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# Chaotic Field Line Diffusion In Tokamaks

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**Abstract.** We have studied numerically the chaotic field line diffusion in a Tokamak with ergodic magnetic limiters. The model field is analytically obtained by solving a Grad-Shafranov equation in toroidal polar coordinates, and the limiter field is obtained by supposing its action as a sequence of delta-function pulses. The mean square radial deviation of a bunch of field lines in a predominantly chaotic region is analyzed. Our results show an initially superdiffusive behavior, followed by a subdiffusive regime, with subsequent loss of field lines due to collisions with the inner wall.

The existence of magnetic surfaces is a necessary requirement for plasma confinement in fusion schemes. These surfaces, with the general topology of nested tori, exist provided the system has some spatial symmetry and, accordingly, they may well be destroyed as this symmetry is broken by some means [1]. The more general problem of particle confinement in a plasma is related in a non-obvious way with the problem of determination of magnetic surfaces [2]. Classical and neoclassical transport in a direction perpendicular to these surfaces, for example, are not sufficient to explain the experimental data [3,4]. This phenomenon of anomalous transport has been one of the most studied themes in fusion plasma theory since the sixties [5]. In this case, a layer of stochastic, or chaotic, magnetic field lines would exist. These are volume-filling in an essentially ergodic fashion, so that particles and energy spread uniformly along this region, causing a dramatic enhancement of some transport coefficients.

Magnetic surface destruction, and the subsequent generation of chaotic field lines, is caused by the overlap of two or more chains of magnetic islands. These may result from two helical resonant magnetostatic perturbation, or from just one helical perturbation coupled with toroidal effects. Another way to destroy magnetic surfaces is to break the axisymmetry of a helical perturbation, as in an ergodic magnetic limiter (EML), consisting of one or more rings of external current conductors designed to generate a region of chaotic field lines over a peripheral region of the plasma column. This is believed to create a cold boundary layer that could uniformize particle and energy fluxes at the plasma edge, in order to reduce plasma-wall interactions

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in Tokamaks [6–8].

If the chaotic region contains no islands, the field line mean square deviation would grow linearly with time. However, as remnants of islands coexist with chaotic field lines, this diffusion regime is unlikely to be observed [9,10]. Furthermore, field line diffusion is limited by the finite size of the chaotic region, since we are ergodizing only the plasma edge, otherwise the plasma itself would be destroyed.

In this work we consider chaotic field line diffusion in a Tokamak with ergodic magnetic limiter. We have used a polar toroidal coordinate system $(r_t, \theta_t, \varphi_t)$ that has been introduced to evidence the toroidal character of the field line geometry in Tokamaks [11]. In the large aspect ratio limit $(r_t, \theta_t, \varphi_t)$ reduce to the well-known local coordinates $(r, \theta, \varphi)$. For arbitrary aspect ratio they may be defined in terms of the toroidal coordinates $(\xi, \omega, \varphi)$ by

$$r_t = \frac{R_0'}{\cosh(\xi) - \cos(\omega)}, \qquad \theta_t = \pi - \omega, \qquad \varphi_t = \varphi, \tag{1}$$

where $R_0'$ is the plasma major radius. The equilibrium magnetic field $\mathbf{B_0}$ was obtaining by solving the Grad-Shafranov equation for the poloidal flux function in these coordinates: $\Psi_p = \Psi_p(r_t, \theta_t)$, from which the contravariant field components are:

$$B_0^1 = -\frac{1}{R_0' r_t}\frac{\partial \Psi_p}{\partial \theta_t}, \qquad B_0^2 = \frac{1}{R_0' r_t}\frac{\partial \Psi_p}{\partial r_t}, \qquad B_0^3 = -\frac{\mu_0 I}{{R_0'}^2}, \tag{2}$$

where $I$ is the poloidal current function. In addition, we have considered the following current density profile

$$J_{30}(r_t) = \frac{I_p R_0'}{\pi a^2}(\gamma + 1)\left(1 - \frac{r_t^2}{a^2}\right)^{\gamma}, \tag{3}$$

