

Rarefied Gas Dynamics: Theory and Applications to Vacuum

Lecture 1: Fundamentals of kinetic theory

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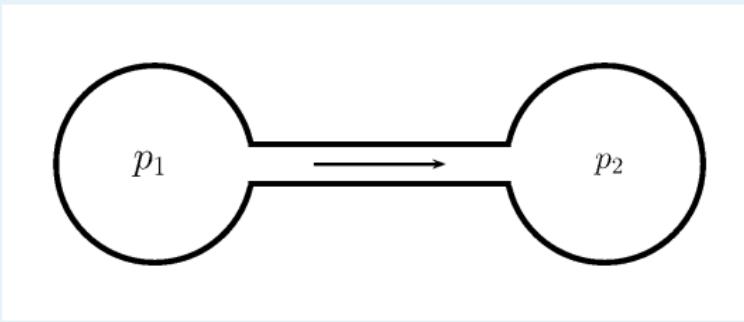
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Outline

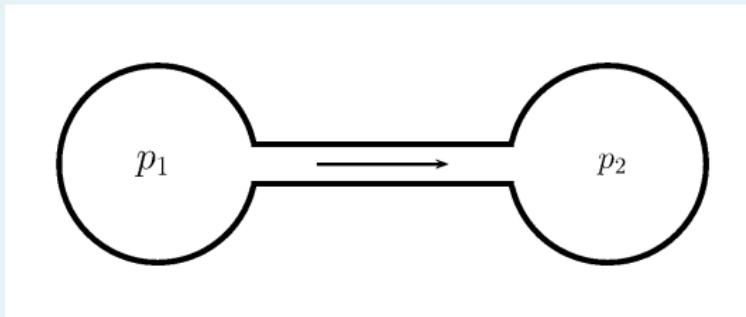
- Typical problems.
- Flow regimes.
- Intermolecular potentials.
- Equivalent free path.
- Velocity distribution function
- Boltzmann equation.
- Model equations.
- General form of gas-surface interaction.
- Velocity slip and temperature jump.
- Viscosity.
- Bibliography.

Poiseuille flow



$$p_1 > p_2$$

Poiseuille flow

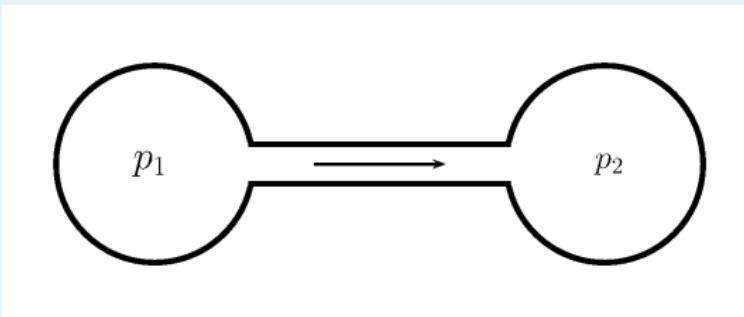


$$p_1 > p_2$$

To be calculated:

- \dot{M} mass flow rate?
- density (or pressure) distribution?

Poiseuille flow

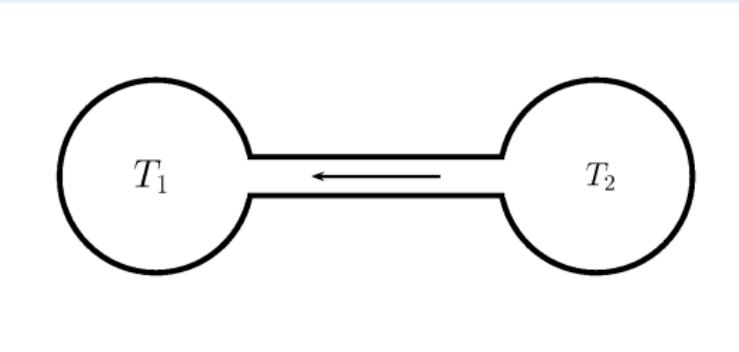


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To be calculated:

- \dot{M} mass flow rate?
- density (or pressure) distribution?
- over the whole range of Kn

Thermal creep

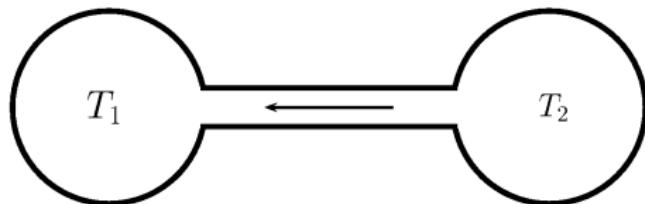


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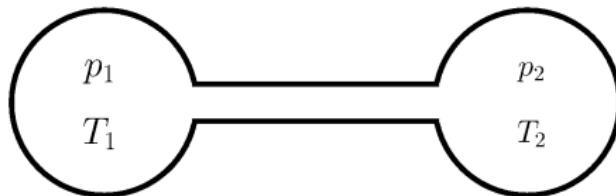


