Física de Sistemas Complexos: de Turbulência à Econofísica (e Mais)

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V ESCOLA DE VERÃO DE FÍSICA DE CURITIBA

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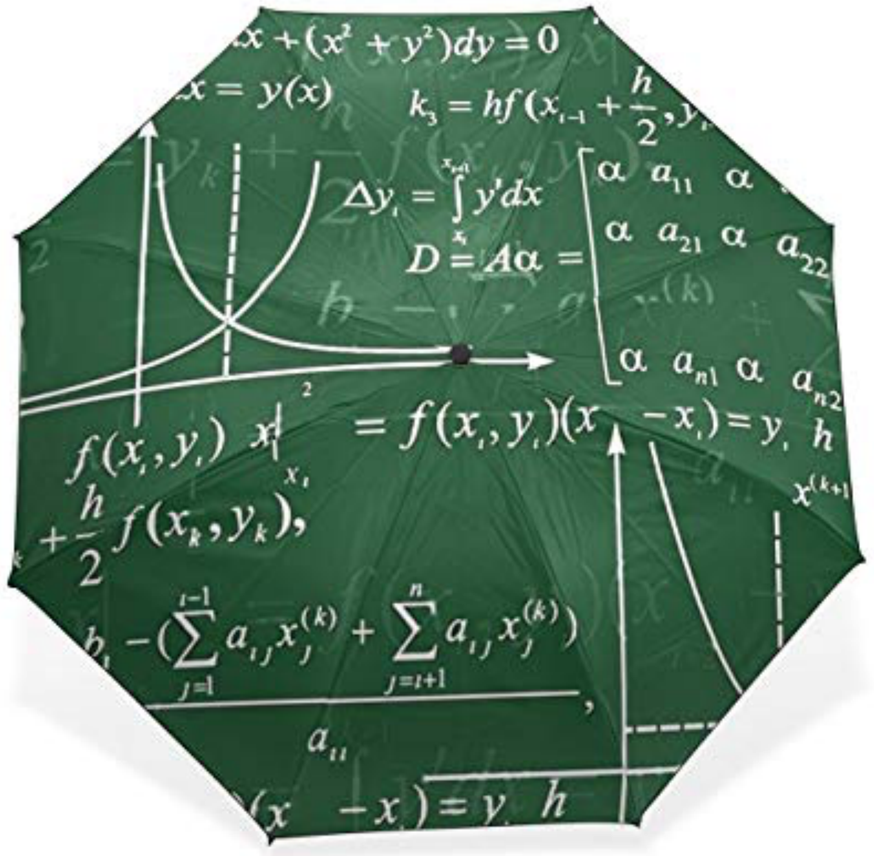
Grupo de pesquisa

Fenômenos de Flutuação, Dinâmica de Fluidos e Sistemas Complexos

Endereço para acessar este espelho: dgp.cnpq.br/dgp/espelhogrupo/3923346117734401



Overview: Ubiquity of Fluctuating Phenomena

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Emergence of skewed non-Gaussian distributions of velocity increments in isotropic turbulence

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PHYSICAL REVIEW LETTERS **128,** 236803 (2022)

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Turbulence Hierarchy and Multifractality in the Integer Quantum Hall Transition

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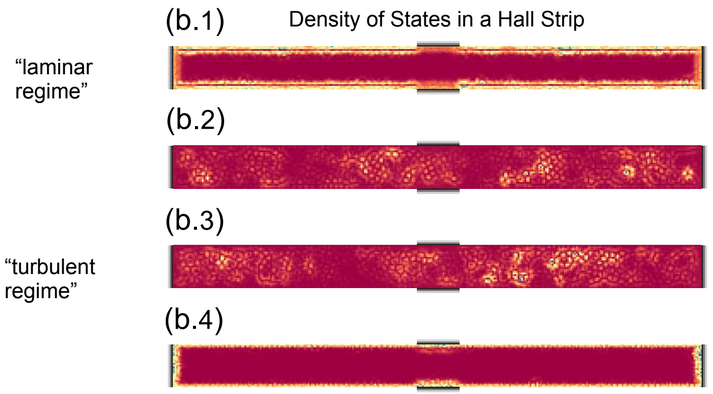

nature COMMUNICATIONS

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ARTICLE

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Turbulence hierarchy in a random fibre laser

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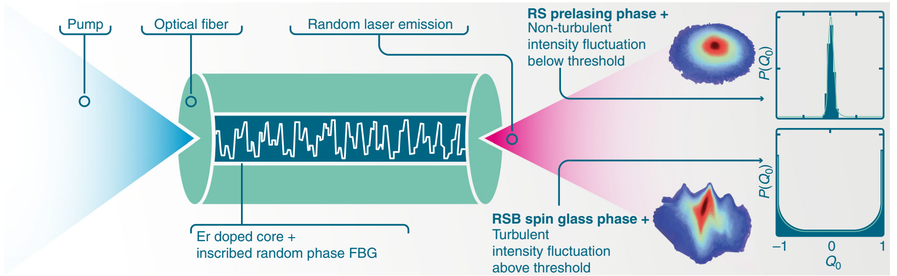

Physical Review E, accepted ("almost"), February 2024

Turbulence hierarchy in foreign exchange markets

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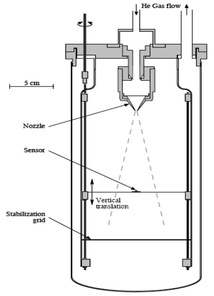
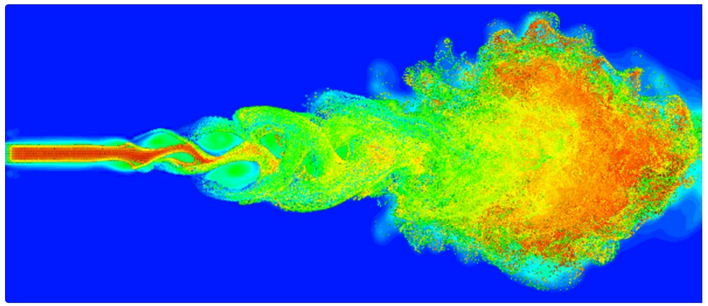
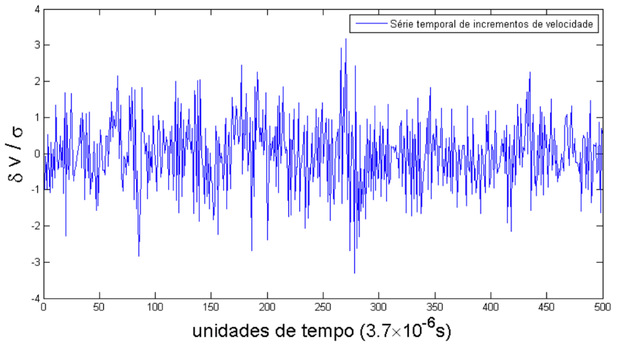
What Fluctuates?

Velocity fluctuations

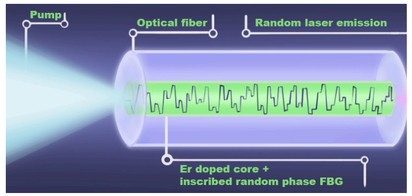
Classical



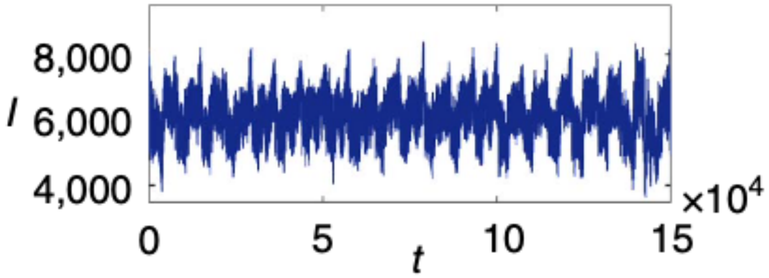




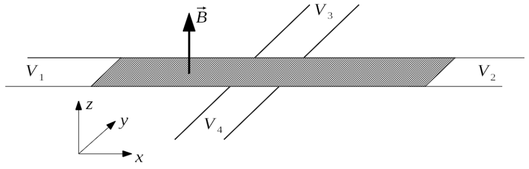
Quantum

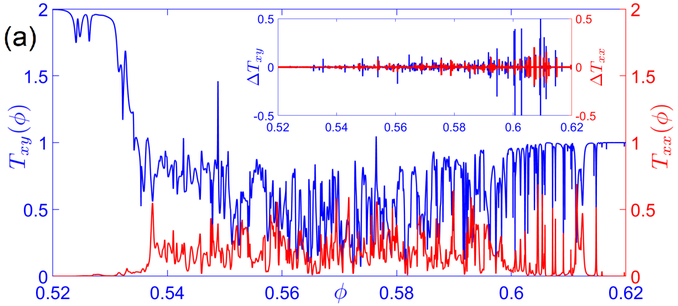


Laser intensity fluctuations



Hall conductance fluctuations

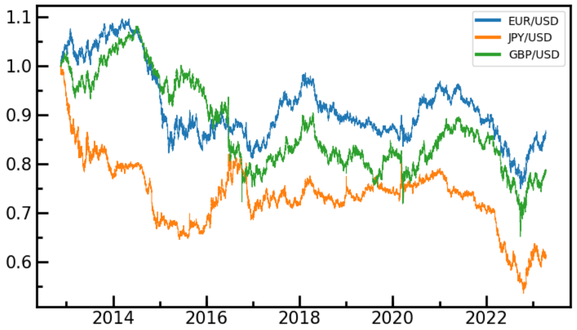




'Immaterial'