in which $I_p$ and $a$ are the total current and plasma radius, respectively, and $\gamma$ is a positive constant. For our purposes it is sufficient to describe the equilibrium using the lowest order poloidal flux $\Psi_{p0} = \Psi_{p0}(r_t)$. Since $r_t$ embodies the toroidal character of the coordinate system, the intersections of the constant $\Psi_{p0}$ curves with a toroidal plane are not concentrical circles, but present a Shafranov shift toward the exterior equatorial region. The corresponding poloidally averaged safety factor $Q(r_t)$ is parabolic with $Q \approx 1$ at the magnetic axis and $Q \approx 5$ at plasma edge.

The EML is composed by $N_a$ current rings located symmetrically along the torus, each of them made of $m_0$ pairs of toroidally oriented segments of length $\ell$ wound around the Tokamak, and conducting a current $I_h$ in opposite directions for adjacent segments. In order to best follow the actual paths traced out by field lines, we have considered a poloidal winding law characterized by a tunable parameter $\lambda > 0$:

$$r_t = b_t, \qquad u_t \equiv m_0(\theta_t - \lambda \sin\theta_t) - n_0\varphi_t = const., \tag{4}$$

where $b_t$ is the minor radius. The magnetic field of the EML, supposed to be a vacuum field, is given by $\mathbf{B}_1 = \nabla \times \mathbf{A}_1$, with

$$A_{13}(r_t, \theta_t, \varphi_t) = -\frac{\mu_0 I_h R'_0}{\pi} \sum_{k=-m_0}^{+m_0} J_k(m_o\lambda)\left(\frac{r_t}{b_t}\right)^{m_0+k} e^{i[(m_0+k)\theta_t - n_0\varphi_t]}. \tag{5}$$

Excluding marginal stability states, where the plasma response would have to be taken into account, the model field will be the superposition of the equilibrium and EML fields: $\mathbf{B} = \mathbf{B}_0 + \mathbf{B}_1$.

The magnetic field line equations:

$$\frac{dr_t}{B^1} = \frac{r_t d\theta_t}{B^2} = \frac{d\varphi_t}{B^3} \tag{6}$$

may be written in a canonical form

$$\frac{d\mathcal{J}}{dt} = -\frac{\partial H}{\partial \vartheta}, \qquad \frac{d\vartheta}{dt} = \frac{\partial H}{\partial \mathcal{J}}, \tag{7}$$

after defining $t = \varphi$, and the following action-angle variables:

$$\mathcal{J} = \frac{1}{4}\left[1 - \left(1 - 4\frac{r_t^2}{{R'_0}^2}\right)^{1/2}\right], \tag{8}$$

$$\vartheta = 2\arctan\left[\frac{1}{\Omega(r_t)}\left(\frac{\sin\theta_t}{1+\cos\theta_t}\right)\right], \tag{9}$$

where

$$\Omega(r_t) = \left(1 - 2\frac{r_t}{R'_0}\right)^{1/2}\left(1 + 2\frac{r_t}{R'_0}\right)^{-1/2}, \tag{10}$$

and the hamiltonian simulates the finite size of the EML by consisdering its action as a sequence of delta-function pulses [12,13]:

$$H(\mathcal{J}, \vartheta, t) = H_0(\mathcal{J}) + H_1(\mathcal{J}, \vartheta, t) \tag{11}$$

$$= \frac{1}{B_T {R'_0}^2}\Psi_{p0}(\mathcal{J}) + \frac{\ell}{R'_0}\frac{1}{B_T {R'_0}^2} A_{31}(\mathcal{J}, \vartheta, t) \sum_{k=-\infty}^{+\infty} \delta\left(t - k\frac{2\pi}{N_a}\right), \tag{12}$$

where $B_T \approx -\mu_0 I / R'_0$ is the toroidal field at the magnetic axis.