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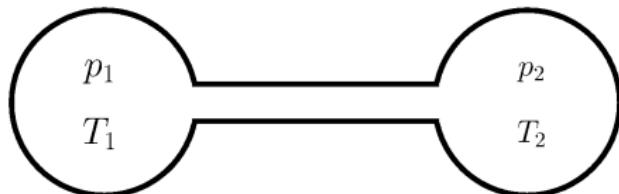
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Thermomolecular pressure difference



net flow rate = 0

Thermomolecular pressure difference

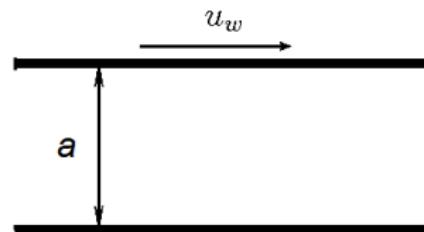


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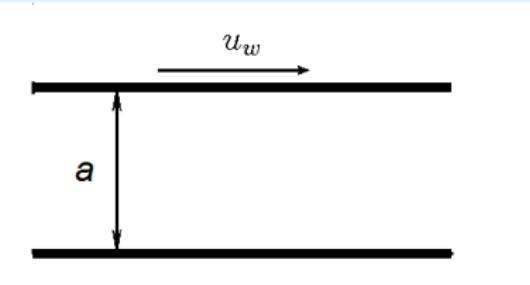
To be calculated:

$$\frac{p_1}{p_2} = \left(\frac{T_1}{T_2} \right)^\gamma, \quad \gamma = ? \quad (1)$$

Couette flow



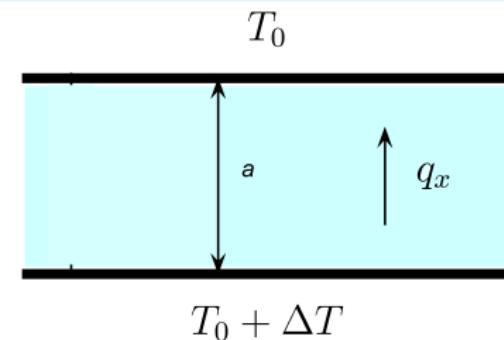
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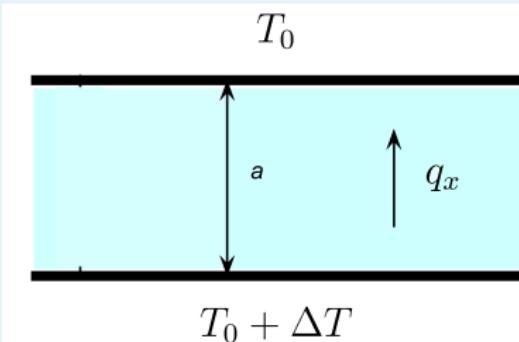
To be calculated:

- P_{xy} shear stress
- $u_x(y)$ velocity profile

Heat transfer between two plates



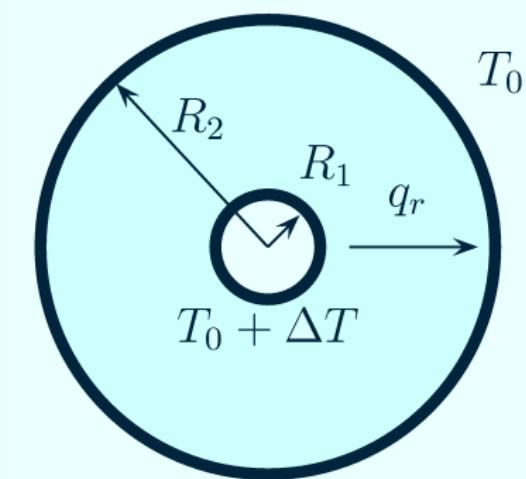
Heat transfer between two plates



To be calculated:

- q_x heat flux
- $T(x)$ temperature distribution

Heat transfer between two cylinders (Pirani sensor)



To be calculated:

- q_r heat flux
- $T(r)$ temperature distribution

Definition

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10^5 Pa > pressure > 10^{-9} Pa

10^{-8} m < mean free path < 10^6 m

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Knudsen number variation

$$10^{15} > \text{Kn} > 10^{-8}$$

Free molecular regime

$$Kn \gg 1 \quad (4)$$

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- Test particle Monte Carlo method (afternoon lecture)

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- Method of angle elements

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- The methods are well developed and well known.