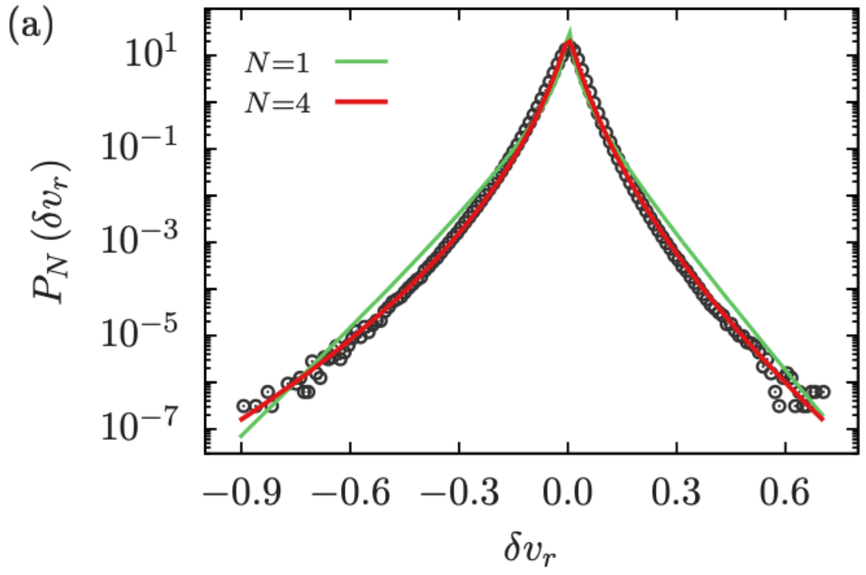
Price fluctuations



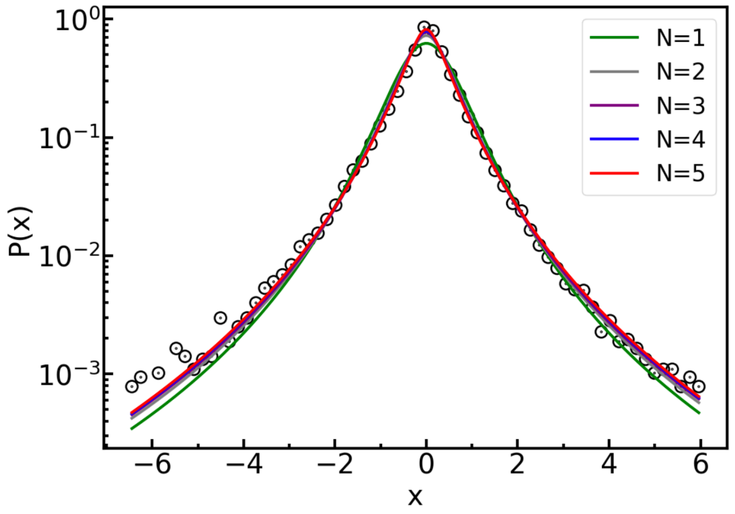


Nature of the Fluctuations: Non-Gaussian (Heavy Tailed) Distributions

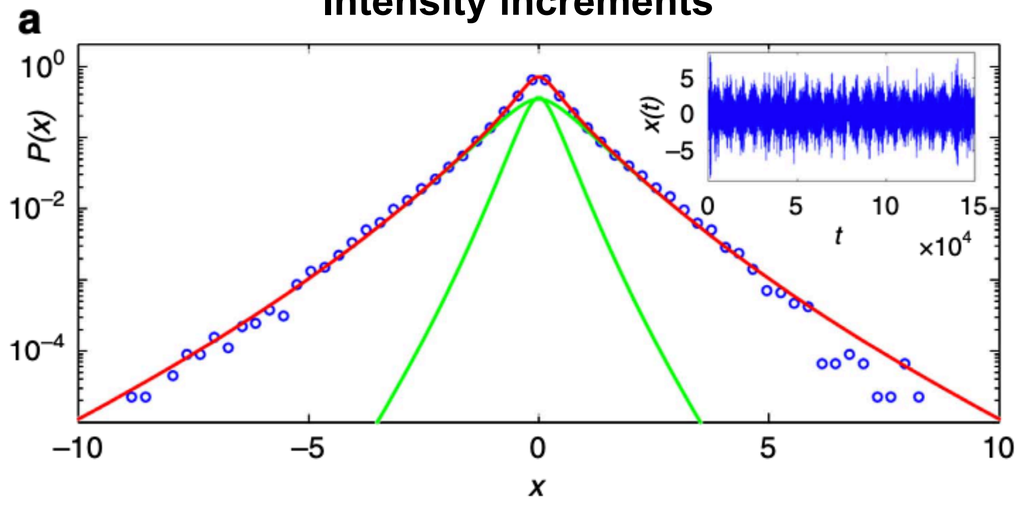
Velocity increments



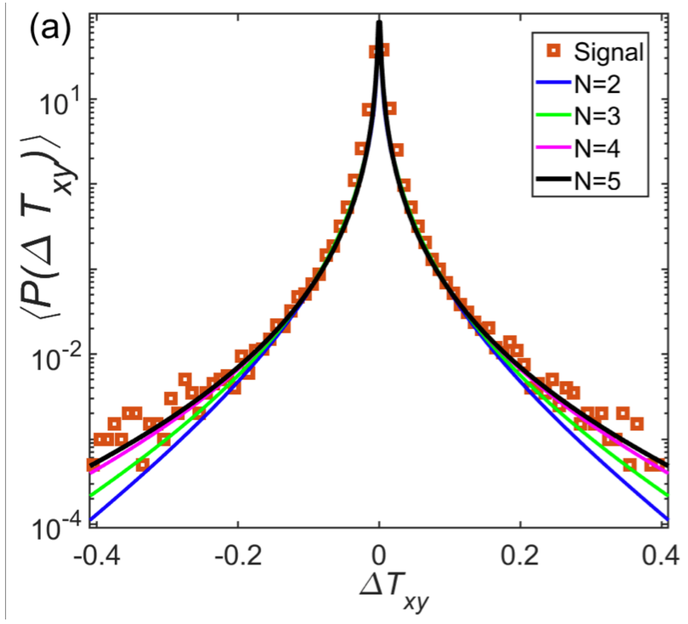
Price increments



Intensity increments

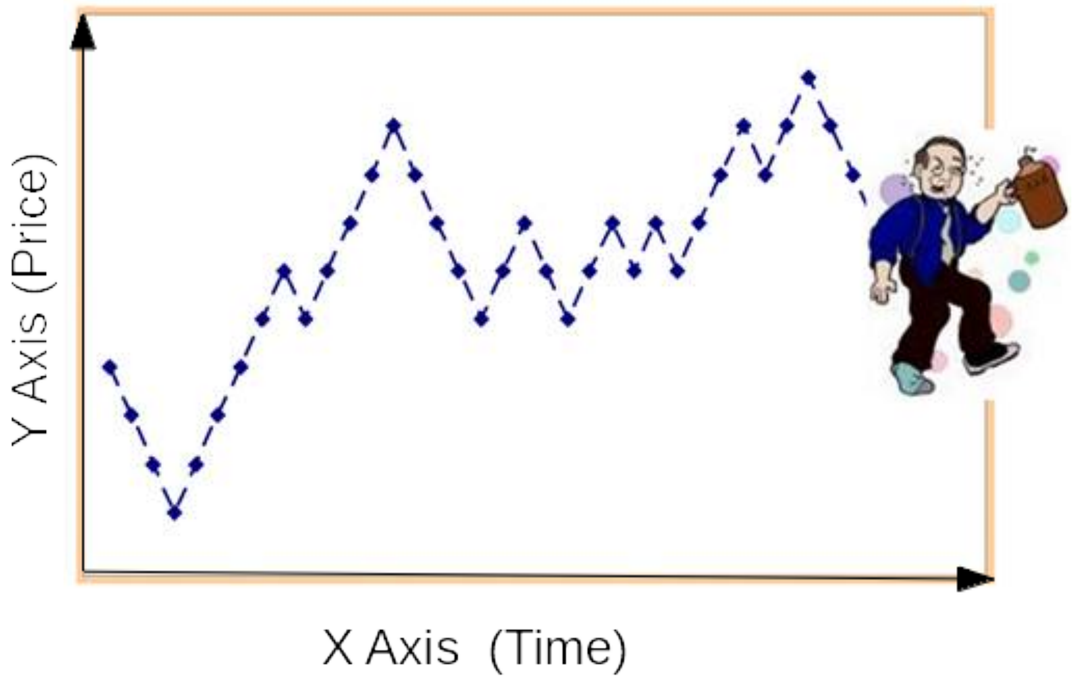


Conductance increments



Caminhada Aleatória

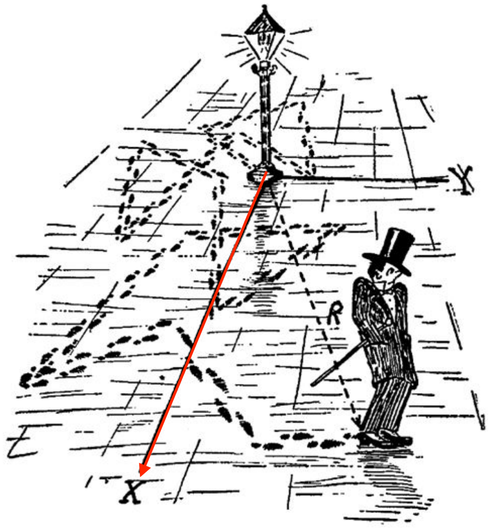
- **A caminhada do bêbado:** a cada intervalo de tempo τ, o bêbado "joga uma moeda" e dá um passo de tamanho l para a frente (se "cara") ou para trás (se "coroa") com igual chance.



- **Incerteza e probabilidade:** como há um elemento aleatório na dinâmica, não sabemos mais ao certo onde o bêbado vai estar depois de um certo tempo. O melhor que podemos fazer é falar sobre a **probabilidade** de ele estar em uma dada posição em um dado tempo.

Random Walk: Non-Stationary Distributions

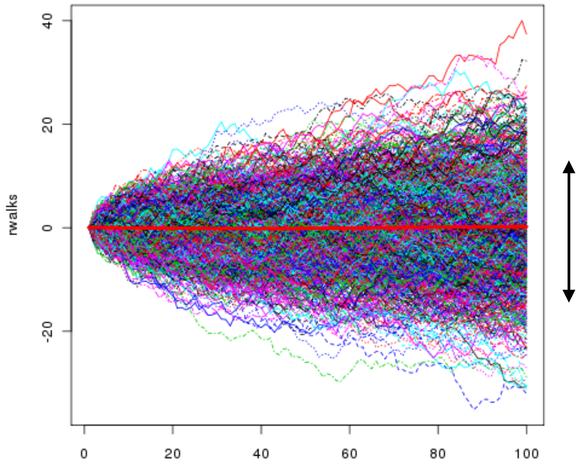
- **Brownian motion:** in the limit of infinitesimal time and length steps the **trajectory of a random walk is continuous** and the probability distribution is a Gaussian whose variance increases with time.



Drunkard's walk

Courtesy of George Gamow

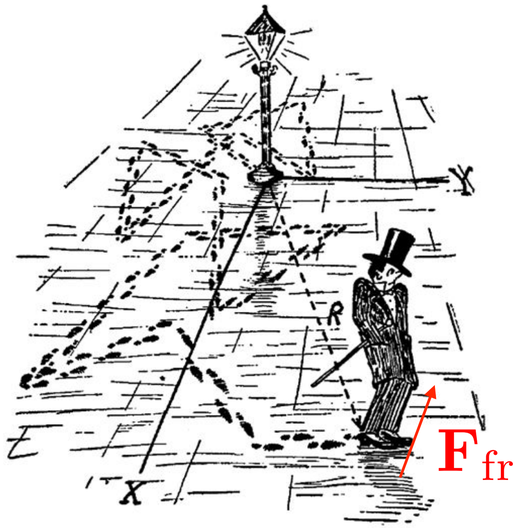
$$p(x,t) = \frac{1}{\sqrt{2\pi\sigma^2 t}} \exp\left(-\frac{x^2}{2\sigma^2 t}\right)$$



$$R \equiv \sqrt{\langle x^2 \rangle} = \sigma\sqrt{t}$$

Damped Random Walk: Stationary Distribution

- **Damped Brownian motion** reaches a stationary a Gaussian distribution.



