Multiple short-term memories in coupled weakly nonlinear map lattices

Antônio M. Batista,[1,2] Ricardo L. Viana,[2] and Sérgio R. Lopes[2]
[1]Departamento de Matemática, Universidade Estadual de Ponta Grossa, 84010-330, Ponta Grossa, PR, Brazil
[2]Departamento de Física, Universidade Federal do Paraná, 81531-990, Curitiba, PR, Brazil

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We investigate short-time memory storage in coupled map lattices with a periodic external input. In the case of linear coupled maps, the transient length necessary to achieve permanent memory is studied. We present numerical evidence that coupled weakly nonlinear maps are able to store multiple short-time memories, and use this fact to encode symbols in a matrix of pixels, using suitable control laws.

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An intensively studied model for neural networks is the Hopfield model [1], which assumes a network of neurons in the form of an Ising spin system, with a symmetric connectivity matrix describing synaptic activity. A common feature of this kind of model is that the state variable is discrete and binary, assuming only two possible states: active or idle [2]. Since binary neurons are bound to carry only a limited amount of information, the memory capacity of the network is proportional to the number of neurons, or network size. If more information could be stored in each network unit we could design smaller networks, at least in principle, while retaining the overall memory capacity. The use of continuous state variables as network units is thus very appealing, since it allows the storage of any real number. Coupled map lattices could provide the conditions for their use as networks, in which the units are discrete time maps with a continuous state variable [3,4]. A pioneering study of coupled map lattices as neural network models was done by Nozawa [5], who considered a discretized version of a Hopfield model.

This possibility was recently explored to explain results of a charge density wave (CDW) experiment in $NbSe_3$ [6], in which the memory encoding manifested itself as a synchronization of the responses to a periodic train of driving electric pulses in a crystal. A coupled map lattice with external periodic input was proposed to explain the existence of short-term memory formation. ''Short term'' means that the lattice memorizes a sequence of inputs, provided they continue to be applied to the system. After the external input ceases, the lattice loses almost all information.

In this paper we explore some of the consequences of the coupled lattice map used in Ref. [6] in the modeling of the above mentioned CDW experiment. In particular, we analyze the influence of some coupled map parameters on the duration of the transient necessary to achieve memory storage. We also design a coupled lattice map for storage patterns that can be used to encode a given piece of information as a symbol in a pixel matrix. The rule for storing a given sequence of pixels is translated into an analytical formula for some control parameter, such as the input amplitude or the coupling strength.

Let x_n be the continuous dynamical neuron state at discrete time $n=0,1,2,\ldots$. A unidimensional lattice is formed with these maps, where the variable related to the ith site $(i=1,2,3,\ldots,N)$ at time n is represented by $x_n^{(i)}$. Each unit has an evolution described by a map $x\mapsto f(x)$, and the coupling is with the nearest neighbors only [3]. Note that a modeling based on the usual neural network architecture would need a global rather than a local coupling [5]. The model to be treated in this work is

$$x_{n+1}^{(i)}=f(x_n^{(i)})+\mathrm{int}\{k[f(x_n^{(i-1)})-2f(x_n^{(i)})+f(x_n^{(i+1)})]-(1+A_n)\}, \tag{1}$$

where k is the coupling strength and $\mathrm{int}\{z\}$ is the largest integer less than or equal to z. A_n represents an external input signal that is periodically applied to lattice sites, and it constitutes the pattern that the network is supposed to memorize. It could be, for example, an input cycle of period 2: $A_1=9, A_2=10, A_3=9, A_4=10, \ldots$.

This coupled lattice map model describes the dynamics of an overdamped chain of N particles in a deep periodic potential, with nearest neighbors connected by springs with elastic constant k, and subjected to external force kicks of amplitude $1+A_n$. This is related to the dynamics of sliding charge-density waves [7]. In Ref. [6] a simple linear map $f(x)=x$ was used, but here we will also analyze the influence of a small nonlinearity, in the form $f(x)=x+rx^2$, where $r\ll 1$.

Starting from an initial configuration for the network units $x_0^{(i)}$, with $i=1,2,\ldots,N$, the system evolves in time by a sequence of patterns that may or may not settle down into a stable configuration, or attractor. The memorized pattern following the ''learning input'' A_n may be recovered from the lattice pattern by using a ''curvature'' variable $c_n^{(i)}$, defined (for site i and time n) as

$$c_n^{(i)}=k[f(x_n^{(i-1)})-2f(x_n^{(i)})+f(x_n^{(i+1)})], \tag{2}$$

such that memory storage is characterized by clustering of curvature variables with the same value. In histograms of the fractional part of $c_n^{(i)}$, these memories show up as sharp frequency peaks [6]. However, these memories are somewhat different from those displayed by a Hopfield-type neural network. In the latter case, a given configuration is learned and stored for long times without further inputs, because the configuration minimizes some energy functional. Here we deal with memories that persist only if the external input is being continuously applied.