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- Direct simulation Monte Carlo method is applied.
- Kinetic Boltzmann equation is solved

Hard spheres

$$U = \begin{cases} \infty & r < d \\ 0 & r > d \end{cases} \quad (7)$$

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$k_B = 1.380\ 649 \times 10^{-23}$ J/K - Boltzmann constant

m - mass of molecule

T - temperature of gas

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T - temperature of gas

$$\lambda = \frac{1}{\sqrt{2}n\pi d^2} \quad \text{mean-free-path} \quad (9)$$

n - number density of gas

Lennard-Jones (LJ) potential

$$U = 4\epsilon \left[\left(\frac{d}{r} \right)^{12} - \left(\frac{d}{r} \right)^6 \right] \quad 0 \leq r < \infty \quad (10)$$

Viscosity can be written down as

$$\mu = \frac{\sqrt{T}}{d^2} \phi \left(\frac{k_B T}{\epsilon} \right) \quad (11)$$

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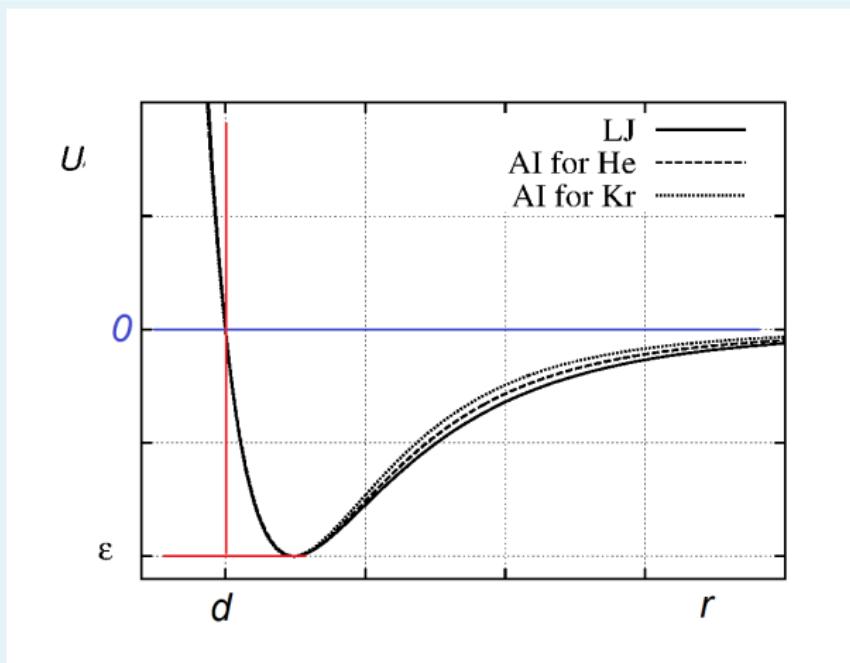
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a_1, a_2, b, A, C_{2n} are interpolating coefficients calculated *ab initio*

Nothing is extracted from experimental data

Comparison LJ and AI



AI for He: Cencek et al., *J. Chem. Phys.* **136** 224303 (2012)

AI for Kr: Jäger et al., *J. Chem. Phys.* **144** 114304 (2016)

Definition

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It is replaced by equivalent-free-path defined in terms of macroscopic quantities: μ - viscosity, p - pressure

$$\ell = \frac{\mu v_m}{p} \quad (13)$$

$$v_m = \sqrt{\frac{2k_B T}{m}} \quad \text{- most probable molecular speed} \quad (14)$$

Rarefaction parameter

$$\delta = \frac{a}{\ell} = \frac{pa}{\mu v_m} \sim \frac{1}{Kn} \quad (15)$$

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$\delta \ll 1$

Transitional regime

$\delta \sim 1$

Definition of velocity distribution function

$$f(t, \mathbf{r}, \mathbf{v})$$

t - time

\mathbf{r} - position vector

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$f(t, \mathbf{r}, \mathbf{v}) d^3\mathbf{r} d^3\mathbf{v}$ - number of particles in $(d^3\mathbf{r} d^3\mathbf{v})$ near (\mathbf{r}, \mathbf{v}) at t

Macroscopic quantities or moments

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$$\mathbf{V} = \mathbf{v} - \mathbf{u} \text{ - peculiar velocity}$$