Drunkard's walk

Courtesy of George Gamow

- **Langevin equation**: stochastic differential equation given by

$$dx(t) = -\mu x dt + \sqrt{2\mu\varepsilon}dW(t)$$

ε: noise amplitude

- For $t \to \infty$, the stationary distribution is

$$p(x) = \frac{1}{\sqrt{2\pi\varepsilon}} \exp\left(-\frac{x^2}{2\varepsilon}\right)$$

Quantum Mechanics from Brownian Motion

PHYSICAL REVIEW VOLUME 150, NUMBER 4 28 OCTOBER 1966

Derivation of the Schrödinger Equation from Newtonian Mechanics*

EDWARD NELSON
Department of Mathematics, Princeton University, Princeton, New Jersey
(Received 21 April 1966; revised manuscript received 21 June 1966)

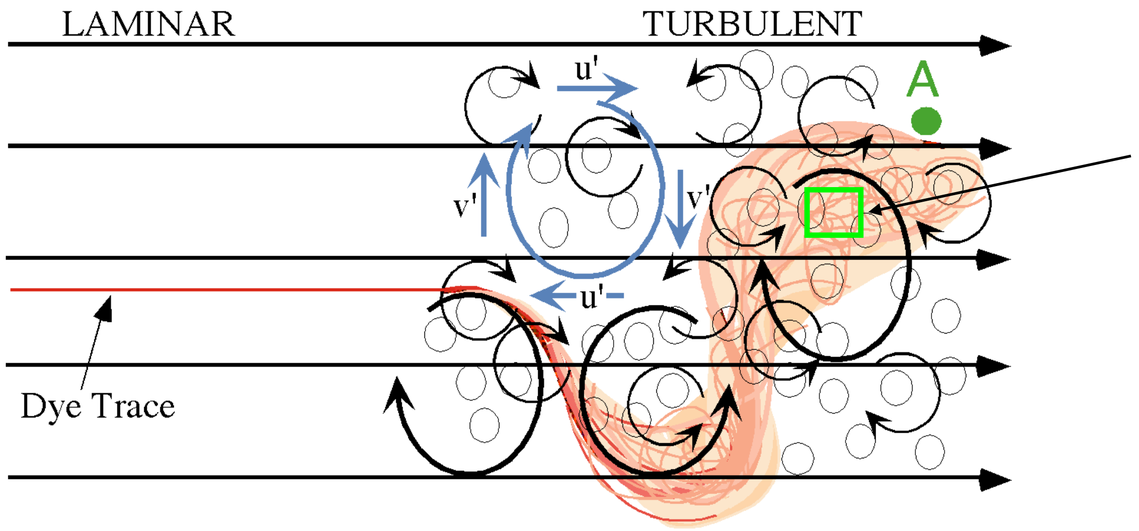
We examine the hypothesis that every particle of mass m is subject to a Brownian motion with diffusion coefficient $\hbar/2m$ and no friction. The influence of an external field is expressed by means of Newton's law $\mathbf{F}=m\mathbf{a}$, as in the Ornstein-Uhlenbeck theory of macroscopic Brownian motion with friction. The hypothesis leads in a natural way to the Schrödinger equation, but the physical interpretation is entirely classical. Particles have continuous trajectories and the wave function is not a complete description of the state. Despite this opposition to quantum mechanics, an examination of the measurement process suggests that, within a limited framework, the two theories are equivalent.

I. INTRODUCTION

WE shall attempt to show in this paper that the radical departure from classical physics produced by the introduction of quantum mechanics forty years ago was unnecessary. An entirely classical derivation and interpretation of the Schrödinger equation will be given, following a line of thought which is a natural development of reasoning used in statistical mechanics and in the theory of Brownian motion.

Origin of Non-Gaussian Distributions I : Local Equilibrium

- **Fluctuating Environment and Separation of time Scales:** the particle velocity fluctuates much more rapidly than its environment (‘energy reservoir’).



Short time scales: local equilibrium!

- **Local equilibrium:** during short times the systems relaxes to a quasi equilibrium.

- Hence, the **velocity distribution** (conditioned on a fixed background) is a **Gaussian**:

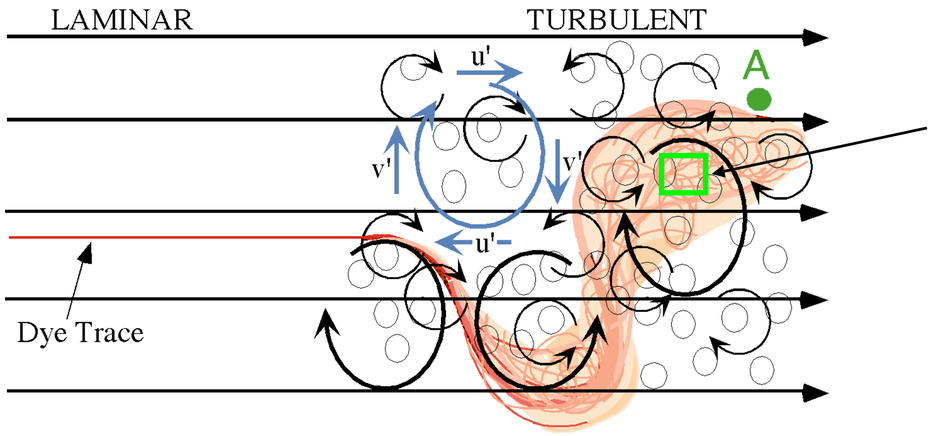
$$p(x|\varepsilon) = \frac{1}{\sqrt{2\pi\varepsilon}} \exp\left(-\frac{x^2}{2\varepsilon}\right)$$

ε: background variable

Origin of Non-Gaussian Distributions II: Intermittency

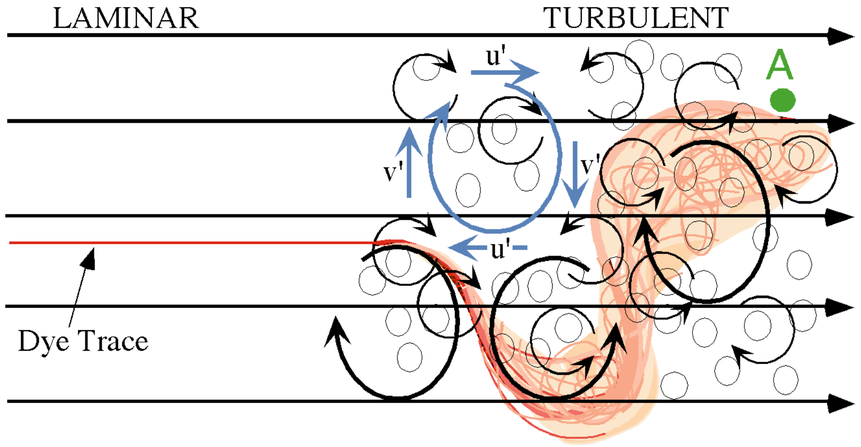
- **Slowly varying background**: the local equilibrium parameter (i.e., local noise amplitude) varies from place to place.
- In other words: the **local variance ε is a (slow) random variable**!
- Hence, the marginal distribution is a **mixture of Gaussians**:

$$p(x) = \int_0^\infty p(x|\varepsilon) f(\varepsilon) d\varepsilon$$
$$= \int_0^\infty \frac{1}{\sqrt{2\pi\varepsilon}} \exp\left(-\frac{x^2}{2\varepsilon}\right) f(\varepsilon) d\varepsilon$$

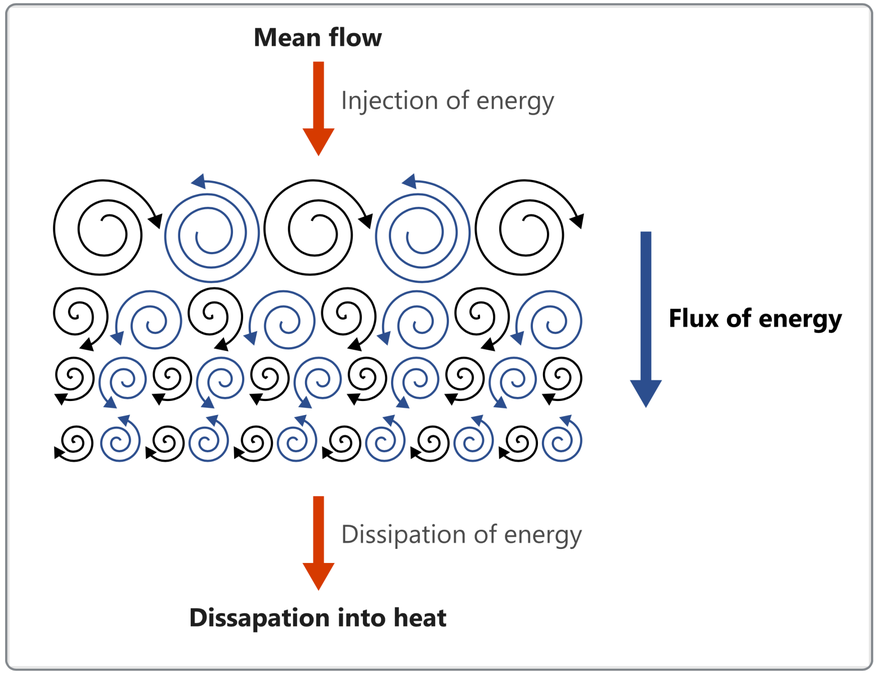


Over long time scales the background varies!

Intermittency Hierarchical Models



Energy Cascade



Energy flux down the cascade:

$$d\epsilon_1 = -\gamma_1(\epsilon_1 - \epsilon_0)dt + k_1\epsilon_1 dW_1$$

$$d\epsilon_2 = -\gamma_2(\epsilon_2 - \epsilon_1)dt + k_2\epsilon_2 dW_2$$

$$\vdots$$

$$d\epsilon_N = -\gamma_N(\epsilon_N - \epsilon_{N-1})dt + k_N\epsilon_N dW_N$$

Stationary background distribution:

$$f(\varepsilon_N) = \frac{1}{\varepsilon_0\omega\Gamma(\boldsymbol{\beta}+\mathbf{1})} G^{0,N}_{N,0}\left(\begin{matrix}-\boldsymbol{\beta}-\mathbf{1} \\ - \end{matrix}\middle|\, \frac{\varepsilon_N}{\varepsilon_0\omega}\right)$$

β : 'temperature' of the temperatures

Marginal Distributions

- Compound distribution:

$$P(\delta v) = \int_0^\infty \frac{e^{-\frac{(\delta v)^2}{2\varepsilon_N}}}{\sqrt{2\pi\varepsilon_N}} f(\varepsilon_N) d\varepsilon_N$$

- Analytical solution (also in terms of G functions):

$$P(\delta v) = \frac{1}{\sqrt{2\pi\omega}\Gamma(\boldsymbol{\beta}+\mathbf{1})} G^{1,N}_{N,1}\left(\begin{matrix} -\boldsymbol{\beta}-\mathbf{1/2} \\ 0 \end{matrix} \middle| \frac{(\delta v)^2}{2\omega\varepsilon_0} \right)$$