The impulsive nature of the EML perturbation enables us to derive analytically a stroboscopic map for the field line dynamics, defining $\mathcal{J}_n$ and $\varphi_n$ as action-angle variables just after the n-th crossing of a field line with the planes $t = k(2\pi/N_a)$, with $k$ a positive integer. Due to the explicit "time"-dependence of the hamiltonian it is a near-integrable symplectic map in the form [14]:

$$\mathcal{J}_{n+1} = \mathcal{J}_n + \epsilon f(\mathcal{J}_{n+1}, \vartheta_n, t_n), \tag{13}$$

$$\vartheta_{n+1} = \vartheta_n + \frac{2\pi}{N_a Q(\mathcal{J}_{n+1})} + \epsilon g(\mathcal{J}_{n+1}, \vartheta_n, t_n), \tag{14}$$

$$t_{n+1} = t_n + \frac{2\pi}{N_a}, \tag{15}$$

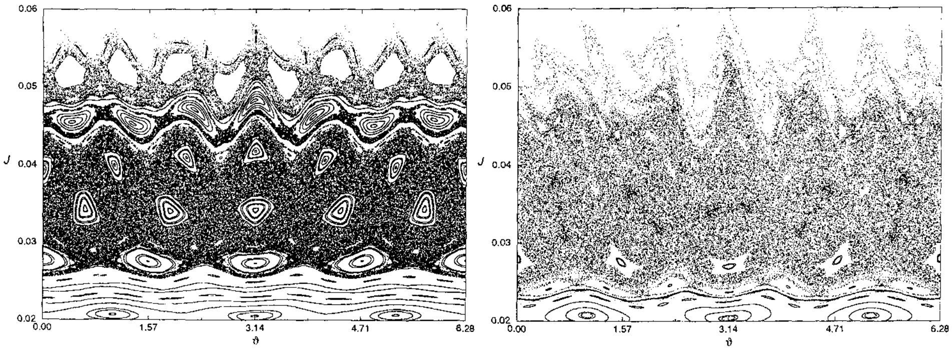


**FIGURE 1.** Stroboscopic field line map for $N_a = 4$ limiters with $I_h = 0.021 I_p$ (a), and $I_h = 0.045 I_p$ (b), with $(m_0, n_0) = (4, 1)$, and $\lambda = 0.4827$.

where $f = -\partial H_1/\partial\vartheta$, $g = \partial H_1/\partial\mathcal{J}$, and $\epsilon = 2(\ell/R_0')(I_h/I_p)$ is typically a small parameter.

In Fig. 1a, we show a phase portrait of a large number of iteration of the above map for a EML with $N_a = 4$ current rings, $(m_0, n_0) = (4, 1)$, $\lambda = 0.4827$, and a current $(I_h/I_p) = 0.021$. There is a main chain of four magnetic islands at $\mathcal{J} \approx 0.027$ surrounded by many satellite chains caused by toroidicity effects. The overlap of these chains may generate a sizeable chaotic field line region, as shown in Fig. 1b, where the EML current was raised to $(I_h/I_p) = 0.045$. To study field line diffusion within such a predominantly chaotic region, we have taken $N_\vartheta$ initial conditions $(\mathcal{J}_{0i} = \bar{\mathcal{J}}, \vartheta_{0i} = 2\pi i/N_\vartheta)$, with $i = 1, 2, \ldots N_\vartheta$, and $\bar{\mathcal{J}}$ is picked up from the centre of the region. We iterate the field line map and compute for each "time" the mean square action deviation

$$< (\delta\mathcal{J}_n)^2 > = \frac{1}{N_\vartheta} \sum_{i=1}^{N_\vartheta} (\mathcal{J}_{ni} - \mathcal{J}_{0i})^2 \tag{16}$$