Pressure

$$p = \frac{1}{3} (P_{11} + P_{22} + P_{33}) \quad (16)$$

State equation

$$p = n k_B T \quad (17)$$

valid for any state of gas,

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valid for any state of gas,

even out of equilibrium

Equilibrium. Global Maxwellian

$$f^M = n_0 \left(\frac{m}{2\pi k_B T_0} \right)^{3/2} \exp \left(-\frac{mv^2}{2k_B T_0} \right) \quad (18)$$

$$n = \int f^M d^3v = n_0 = \text{const} \quad (19)$$

$$\mathbf{u} = \frac{1}{n} \int \mathbf{v} f^M d^3v = 0 \quad \text{no macroscopic motion} \quad (20)$$

$$T = \frac{m}{3nk} \int V^2 f^M d^3v = T_0 = \text{const} \quad (21)$$

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$$P_{ij} = m \int V_i V_j f^M d^3 v = 0, \quad i \neq j \quad \text{no shear stress} \quad (23)$$

$$P_{ii} = m \int V_i V_j f^M d^3 v = p, \quad i = 1, 2, 3 \quad (24)$$

$$\mathbf{q} = \frac{m}{2} \int V^2 \mathbf{V} f^M d^3 v = 0 \quad \text{no heat flux} \quad (25)$$

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In equilibrium, Global Maxwellian remains
at ANY gas rarefaction
even without intermolecular collisions
with gas-surface interaction different from specular.

New definition of kelvin unity

Old: 1 K is $1/273.16$ part of the thermodynamic temperature of the triple point of water.

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Average energy of gaseous particles in equilibrium

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1 K is a variation of temperature T that results in a change of average energy of gaseous particles by $\frac{3}{2} \times 1.380\ 649 \times 10^{-23}$ J.

Boltzmann equation

$$\frac{\partial f}{\partial t} + \mathbf{v} \cdot \frac{\partial f}{\partial \mathbf{r}} = Q \quad (28)$$

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$$Q = \int (f' f'_* - f f_*) g b db d\varepsilon d\mathbf{v}_* \quad (29)$$

$$g = |\mathbf{v} - \mathbf{v}_*| \quad \text{relative speed of collision} \quad (30)$$

$$f = f(\mathbf{v}), \quad f_* = f(\mathbf{v}_*) \quad f' = f(\mathbf{v}') \quad f'_* = f(\mathbf{v}'_*) \quad (31)$$

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\mathbf{v} and \mathbf{v}_* - molecular velocities before collision

\mathbf{v}' and \mathbf{v}'_* - molecular velocities after collision

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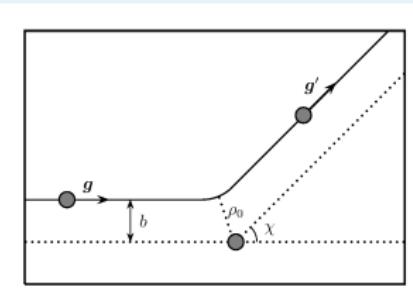
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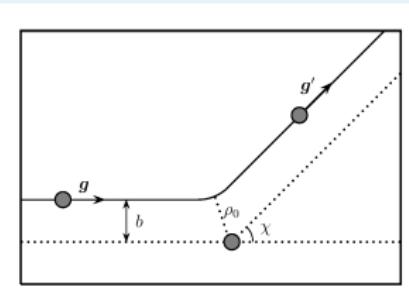
b, ε - impact parameters

Scheme of binary collision



azimuthal impact angle $0 \leq \varepsilon \leq 2\pi$ determines the collision plane

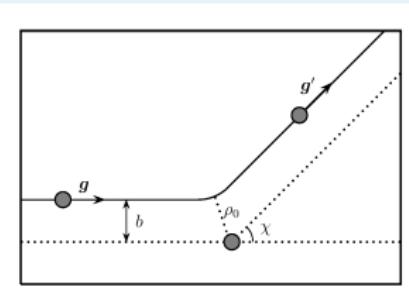
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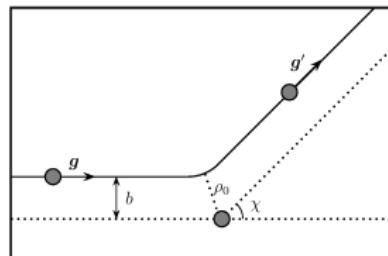


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$$\mathbf{v}, \mathbf{v}_* \Rightarrow b, \varepsilon, U \Rightarrow \mathbf{v}', \mathbf{v}'_*, \quad (32)$$

Main properties of collision integral Q

- It vanishes in equilibrium

$$Q(f^M) = 0 \quad (33)$$

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- It vanishes in equilibrium

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- Conservation laws

$$\int Q\psi(\mathbf{v}) d\mathbf{v} = 0, \quad (34)$$

where

$$\psi = m, m\mathbf{v}, \frac{mv^2}{2} \quad (35)$$

Main properties

- Irreversibility, H - theorem

$$\int Q \ln f \, dv \leq 0 \quad (36)$$

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- Irreversibility, H - theorem

$$\int Q \ln f \, dv \leq 0 \quad (36)$$

- Prandtl number

$$Pr = c_p \frac{\mu}{\kappa} \simeq \frac{2}{3} \quad (37)$$

c_p - specific heat at $p = \text{const}$

μ and κ - viscosity and heat conductivity

Discrete velocity method:

$$\mathbf{v}_1, \mathbf{v}_2, \dots, \mathbf{v}_N, \quad (38)$$

The BE is split into N differential eqs.

coupled via the collisions integral (next lecture)

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A numerical solution of the exact Boltz. Eq. requires significant computational effort.