- Power-law tail :

$$P_N(x) \sim \sum_{i=1}^{N} \frac{c_i}{x^{2\beta_i+3}}, \quad \text{for} \quad |x| \to \infty$$

Particular Case N=1 ('Superstatistics')

- The first members ($N = 1$) of the two classes are well-known distributions.
- $\mathbf{s = 1}$:

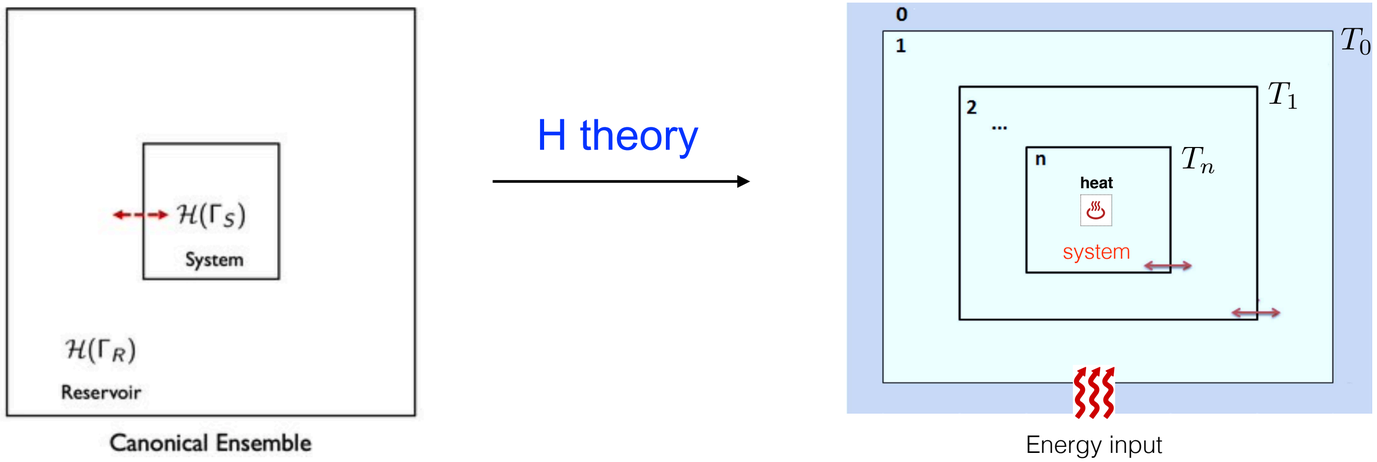
$$P(x) \sim G_{1,1}^{1,1}\left(x^2\right) \sim \exp_q\left(-x^2\right) \qquad (q\text{-Gaussian distribution})$$

- $\mathbf{s = 1/2}$:

$$P(x) \sim G_{0,2}^{2,0}\left(x^2\right) \sim x^{2(\nu+1)} K_\nu\left(x^2\right) \qquad (K \text{ distribution})$$

H Theory: Hierarchical Statistical Mechanics

- **Multiscale heat reservoir:** "local temperature" at each scale.



- **Local equilibrium:** conditional probabilities of states follow a Boltzmann distribution.

$$p(E|T_N) = \frac{1}{Z(T_N)} \exp\left(-\frac{E}{k_B T_N}\right)$$

T_N: background temperature

- **Marginal distribution:** average over the temperature distribution.

$$P(E) = \int_0^\infty \frac{\exp\left[-E/k_B T_N\right]}{Z(T_N)} f(T_N) dT_N$$

Applications

Fluid Turbulence

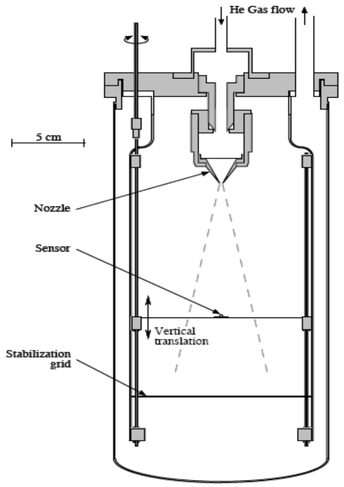
PHYSICAL REVIEW E **95**, 032315 (2017)

Universality classes of fluctuation dynamics in hierarchical complex systems

A. M. S. Macêdo,[1] Iván R. Roa González,[1] D. S. P. Salazar,[2] and G. L. Vasconcelos[1]

Experiments by Chanal et al., EPJB 2000

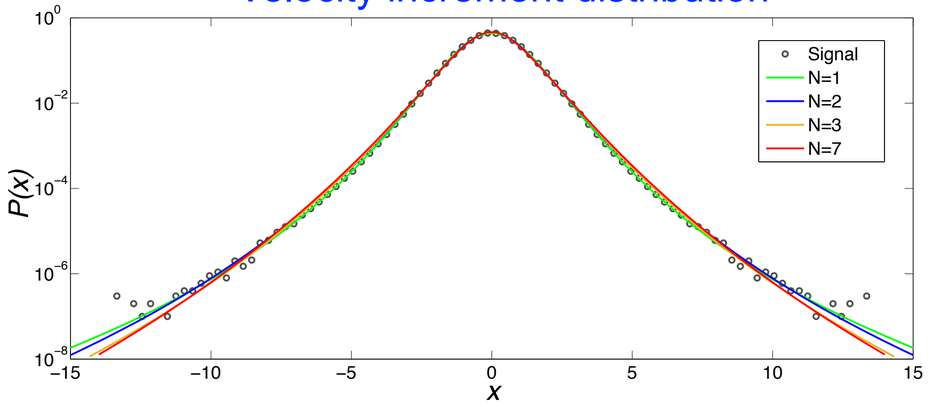
(Courtesy of B. Chabaud and P-E Roche)



$$x \equiv \delta_r u = u(x+r) - u(x)$$

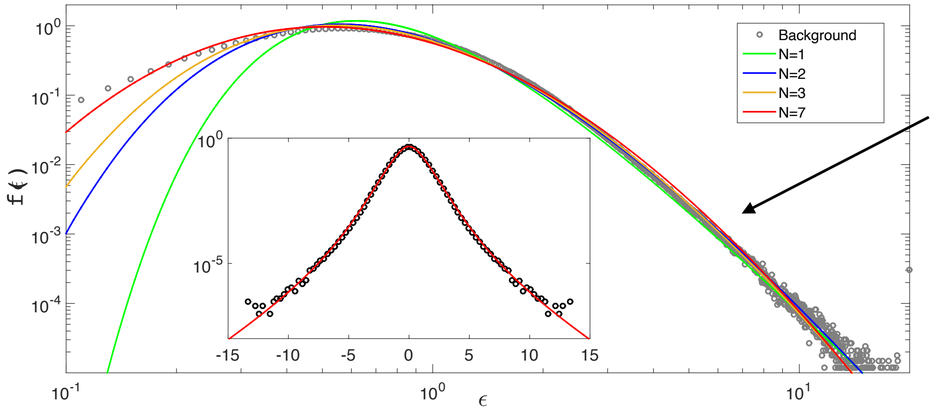
$$P(x) = \int_{-\infty}^{\infty} \frac{1}{\sqrt{2\pi\epsilon}} \exp\left(-\frac{x^2}{2\epsilon}\right) f(\epsilon) d\epsilon$$

Velocity increment distribution



s=1
(power-law tail)

Background distribution



fitting done here!

N=7
(best fit)

Turbulence in Random Lasers

nature COMMUNICATIONS

ARTICLE

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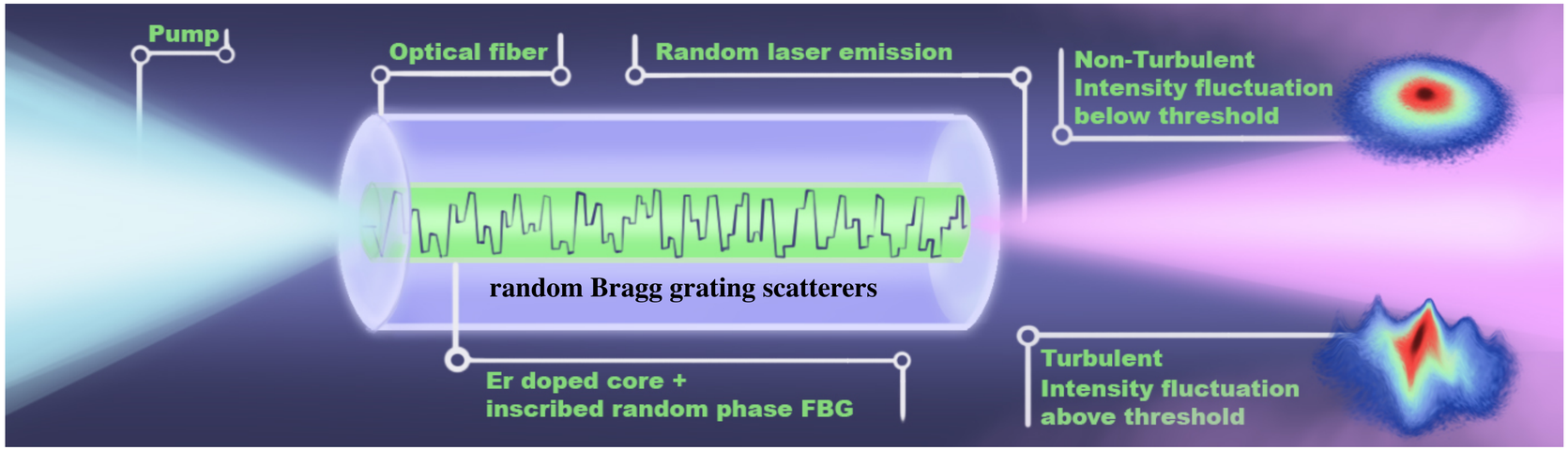
Turbulence hierarchy in a random fibre laser

Iván R. Roa González[1], Bismarck C. Lima[2], Pablo I.R. Pincheira[2], Arthur A. Brum[1], Antônio M.S. Macêdo[1], Giovani L. Vasconcelos[1], Leonardo de S. Menezes[2], Ernesto P. Raposo[1], Anderson S.L. Gomes[2] & Raman Kashyap[3,4]



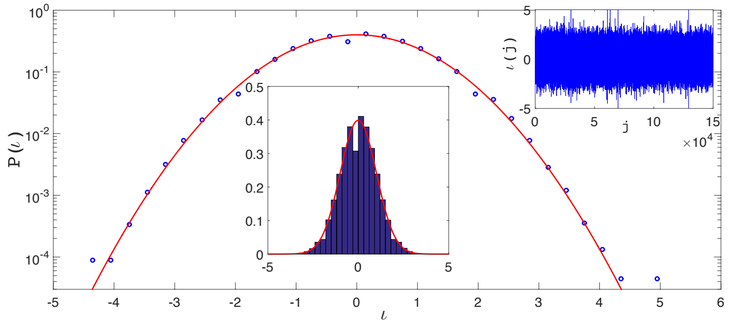
Turbulence in Random Fibre Lasers

- $P_N(x)$ is from the stretched-exponential class ($s = 1/2$):

$$P_N(x) = pP_N(\beta, \varepsilon_0, x) + (1-p)P_N(\beta', \varepsilon_0', x)$$

$$P_N(x) \sim G_{0,N+1}^{N+1,0}\left(\begin{matrix} - \\ \boldsymbol{\beta} - \mathbf{1/2}, 0 \end{matrix} \middle| \frac{\omega x^2}{2\varepsilon_0} \right)$$