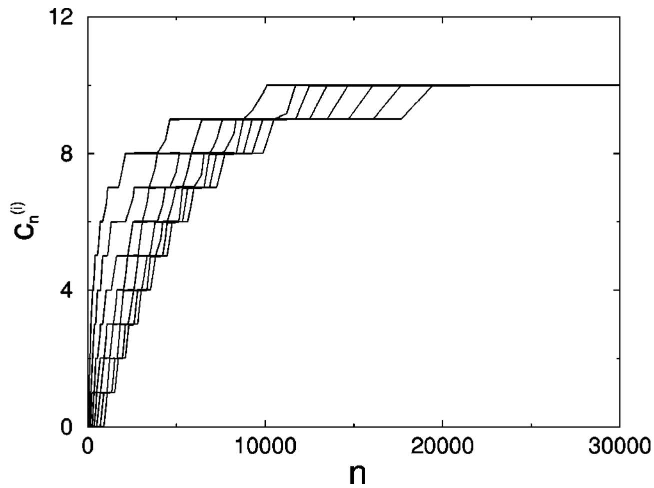

FIG. 1. Temporal evolution of the curvature variable for sites of a coupled identity map lattice with $N=10$, $k=0.01$, and a two-cycle input $A_1=9$, $A_2=10$. We have used a random initial condition $x_0^{(i)}$ and mixed boundary conditions $x_n^{(1)}=0$, $x_n^{(N-1)}=x_n^{(N)}$. Each curve corresponds to a given site ($i=2,3,4,\ldots,9$).

As an example (Fig. 1), we have computed the curvature variables for $N=10$ linear maps ($r=0$) in a lattice with mixed boundary conditions (one end nailed, one end free) $x_n^{(1)}=0$, $x_n^{(N-1)}=x_n^{(N)}$ and the two-cycle input $A_1=9$, $A_2=10$, etc. It turns out that the input $A_n=9$ has a transient memory, whereas the other ($A_n=10$) is a fixed point and persists as long as the input continues to be applied. Note that each effective site (eight out of the ten) eventually attains the same memory value for the curvature variable.

To understand how these memories are formed, we begin by considering the effect of the first pulse A_1, which is applied to all lattice sites. Thus the increment in the state variable $x_n^{(i)}$ is the same for all sites, except for that located at the fixed end $x_n^{(1)}=0$. This causes an increase of the curvature in the vicinity of the fixed end. It will further increase with time until the coupling between the fixed end and the next site becomes large enough to hold the curvature constant. This happens for the other sites toward the free end of the lattice until the saturation value is reached, which corresponds to the permanent memory.

There is a transient time to achieve this permanent memory. In the example of Fig. 1 it is less than 2×10^4 units of time, and this value depends on various map and lattice parameters. The dependence of transient memory duration on the lattice position i is depicted in Fig. 2 for a larger lattice ($N=50$) with single input $A=10$. The numerical curve seems to be well fitted by a fourth-order polynomial whose coefficients depend on the lattice size. Sites closer to the fixed end attain permanent memory values faster than distant ones, because of the mechanism just mentioned. The transient time also depends on the coupling strength k. Actually, they were found to be inversely proportional to each other, since the lesser the coupling, the weaker its diffusive effect over the lattice, and the more time would be necessary to propagate information. So the memory transient would be considerably higher in this case. Finally, it has been found that the transient length increases with the fifth power of the input amplitude A.