Discrete velocity method:

$$\mathbf{v}_1, \mathbf{v}_2, \dots, \mathbf{v}_N, \quad (38)$$

The BE is split into N differential eqs.

coupled via the collisions integral (next lecture)

A numerical solution of the exact Boltz. Eq. requires significant computational effort.

Model equation is a simplified Boltzmann equation

Bhatnagar, Gross, Krook (BGK) model

$$Q = \nu (f_{loc}^M - f) \quad (39)$$

Bhatnagar, Gross, Krook (BGK) model

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ν - frequency of intermolecular collisions

f_{loc}^M local Maxwellian

$$f_{loc}^M = n \left(\frac{m}{2\pi k_B T} \right)^{3/2} \exp \left[-\frac{m(\mathbf{v} - \mathbf{u})^2}{2k_B T} \right] \quad (40)$$

$n(t, \mathbf{r})$, $\mathbf{u}(t, \mathbf{r})$ and $T(t, \mathbf{r})$ are unknown

and calculated via f

Main properties

$$Q = \nu (f^M - f) \quad (41)$$

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- $Q(f^M) = 0$: Yes!
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- H - theorem: Yes!
- Prandtl number: =1 NOT $2/3$

Mass **or** heat transfer is described.

S model

$$Q = \frac{p}{\mu} \left\{ f^M \left[1 + \frac{2m(\mathbf{q} \cdot \mathbf{V})}{15n(kT)^2} \left(\frac{mV^2}{2kT} - \frac{5}{2} \right) \right] - f \right\} \quad (42)$$

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- H - theorem: Not proved 

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Main properties

- $Q(f^M) = 0$: Yes!
- Conservation laws: Yes!
- H - theorem: Not proved
- Prandtl number:=2/3 Yes!

Mass **and** heat transfer are described.

Ellipsoidal model

$$Q = \frac{2p}{3\mu} \left\{ n \sqrt{\frac{\det A}{\pi^3}} \exp \left[- \sum_{i,j=1}^3 A_{ij} (v_i - u_i)(v_j - u_j) \right] - f \right\} \quad (43)$$

$$A = B^{-1}, \quad B_{ij} = (kT/m) (3\delta_{ij} - P_{ij}/p) \quad (44)$$

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Main properties

- $Q(f^M) = 0$: Yes! 
- Conservation laws: Yes! 
- H - theorem: **Probably proved** 
- Prandtl number:=2/3 Yes! 

Mass **and** heat transfer are described.

Accommodation coefficient

$$\alpha(\psi) = \frac{J^-(\psi) - J^+(\psi)}{J^-(\psi) - J_{diff}^+(\psi)} \quad (45)$$

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if $\psi = \frac{1}{2}mv^2$, then α is energy accommodation coefficient

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ACs depend on the incident distribution function *via* J^- .



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if $\psi = m\mathbf{v}$, then α is momentum accommodation coefficient

if $\psi = \frac{1}{2}mv^2$, then α is energy accommodation coefficient

ACs depend on the incident distribution function *via* J^- .
 AC is dependent on gas flow.



Accommodation coefficient

$$\alpha(\psi) = \frac{J^-(\psi) - J^+(\psi)}{J^-(\psi) - J_{diff}^+(\psi)} \quad (48)$$

$$J^+(\psi) = \int_{v_n > 0} |v_n| f(\mathbf{v}) \psi(\mathbf{v}) \, d\mathbf{v} \quad (49)$$

$$J^-(\psi) = \int_{v_n < 0} |v_n| f(\mathbf{v}) \psi(\mathbf{v}) \, d\mathbf{v} \quad (50)$$

specular reflection $\Rightarrow J^+ = J^- \Rightarrow \alpha = 0$

Accommodation coefficient

$$\alpha(\psi) = \frac{J^-(\psi) - J^+(\psi)}{J^-(\psi) - J_{diff}^+(\psi)} \quad (48)$$

