deviation. After this value the standard deviation increases in a linear fashion. Moreover the transient time elapsed before stationary memories depends on the amplitude of the stochastic perturbation.

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