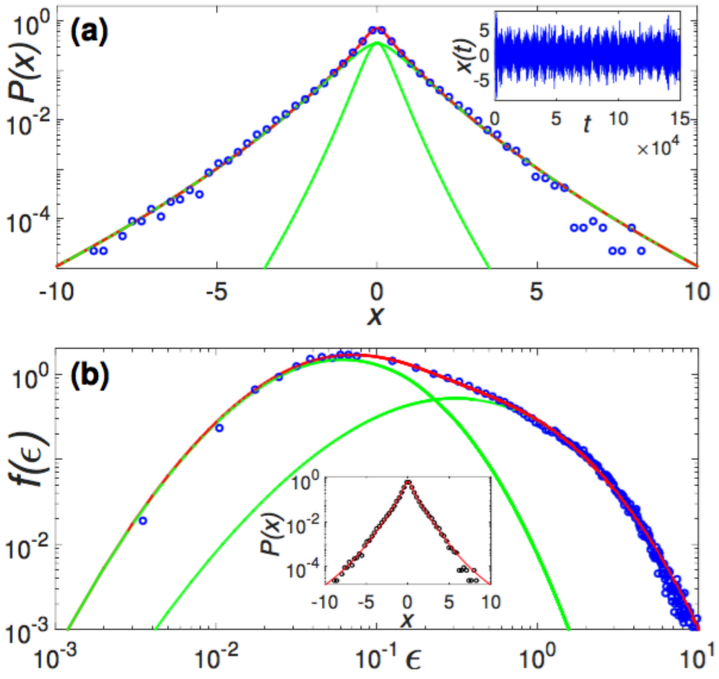
Below threshold



N = 0

(Gaussian)

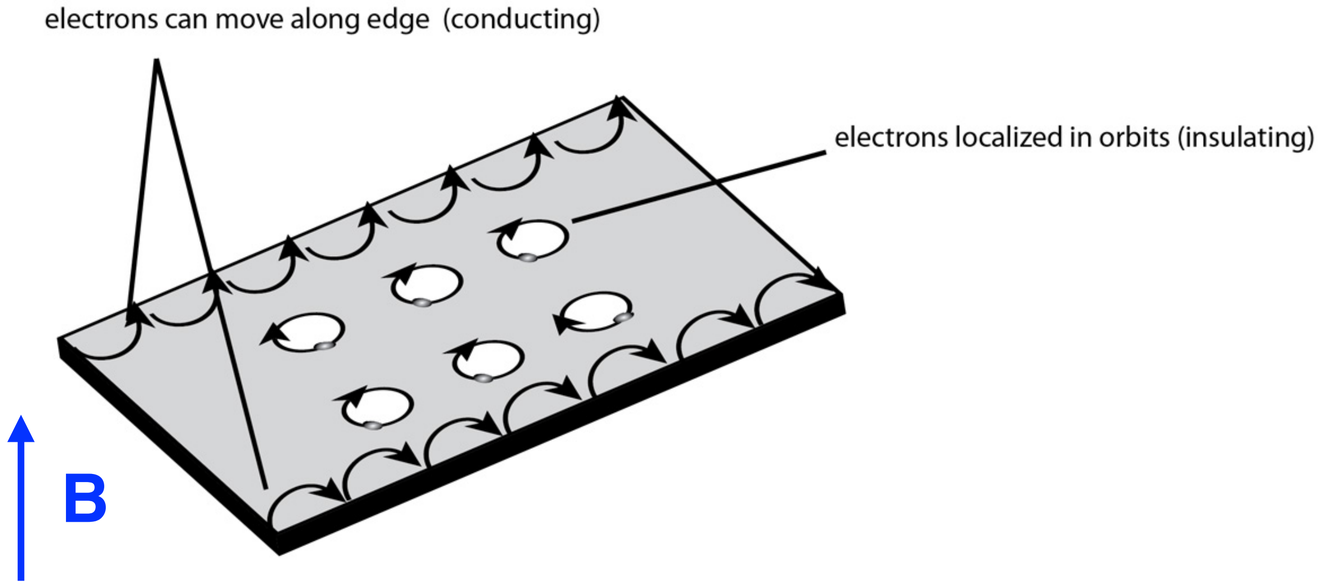
Above threshold

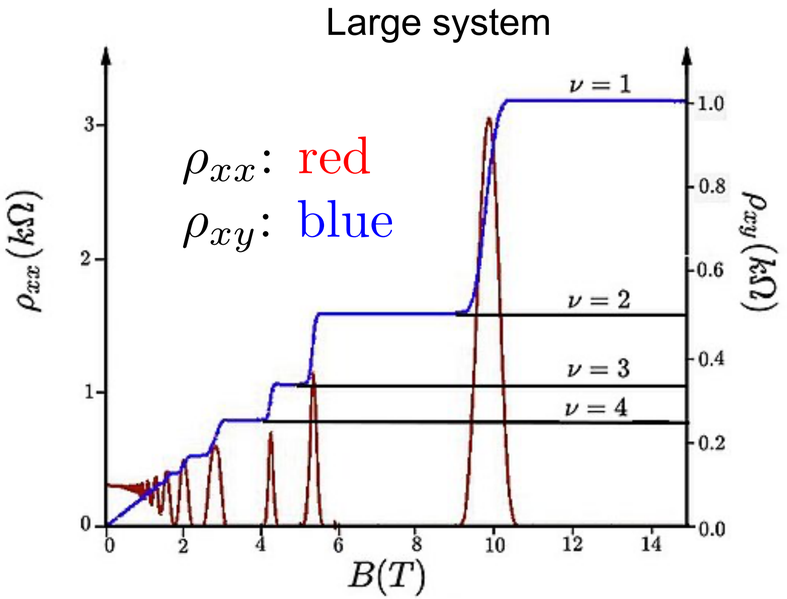


N = 6

Quantum Hall Effect

- **Quantum Hall effect:** the longitudinal conductivity vanishes for all energies except at the critical energy. **The quantum Hall plateau transition is a special case of a metal-insulator transition**.



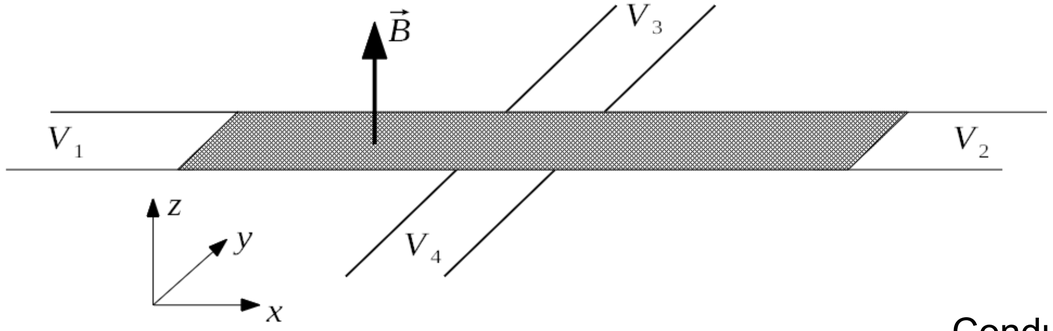


Turbulence in the Quantum Hall Effect

PHYSICAL REVIEW LETTERS **128,** 236803 (2022)

Turbulence Hierarchy and Multifractality in the Integer Quantum Hall Transition

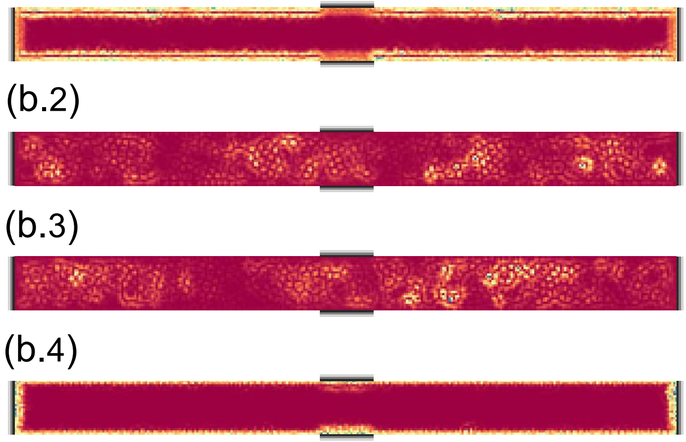
Anderson L. R. Barbosa,[1,*] Tiago H. V. de Lima,[1] Iván R. R. González,[2,3] Nathan L. Pessoa,[2,4] Antônio M. S. Macêdo,[2,†] and Giovani L. Vasconcelos[5,‡]



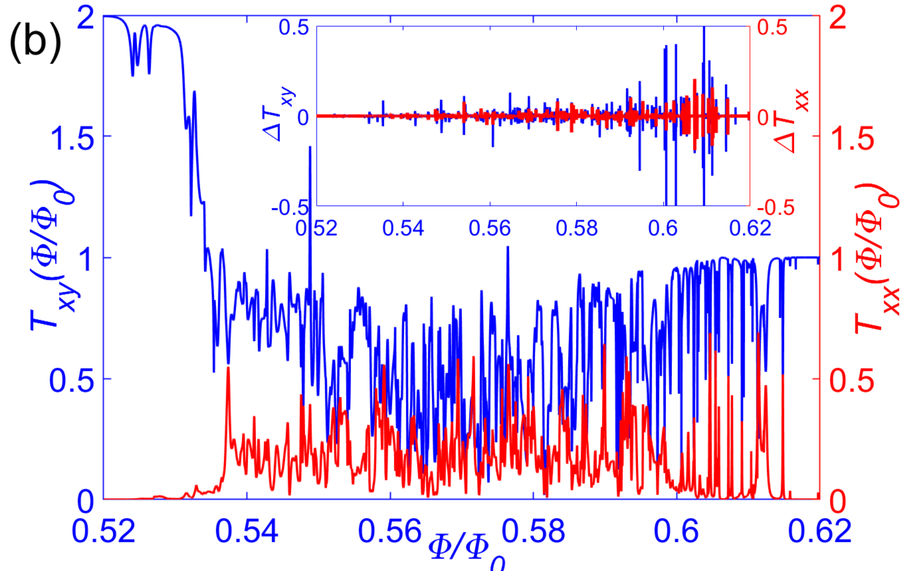
“laminar regime”

“turbulent regime”