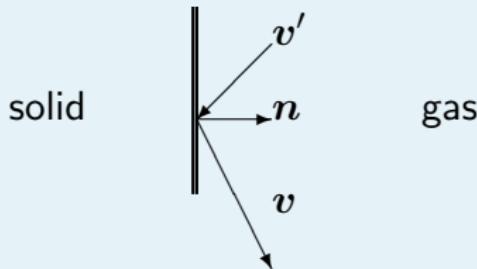
$$J^+(\psi) = \int_{v_n > 0} |v_n| f(\mathbf{v}) \psi(\mathbf{v}) \, d\mathbf{v} \quad (49)$$

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specular reflection $\Rightarrow J^+ = J^- \Rightarrow \alpha = 0$

complete accommodation $\Rightarrow J^+ = J_{diff}^+ \Rightarrow \alpha = 1$

General form



$$v_n f(\mathbf{v}) = - \int_{v'_n < 0} v'_n R(\mathbf{v}' \rightarrow \mathbf{v}) f(\mathbf{v}') \, d\mathbf{v}' \quad \text{for } v_n > 0 \quad (51)$$

$R(\mathbf{v}' \rightarrow \mathbf{v})$ - scattering kernel

Normalization / impermeability

$$\int_{v_n > 0} R(\mathbf{v}' \rightarrow \mathbf{v}) d\mathbf{v} = 1 \quad (52)$$

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$$\int_{v_n > 0} R(\mathbf{v}' \rightarrow \mathbf{v}) d\mathbf{v} = 1 \quad (52)$$

Reciprocity

$$|v'_n| \exp\left(-\frac{mv'^2}{2kT_w}\right) R(\mathbf{v}' \rightarrow \mathbf{v}) \quad (53)$$

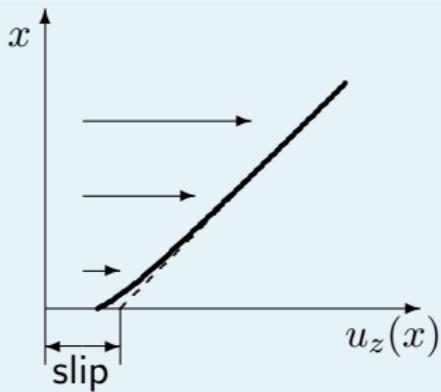
$$= |v_n| \exp\left(-\frac{mv^2}{2kT_w}\right) R(-\mathbf{v} \rightarrow -\mathbf{v}') \quad (54)$$

Details in Lecture 3

Velocity slip and temperature jump boundary conditions

Moderate rarefaction ($\delta > 10$) can be considered by
the Navier-Stokes equations subject to
the velocity slip and temperature jump boundary conditions

Viscous slip coefficient

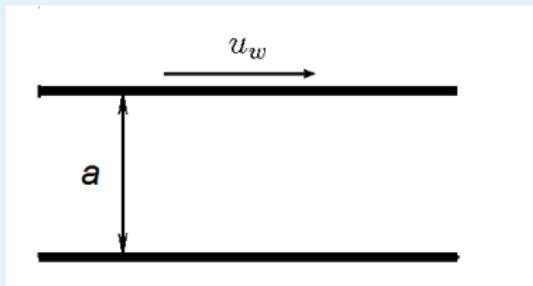


Definition

$$u_z = \sigma_P \ell \frac{du_z}{dx} \quad \text{at} \quad x = 0 \quad (55)$$

σ_P - viscous slip coefficient

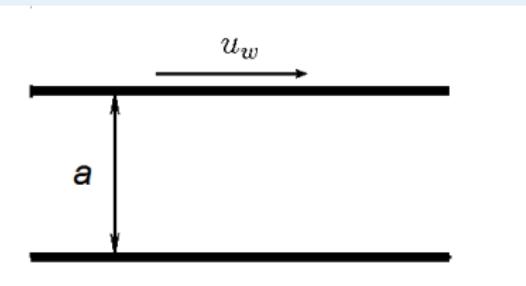
Application to Couette flow



No slip condition

$$P_{xy} = \mu \frac{u_w}{a} \quad (56)$$

Application to Couette flow



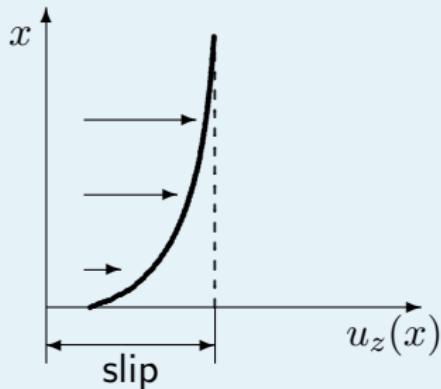
No slip condition

$$P_{xy} = \mu \frac{u_w}{a} \quad (56)$$

Slip condition

$$P_{xy} = \mu \frac{u_w}{a} \left(1 + 2\sigma_P \frac{\ell}{a} \right)^{-1} \quad (57)$$