The addition of a small nonlinearity to the isolated maps

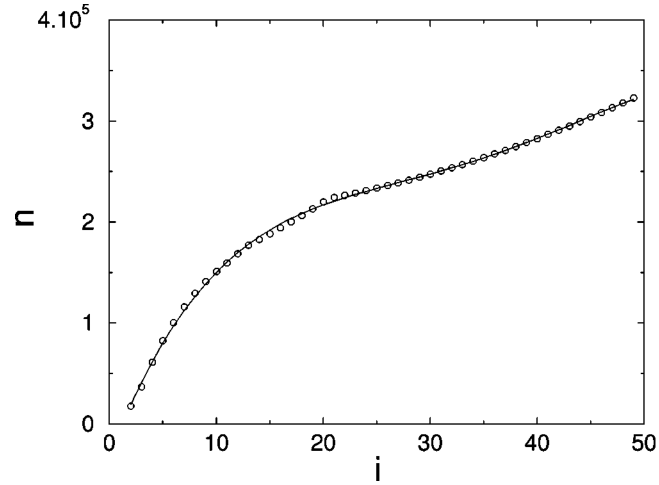

FIG. 2. Duration of the transient for the memorized value $c^{(i)}=10$ as a function of the lattice position, for a coupled identity map lattice with $N=50$, $k=0.01$, $A=10$, and the same initial and boundary conditions as in Fig. 1.

has a significant effect on the capacity for memory storage. For a nonlinearity as tiny as $r=10^{-8}$, it turns out that multiple memories may be stored, instead of only one (as in the linear case) (see Fig. 3). Extensive numerical testing has been done to check that these multiple permanent memories remain as long as we continue to apply the inputs, in contrast with the single memory typically displayed by linear maps. First note that an identity map $f(x)=x$ has unit slope, and thus a continuum of fixed points. With a small nonzero nonlinearity the map has only one fixed point at $x=0$, and the iterations wander very close to the 45°-line, allowing any point to be set out as a memory value influenced by the external kicks. As the nonlinearity term is rx^2, its effect for very small r is only noticeable for high values of x, say, 10^5 or even more. Increasing the nonlinearity causes the curvature variable to vanish, since the maps begin to synchronize in phase, presenting the same value of x for all sites. This effect begins to occur from the free end of the chain.

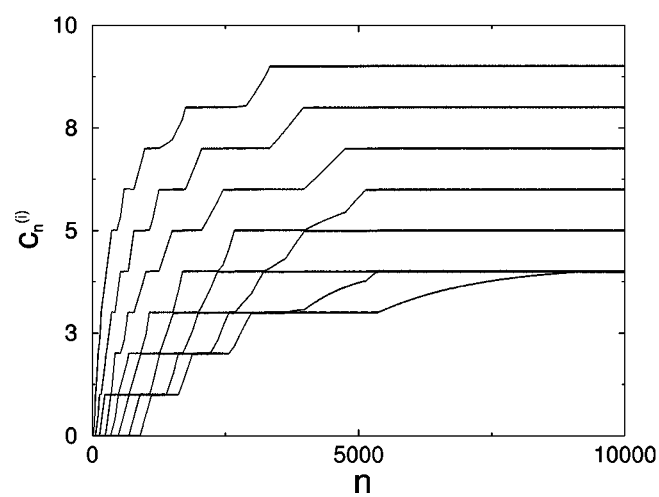

FIG. 3. Temporal evolution of the curvature variable for sites of a coupled weakly nonlinear map lattice with $N=10$, $k=0.01$, $r=10^{-8}$, $A=10$. We have used a random initial condition $x_0^{(i)}$ and mixed boundary conditions $x_n^{(1)}=0$, $x_n^{(N-1)}=x_n^{(N)}$. Each curve corresponds to a given site ($i=2,3,4,\ldots,9$).

[3,4[	[7,8[	[11,12[	[15,16]
[2,3[	[6,7[	[10,11[	[14,15[
[1,2[	[5,6[	[9,10[	[13,14[
[0,1[	[4,5[	[8,9[	[12,13[

FIG. 4. Graphic 4×4 matrix for encoding of symbols, where each pixel is related to the indicated interval of the curvature variable.

Let us consider a specific example to see why multiple short-term memories are found in the weakly nonlinear case. We begin by writing the curvature variable for the quadratic map as

$$c_n^{(i)}=k\{x_n^{(i-1)}-2x_n^{(i)}+x_n^{(i+1)}+r[(x_n^{(i-1)})^2-2(x_n^{(i)})^2+(x_n^{(i+1)})^2]\}. \tag{3}$$

We use a lattice with $N=6$ sites. In the linear case ($r=0$), we have the following stationary values for the state variables: $x^{(2)}=-11\,000$, $x^{(3)}=-15\,000$, $x^{(4)}=-18\,000$, $x^{(5)}=-20\,000$. Substituting these values into Eq. (3) we can obtain the stored memories $c^{(3)}=c^{(4)}=10$. If we add a small nonlinearity ($r=10^{-8}$), different stationary values are obtained (by iterating as much as 4 billion times): $x^{(2)}=-9472$, $x^{(3)}=-12\,709$, $x^{(4)}=-15\,047$, $x^{(5)}=-16\,583$. Using Eq. (3), we have this time not one but two different memories: $c^{(3)}=9.05$ and $c^{(4)}=8.11$.