(b.1) Density of States

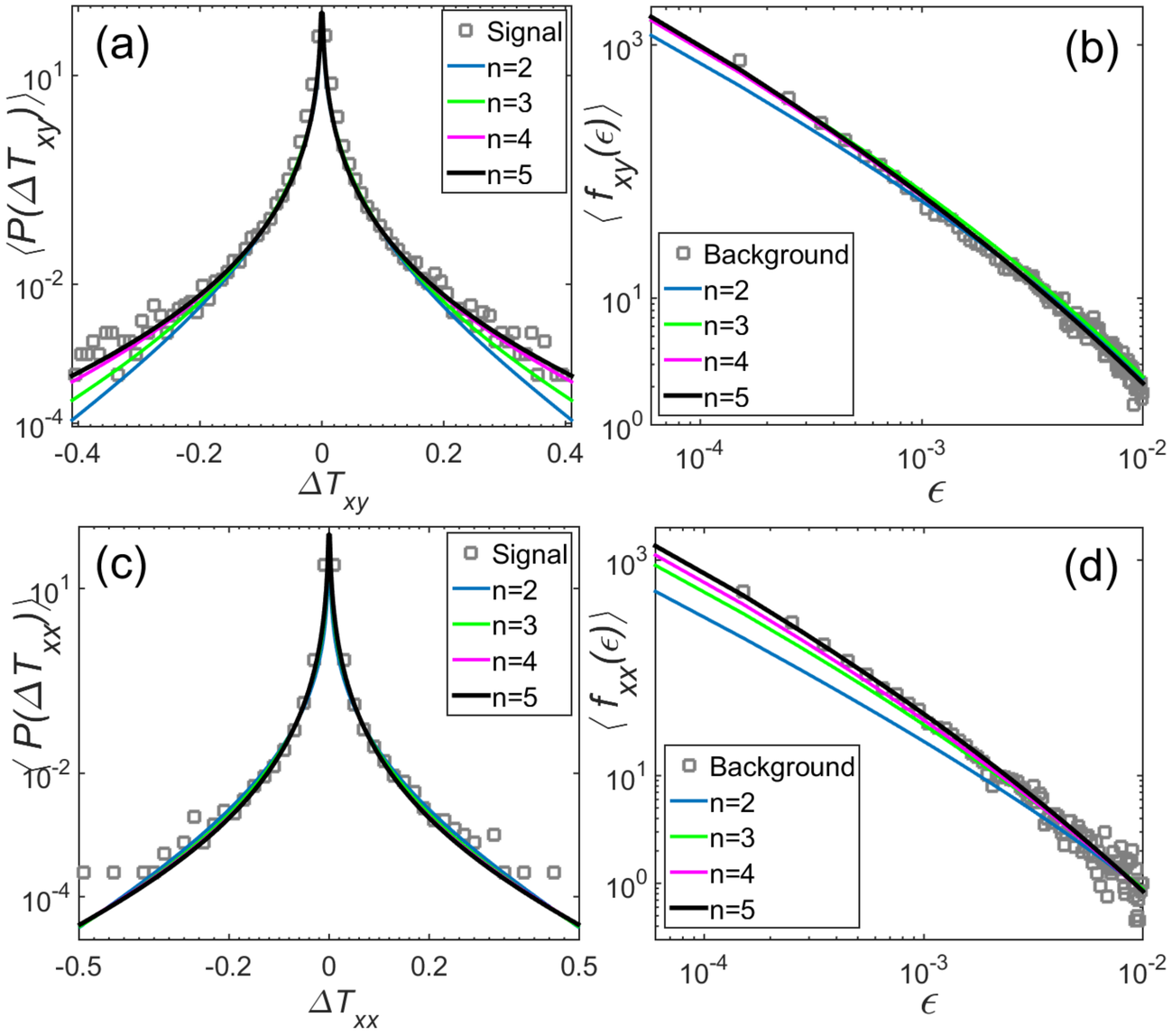


Conductance Fluctuations



Turbulence in the Quantum Hall Effect

- **Hierarchical distributions of transmission coefficients:**



$$P(z) = \int_{-\infty}^{\infty} P(z|\varepsilon) f(\varepsilon) d\varepsilon$$

$$P(z|\varepsilon) = \frac{1}{\sqrt{2\pi\sigma^2}} \exp\left(-\frac{(z-\mu)^2}{2\varepsilon}\right).$$

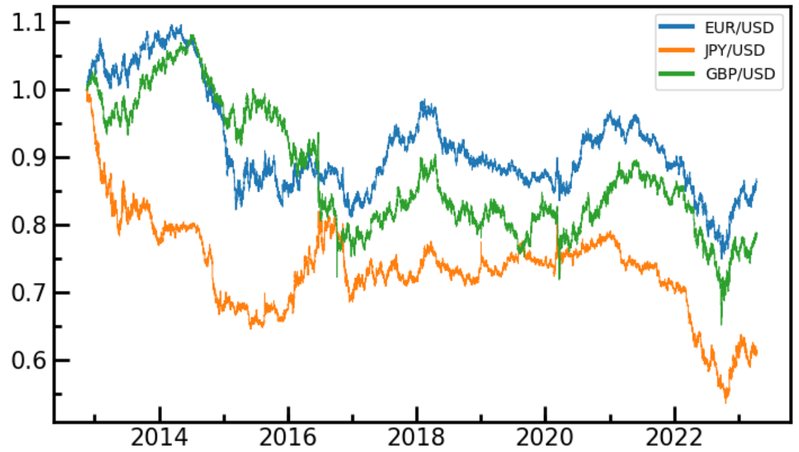
$$z = \ln\left(\frac{1+\Delta T}{1-\Delta T}\right)$$

$$\Delta T = T(B+\Delta B) - T(B)$$

N=5

Turbulence in Financial Markets

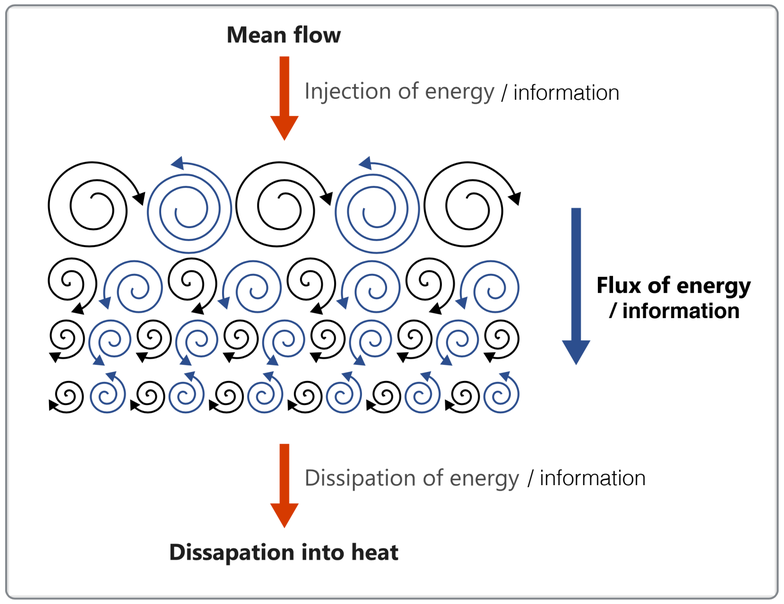
Exchange rates

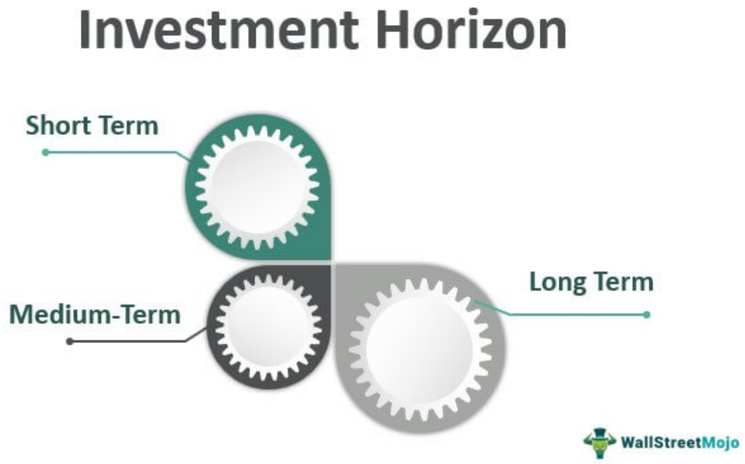


$$x_\tau \equiv \ln S(t+\tau) - \ln S(t)$$

$$dx_\tau = -\mu x_\tau dt + \sqrt{\mu v_\tau} dW$$

Information Cascade





Hierarchical stochastic volatility model:

$$dv = -\gamma_1(v_1 - v_0)dt + k_1 v_1 dW_1$$

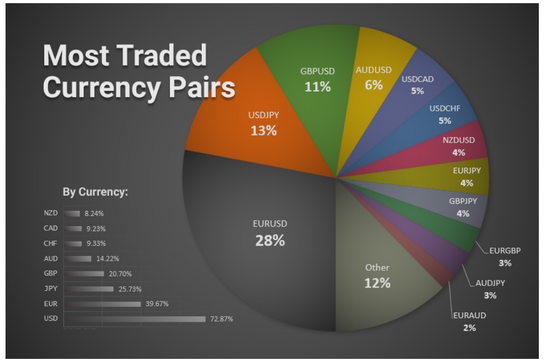
$$dv_2 = -\gamma_2(v_2 - v_1)dt + k_2 v_2 dW_2$$

$$\vdots$$

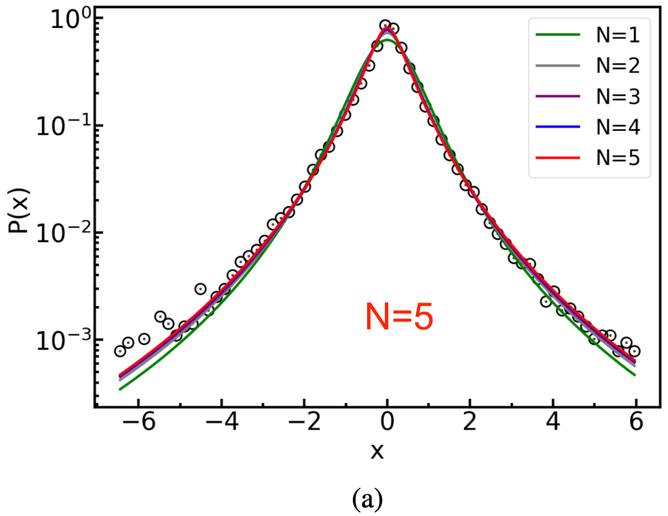
$$dv_N = -\gamma_N(v_N - v_{N-1})dt + k_N v_N dW_N$$

$$p(x) = \int_0^\infty \frac{1}{\sqrt{2\pi v_N}} \exp\left(-\frac{x^2}{2v_N}\right) f(v_N) dv_N$$