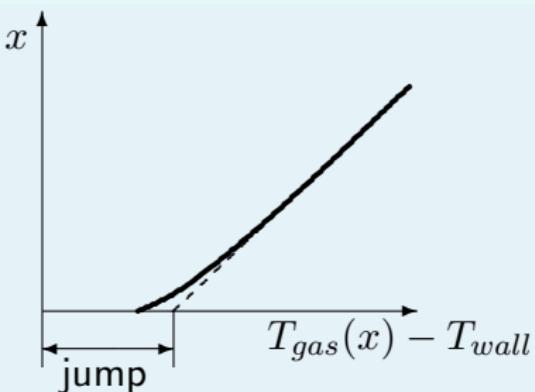
Thermal slip coefficient



Definition

$$u_y = \sigma_T \frac{\mu}{\varrho} \frac{d \ln T}{dz} \quad \text{at} \quad x = 0 \quad (58)$$

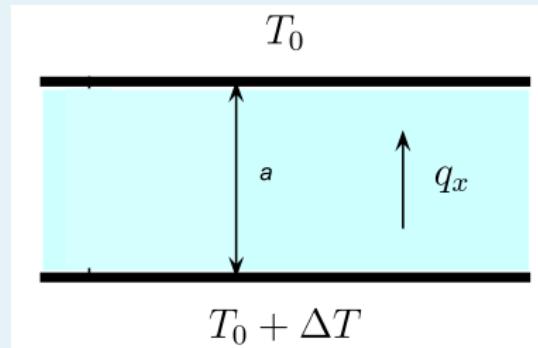
Temperature jump coefficient



Definition

$$T_{gas} - T_{wall} = \zeta_T \ell \frac{dT}{dx} \quad (59)$$

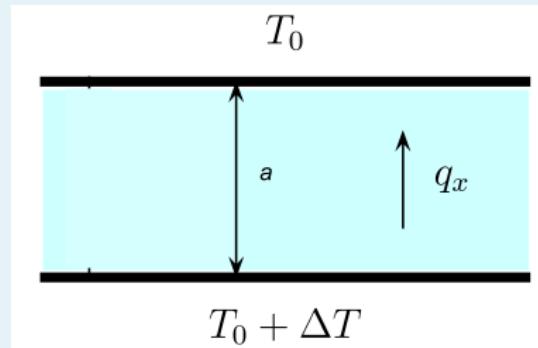
Application to heat transfer



No jump condition

$$q_x = \kappa \frac{\Delta T}{a} \quad (60)$$

Application to heat transfer



No jump condition

$$q_x = \kappa \frac{\Delta T}{a} \quad (60)$$

Jump condition

$$q_x = \kappa \frac{\Delta T}{a} \left(1 + 2\zeta_{\tau} \frac{\ell}{a} \right)^{-1} \quad (61)$$

Diffuse scattering

σ_P calculated from kinetic equation with gas-surface boundary condition

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σ_P calculated from kinetic equation with gas-surface boundary condition

$\sigma_P = 1.016$ solution of BGK model

$\sigma_P = 1.018$ solution of S model

$\sigma_P = 0.985$ solution of Boltzmann Eq for HS

$\sigma_P = 1.00$ solution of Boltzmann Eq for LJ, He at T=300 K

Diffuse scattering

σ_P calculated from kinetic equation with gas-surface boundary condition

$\sigma_P = 1.016$ solution of BGK model

$\sigma_P = 1.018$ solution of S model

$\sigma_P = 0.985$ solution of Boltzmann Eq for HS

$\sigma_P = 1.00$ solution of Boltzmann Eq for LJ, He at T=300 K

$\sigma_P \approx 1.00$ weakly sensitive to intermolecular collisions

Diffuse scattering

σ_T calculated from kinetic equation with gas-surface boundary condition

$\sigma_T = 1.175$ solution of S model

$\sigma_T = 1.02$ solution of Boltzmann Eq. for HS

$\sigma_T = 1.08$ solution of Boltzmann Eq. for LJ, He at T=300 K

Diffuse scattering

σ_T calculated from kinetic equation with gas-surface boundary condition

$\sigma_T = 1.175$ solution of S model

$\sigma_T = 1.02$ solution of Boltzmann Eq. for HS

$\sigma_T = 1.08$ solution of Boltzmann Eq. for LJ, He at T=300 K

$\sigma_T \approx 1.00$ sensitive to intermolecular collisions

Diffuse scattering

ζ_T calculated from kinetic equation with gas-surface boundary condition

$\zeta_T = 1.954$ solution of S model

$\zeta_T = 1.89$ solution of Boltz. eq. for HS

$\zeta_T = 1.91$ solution of Boltz. eq. for LJ, He at 293 K

Diffuse scattering

ζ_T calculated from kinetic equation with gas-surface boundary condition

$\zeta_T = 1.954$ solution of S model

$\zeta_T = 1.89$ solution of Boltz. eq. for HS

$\zeta_T = 1.91$ solution of Boltz. eq. for LJ, He at 293 K

$\zeta_T \approx 1.9$ weakly sensitive to intermolecular collisions

Viscosity *ab initio*

needed to calculate equivalent free path

$$\ell = \frac{\mu v_m}{p} \quad (62)$$

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needed to calculate equivalent free path