This feature suggests the use of coupled weakly nonlinear maps to store more complex information. We use a sequence of pixels in a 4×4 tiled display (Fig. 4), where we assign to each of the 16 pixels a given interval for the curvature variable for each site, and superpose all displays (this would require an $N=18$ lattice). Each site is related to such a display. For a lattice, we superpose N displays, obtaining a single pattern. A symbol may be stored through a predetermined sequence of permanent values for the curvature, and a rule to get to this desired pattern. We may use either the input amplitude A or the coupling strength k as control parameters, to design this learning rule.

As an example, consider the pattern shown in Fig. 5. It

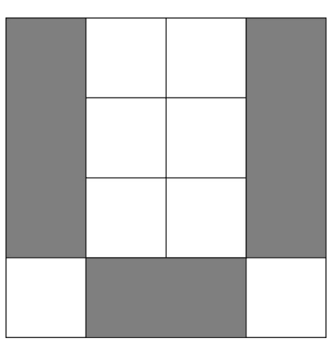

FIG. 5. Encoding of the letter ''V'' using the matrix depicted in Fig. 4, and a coupled weakly nonlinear map lattice with $n=18$, $r=10^{-7}$, $A=10$, and the same initial and boundary conditions as in previous figures.

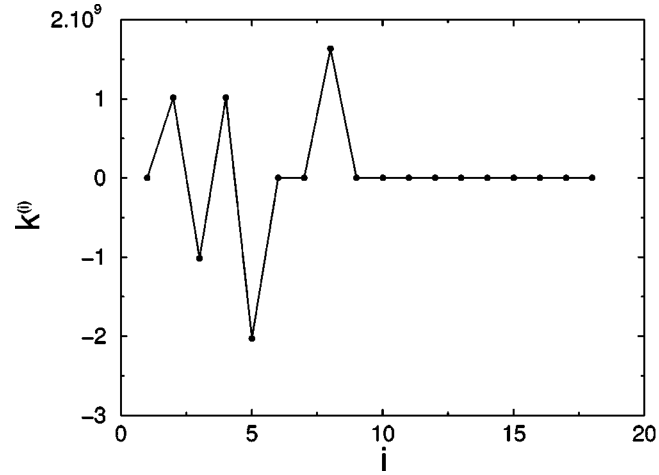

FIG. 6. Lattice profile for the coupling constant k to obtain the target memorized pattern shown in Fig. 5. This pattern is stable for up to 7×10^6 iterations of the coupled map lattice.

may be represented by the following target curvature (note that this choice is not unique, since we have done a coarse-grained partitioning of the interval [0,16]):

$$\mathcal{C}_n^{(i)}=1.5\ \ (1\leqslant i\leqslant6),\quad \mathcal{C}_n^{(i)}=2.5\ \ (7\leqslant i\leqslant9),$$
$$\mathcal{C}_n^{(10)}=\mathcal{C}_n^{(11)}=3.5, \tag{4}$$
$$\mathcal{C}_n^{(12)}=\mathcal{C}_n^{(13)}=4.5,\quad \mathcal{C}_n^{(14)}=8.5,\quad \mathcal{C}_n^{(15)}=13.5,$$
$$\mathcal{C}_n^{(16)}=14.5,\quad \mathcal{C}_n^{(17)}=\mathcal{C}_n^{(18)}=15.5.$$

We use Eq. (3) with the substitution $c_n^{(i)}\rightarrow\mathcal{C}_n^{(i)}$, and allow the coupling strength to be varied for each site and each instant ($k\rightarrow k_n^{(i)}$). The necessary values of $k^{(i)}$ are depicted in Fig. 6, where the stationary values are shown. We remark, however, that this scheme works only for nonzero values of the target curvatures $\mathcal{C}_n^{(i)}$.