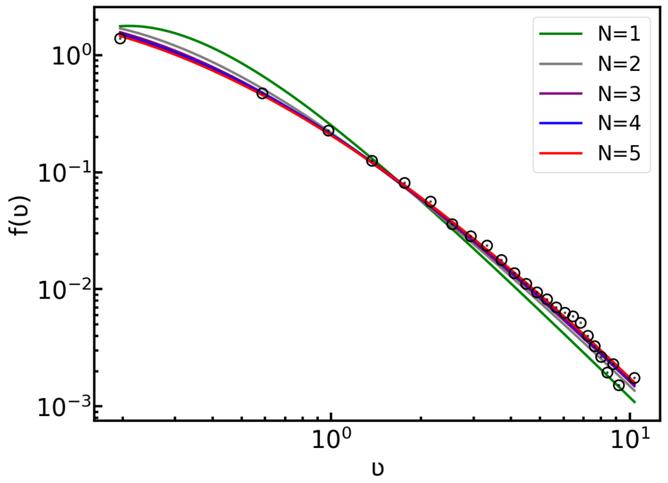
Turbulence in Foreign Exchange Markets



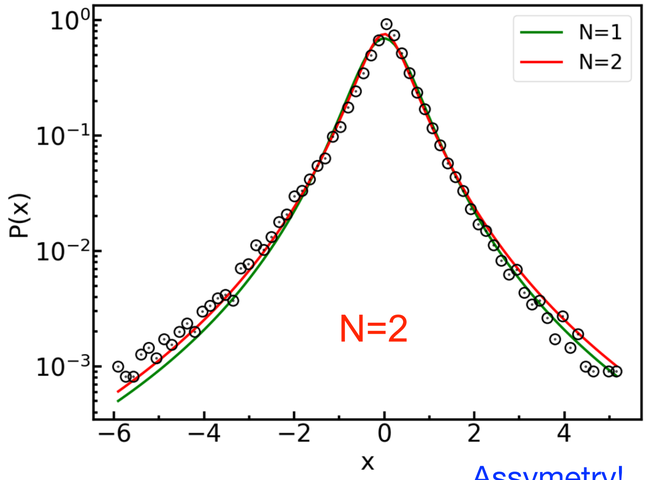
EUR/USD



(a)

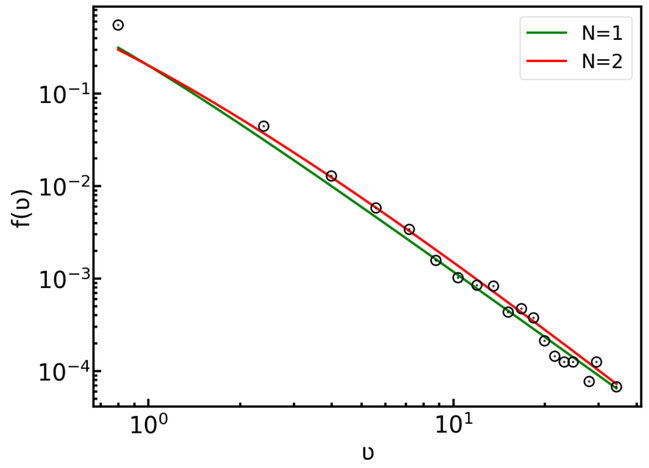


USD/JPY

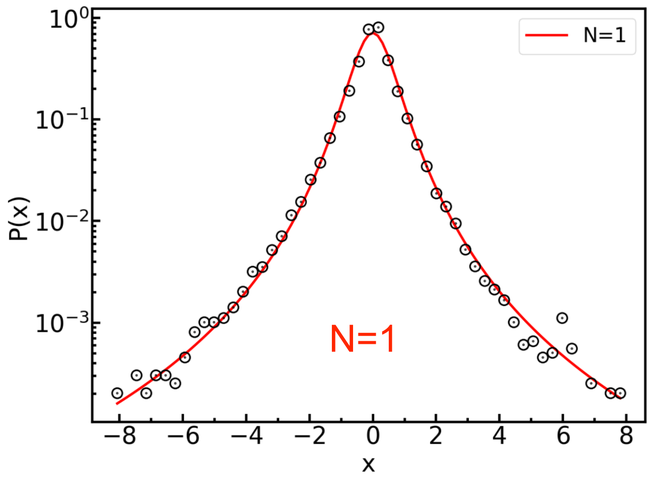


(a)

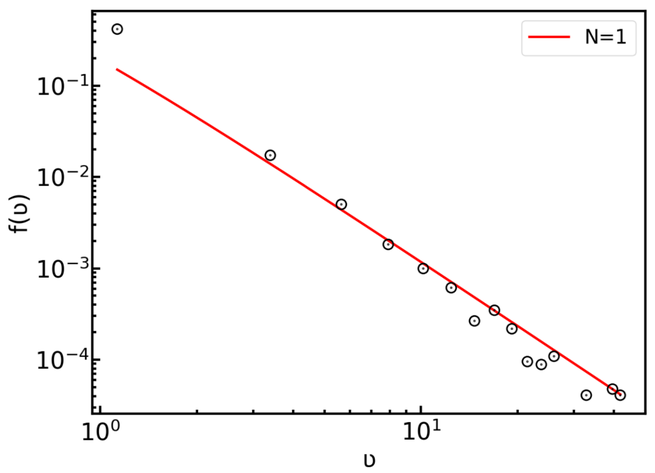
Assymetry!



GBP/USD



(a)



- The larger the market, the greater the number of hierarchical levels in the information cascade.

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Regular-to-Quantum Chaos Transition

Unified stochastic formalism for the regular-to-chaotic transition in quantum billiards

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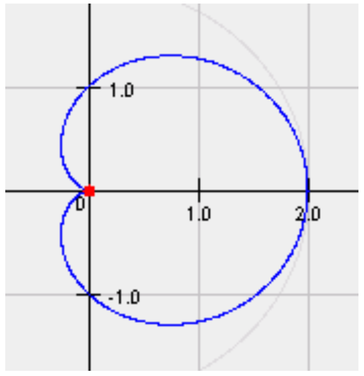
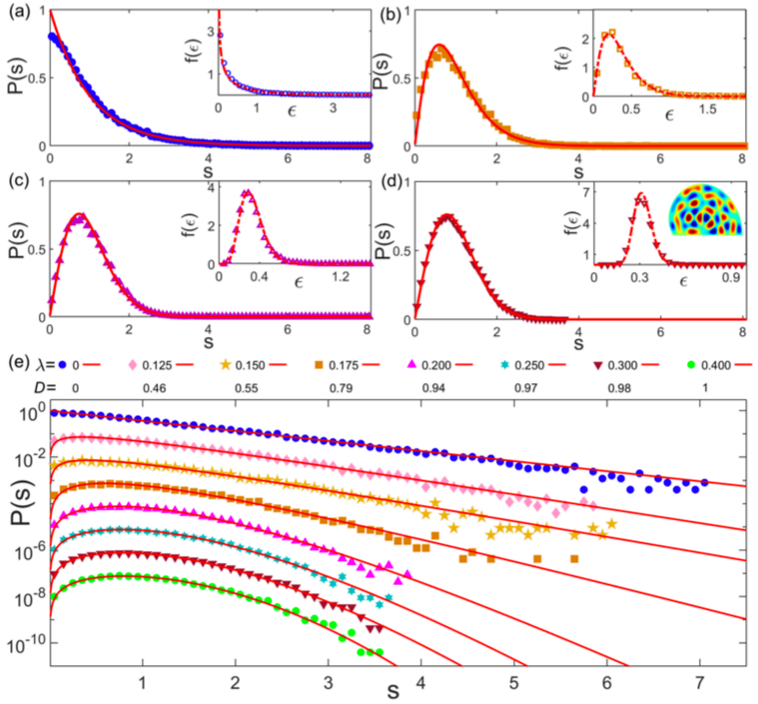
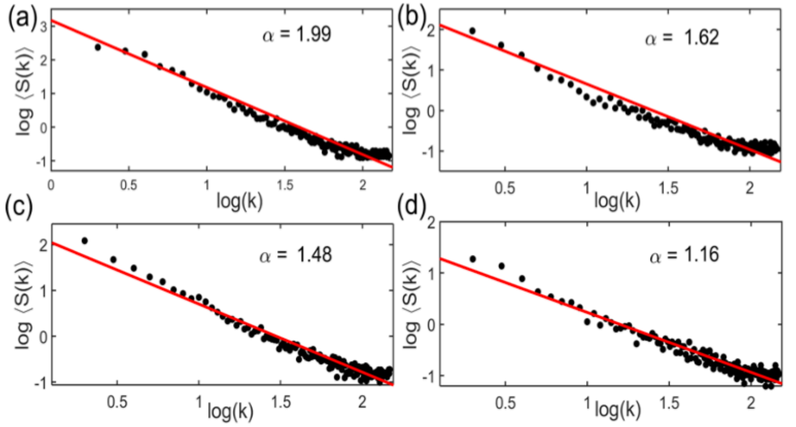
$$ds = -\zeta\left(s - \frac{2\varepsilon}{s}\right)dt + 2\sqrt{\zeta\varepsilon}dW_1,$$
$$d\varepsilon = -\rho(\varepsilon - \varepsilon_0)dt + \kappa\sqrt{\varepsilon\varepsilon_0}dW_2,$$
$$d\zeta = -\tilde{\rho}(\zeta - \zeta_0)dt + \tilde{\kappa}\sqrt{\zeta\zeta_0}dW_3.$$



Spacing distribution



1/f-alpha noise



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Some ongoing and future works

Turbulence in Graphene Membranes

PRL 117, 126801 (2016) PHYSICAL REVIEW LETTERS week ending 16 SEPTEMBER 2016

Anomalous Dynamical Behavior of Freestanding Graphene Membranes

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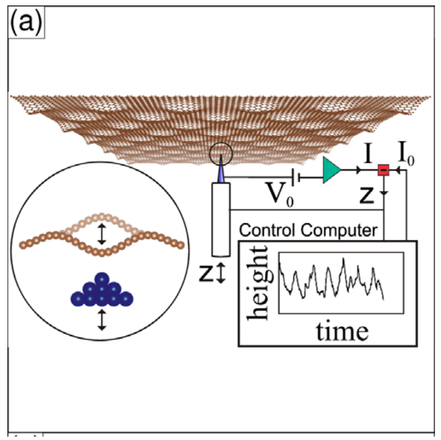
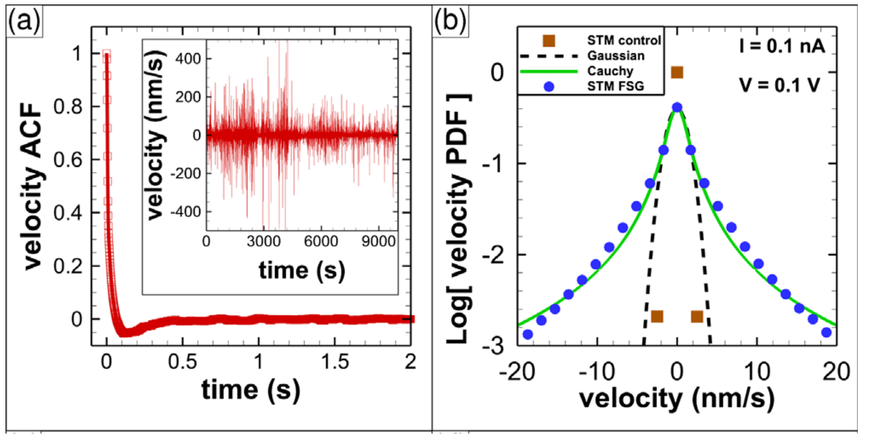
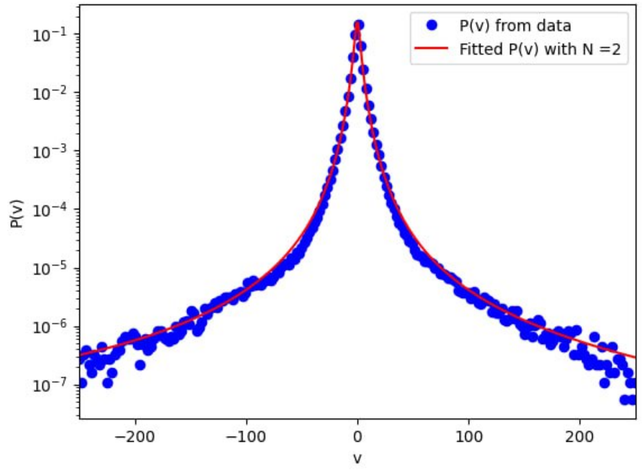
[2]*Department of Physics, Shahid Rajaee Teacher Training University, 16875-163 Lavizan, Tehran, Iran*
[3]*Department of Physics, University of Antwerp, Groenenborgerlaan 171, B-2020 Antwerpen, Belgium*
(Received 6 May 2016; published 13 September 2016)