The time behavior of this quantity is depicted in Fig. 2 for two different values of the EML current. We see that for $I_h = 0.021 I_p$ the process is initially superdiffusive since $< (\delta\mathcal{J}_n)^2 >$ grows with time as $n^\alpha$, with $\alpha > 1$. After only a dozen iterations, however, field line transport becomes subdiffusive, since the scaling exponent now is less than one. The same basic features are present for a higher current $I_h = 0.045 I_p$. The fact that the process is not diffusive (what would correspond to $\alpha = 1$) indicates that the chaotic region contains islands which have a trapping effect on field lines. A chaotic field line that approaches the remnant of an island would stay around it before entering in the neighborhood of another island, and so on. The existence of this subdiffusive regime is also due to the finite size of the chaotic region, which imposes boundaries limiting the excursion of the action variable. Due to collisions between diffusing field lines and the inner wall of the Tokamak, it also turns out that many field lines are lost, making $< (\delta \mathcal{J}_n)^2 >$ to decrease for large times. In

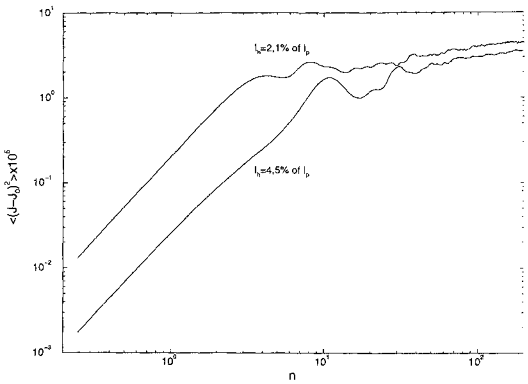


**FIGURE 2.** Time evolution of the mean square action deviation for two different EML currents, and a bunch of initial conditions depicted in Fig. 3a, for the same parameters of Figs. 1a and 1b. n is the number of toroidal turns

order to visualize the effect of the chaotic diffusion on the field line dynamics, we have shown in Fig. 3 the forward images, after 5 and 15 iterations, respectively, of a bunch of initial conditions starting from $\bar{\mathcal{J}} \approx 0.031$. We may observe the stretching and folding nature of the bounded dynamics with positive Lyapunov exponent in the chaotic region. The foldings are modulated by the presence of the island remnants. This highly convoluted set, formed by the forward images of the initial conditions chosen, would interact with the inner wall at $r_t = b_t$ after a large number of iterations, which will cause a considerable loss of field lines. The original EML proposal assumes that these field line collisions with the Tokamak wall would be uniformly spread out due to the ergodic nature of chaotic dynamics.

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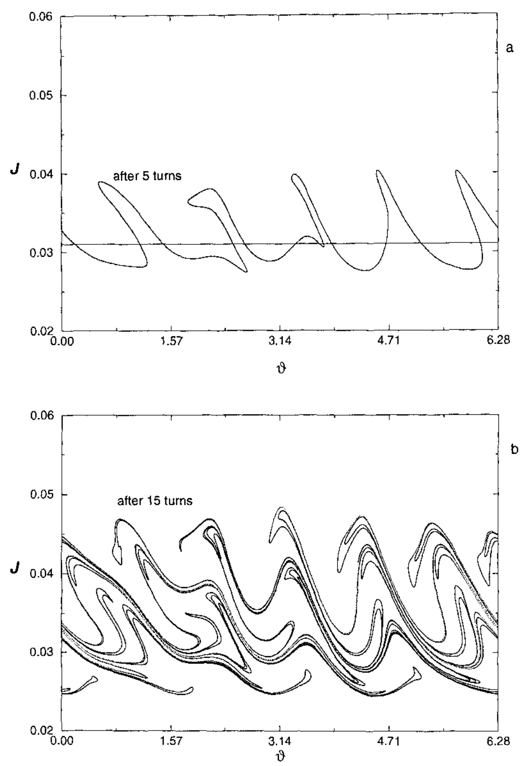


**FIGURE 3.** Forward images, after 5 (a) and 15 (b) iterations, of a bunch of initial conditions within the chaotic region, for the same parameters of Fig. 1b

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