As well as varying k, we find that memory storage is also possible by changing the input strengths $A_n\rightarrow A_n^{(i)}$. We have used this control scheme to obtain the same target pattern of Fig. 5, and the necessary input amplitudes are shown in Fig. 7. This turns to be more feasible to implement, from a physi-

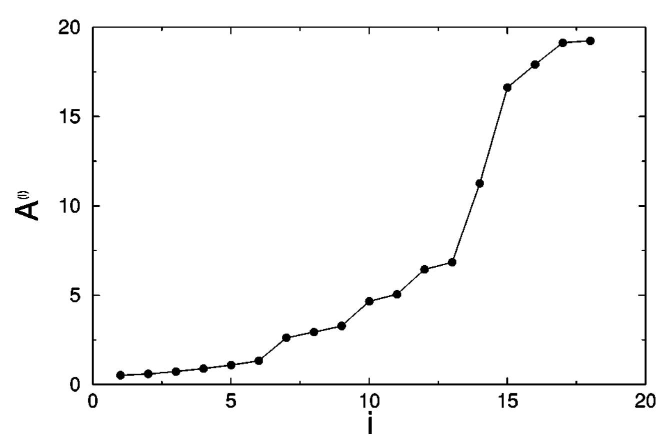

FIG. 7. Lattice profile for the input amplitude A to obtain the target memorized pattern shown in Fig. 5. This pattern is stable for up to 7×10^5 iterations of the coupled map lattice.

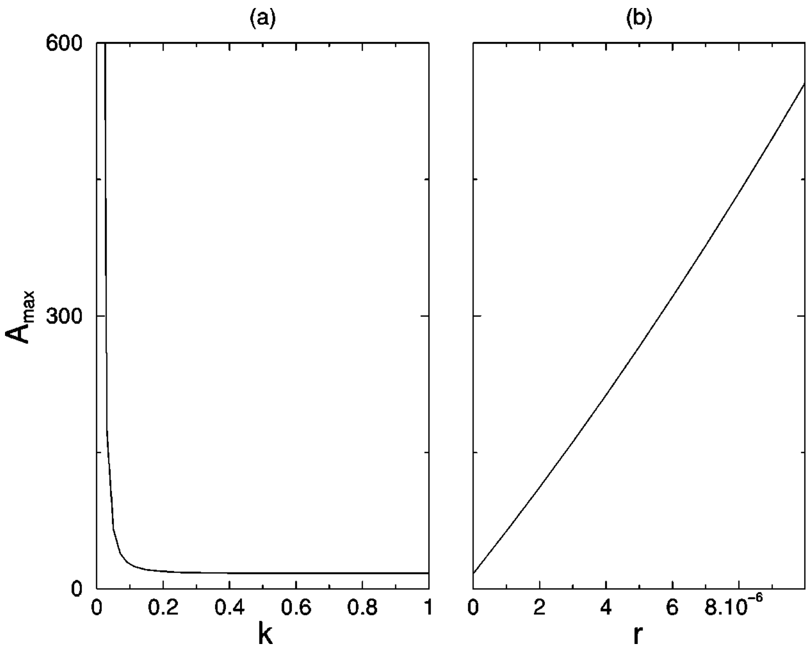

FIG. 8. Largest input amplitude necessary to obtain the target memorized pattern shown in Fig. 5, as a function of (a) coupling strength *k* and (b) nonlinearity parameter *r*.

cal point of view, than adjusting coupling strengths, which are quite difficult to modify. The largest value of the external input A_{max} in order to obtain a given target configuration $\mathcal{C}_n^{(i)}$ depends on both the coupling strength k and the nonlinearity parameter r (Fig. 8). It is found that the input amplitude decreases with k in an inverse-square-law fashion. This scaling comes from the fact that, for weak coupling, a larger input is needed to get the same target than a strongly coupled lattice would require. In intermediate cases, the target pattern is determined by the competition between these two effects: lattice diffusion provided by coupling and external perturbations represented by the kicks applied to the lattice sites.

Summarizing our results, we have explored the fact that a coupled linear map lattice can store an external input signal as a short-term memory. We have found that the memory formation has a transient duration that increases with the lattice size and the input amplitude, but it is inversely proportional to the coupling strength. Thus, for optimal storage one would need small lattices (but not so small as to prevent information encoding when partitioning), large coupling strength, and low signal amplitudes.

The most significant result of this note is the possibility of storage of multiple short-term memories by using a weakly nonlinear map in each lattice site. As a matter of fact, this possibility has already been anticipated in Ref. [6], where the effect of external noise has also been conjectured. This result allows us to use coupled map lattices to store virtually any kind of information. We illustrate this with a matrix display of symbols. Thanks to the form of the lattice coupling, we can choose a given target pattern to memorize and vary the coupling constant or the external input amplitude so as to get the desired result. This works as a kind of control scheme in space and time. Further work is being conducted in order to improve the application of the present technique to the systematic storage of a sequence of symbols, with the purpose of testing the performance of a coupled map lattice as a neural network.

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