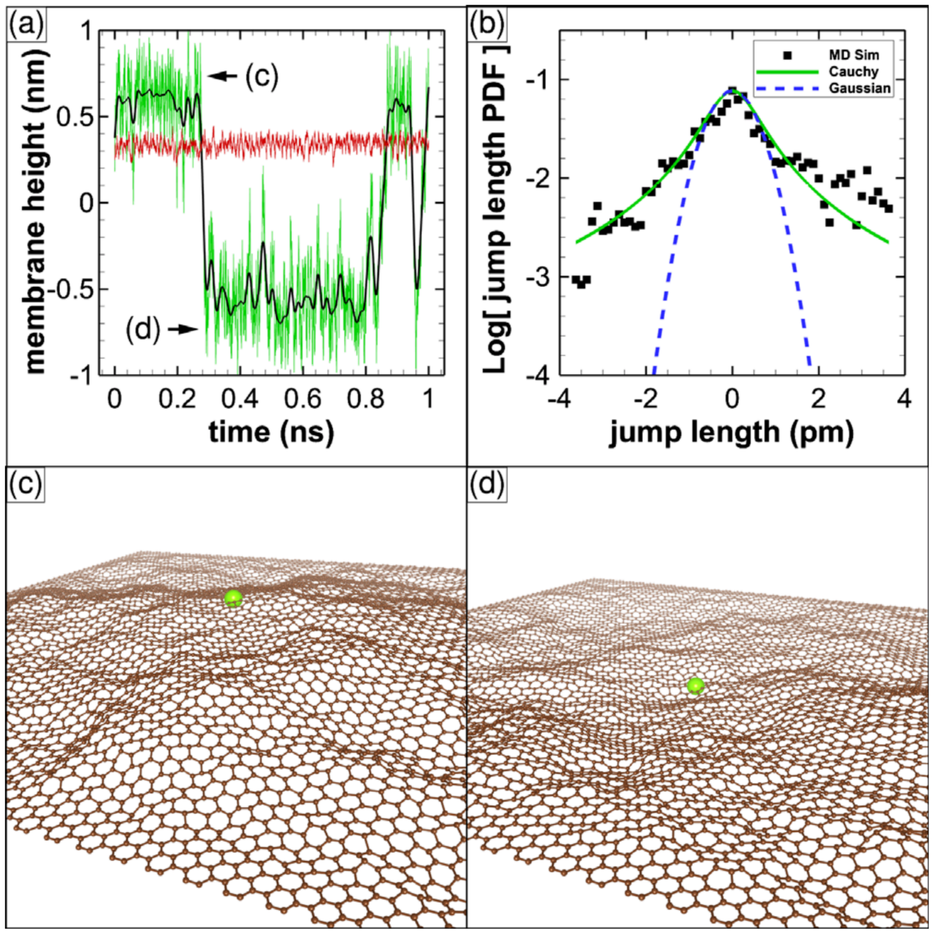


- Our proposal: mixture of Levy distributions!

$$P(v) = \int L_\alpha(v/\varepsilon) f(\varepsilon) d\varepsilon$$



Turbulence in Graphene Membranes: MD Simulations



Ackerman et al.

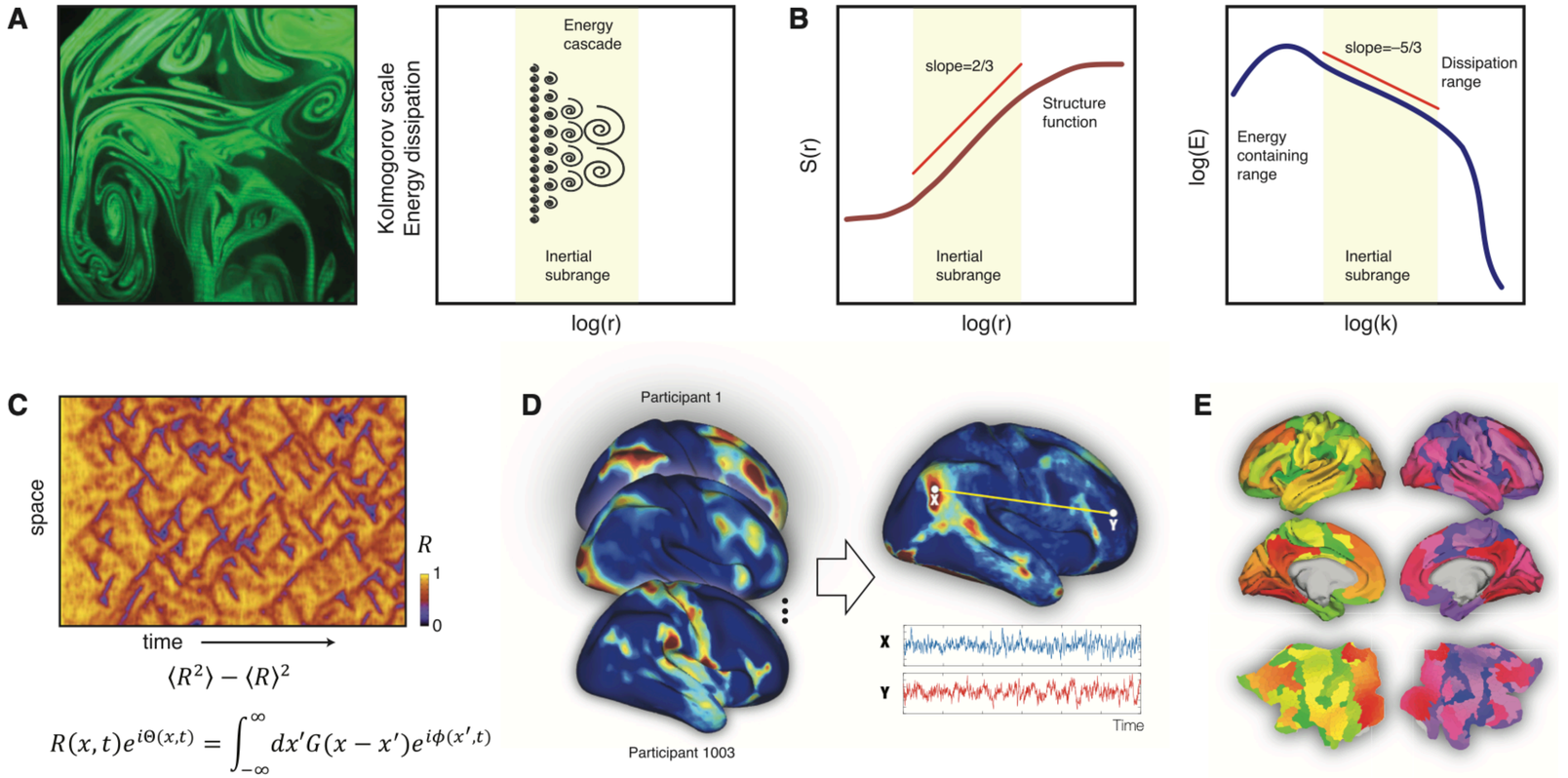
We are (still) trying to reproduce the simulations…

Future Work: Turbulent Brain

Cell Reports
Article

Turbulent-like Dynamics in the Human Brain

Gustavo Deco[1,2,3,4] and Morten L. Kringelbach[5,6,7,8,*]



- Our proposal: apply the H theory to brain dynamics!

(W/ Fernando Santos, Institute for Advanced Studies, Amsterdam)

“What if” Physics: H-Quantum Mechanics (?)

- Nelson’s stochastic quantization:

$$dX^i = b^i\big(X(t),t\big)dt + dw^i$$
$$dw^i dw_i = \hbar dt + \mathrm{o}(dt)$$

4. Successes of stochastic mechanics

Here is a list of the main successes of stochastic mechanics.

- A classical derivation of the Schrödinger equation, by Guerra and Morato [3].
- The probability density ρ of the Markov process agrees with $|\psi|^2$ at all times.
- A stochastic explanation of the relation between momentum and the Fourier transform of the wave function, by David Shucker [5].
- A proof of the existence of the Markov process under the physically natural assumption of finite action, by Eric Carlen [6]. This is perhaps the most technically demanding work in the entire subject.
- A stochastic explanation of why identical particles satisfy either Bose-Einstein or Fermi-Dirac statistics if $d \geq 3$, with parastatistics possible if $d = 2$. This is not contained in §20 of [2], but it follows from the discussion there.
- A stochastic explanation of spin and why it is integral or half-integral, work of Thaddeus Dankel [7], Timothy Wallstrom [8], and of Daniela Dohrn and Francesco Guerra jointly [9].
- If the force is time-independent, the expected stochastic energy $\mathrm{E}_t(\frac{1}{2}u^i u_i + \frac{1}{2}v^i v_i + \varphi)$ is conserved; see §14 of [2].
- A stochastic picture of the two-slit experiment, explaining how particles have trajectories going through just one slit or the other, but nevertheless produce a probability density as for interfering waves; see §17 of [2].

- Our proposal: apply the H theory to Nelson’s stochastic quantization!
- Planck’s constant acquires a stochastic dynamics (?).

Conclusions

- **H-Theory**: a general unified description of fluctuating phenomena in complex hierarchical systems.
- **Analytical expressions** for all probability distributions!
- **Wide range of applications:**
  - Fluid turbulence
  - Financial markets
  - Random lasers
  - Integer quantum Hall transition
  - Graphene membranes
  - Quantum chaos
  - Biological systems*
  - Relativistic complex systems*
  - BEC of photons in a random laser*
  - Other systems*
- **Turbulence provides a 'universal' model** for fluctuation phenomena in complex systems**!**

Obrigado!