$$\ell = \frac{\mu v_m}{p} \quad (62)$$

Viscosity is calculated from the linearized Boltzmann equation with *ab initio* potential over a wide range of temperature with a high accuracy.

Viscosity of He at 25° C: theory vs. Experiment

Experiment: Berg & Burton, *Mol.Phys.* **111** (2013)

$$\mu = (19.8253 \pm 0.0059) \times 10^{-6} \text{ Pa s}, \quad u_r = 3. \times 10^{-4} \quad (63)$$

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Theory: Cencek et al. *J. Chem. Phys.* **136** (2012)

$$\mu = (19.82533 \pm 0.00019) \times 10^{-6} \text{ Pa s}, \quad u_r = 9.6 \times 10^{-6} \quad (64)$$

Relative difference = 1.5×10^{-6}

Viscosity of Ne at 25° C: theory vs. Experiment

Experiment: Berg & Burton, *Mol.Phys.* **111** (2013)

$$\mu = (31.6880 \pm 0.0096) \times 10^{-6} \text{ Pa s}, \quad u_r = 3. \times 10^{-4} \quad (66)$$

Viscosity of Ne at 25° C: theory vs. Experiment

Experiment: Berg & Burton, *Mol.Phys.* **111** (2013)

$$\mu = (31.6880 \pm 0.0096) \times 10^{-6} \text{ Pa s}, \quad u_r = 3. \times 10^{-4} \quad (66)$$

Theory: Bich et al. *Mol. Phys.* **106** (2008).

$$\mu = (31.728 \pm 0.032) \times 10^{-6} \text{ Pa s}, \quad u_r = 1. \times 10^{-3} \quad (67)$$

Relative difference = 1.3×10^{-3}

Viscosity of Ar at 25° C: theory vs. Experiment

Experiment: Berg & Burton, *Mol.Phys.* **111** (2013)

$$\mu = (22.5539 \pm 0.0067) \times 10^{-6} \text{ Pa s}, \quad u_r = 3. \times 10^{-4} \quad (68)$$

Viscosity of Ar at 25° C: theory vs. Experiment

Experiment: Berg & Burton, *Mol.Phys.* **111** (2013)

$$\mu = (22.5539 \pm 0.0067) \times 10^{-6} \text{ Pa s}, \quad u_r = 3. \times 10^{-4} \quad (68)$$

Theory: Vogel et al. *Mol. Phys.* **108** (2010)

$$\mu = 22.552 \times 10^{-6} \text{ Pa s}, \quad (69)$$

Relative difference = 8.4×10^{-5}

Viscosity of Kr at 25° C: theory vs. Experiment

Experiment: Berg & Burton, *Mol.Phys.* **111** (2013)

$$\mu = (25.2812 \pm 0.0076) \times 10^{-6} \text{ Pa s}, \quad u_r = 3. \times 10^{-4} \quad (70)$$

Viscosity of Kr at 25° C: theory vs. Experiment

Experiment: Berg & Burton, *Mol.Phys.* **111** (2013)

$$\mu = (25.2812 \pm 0.0076) \times 10^{-6} \text{ Pa s}, \quad u_r = 3. \times 10^{-4} \quad (70)$$

Theory: Jäger *J. Chem.Phys.* **144** (2016)

$$\mu = (25.279 \pm 0.020) \times 10^{-6} \text{ Pa s}, \quad u_r = 7.9 \times 10^{-4} \quad (71)$$

Relative difference = 8.7×10^{-5}

Viscosity of Xe at 25° C: theory vs. Experiment

Experiment: Berg & Burton, *Mol.Phys.* **111** (2013)

$$\mu = (23.0162 \pm 0.0040) \times 10^{-6} \text{ Pa s}, \quad u_r = 1.7 \times 10^{-4} \quad (72)$$

Viscosity of Xe at 25° C: theory vs. Experiment

Experiment: Berg & Burton, *Mol.Phys.* **111** (2013)

$$\mu = (23.0162 \pm 0.0040) \times 10^{-6} \text{ Pa s}, \quad u_r = 1.7 \times 10^{-4} \quad (72)$$

Sharipov & Benites, *Physical Chemistry Chemical Physics* **23** (2021)

$$\mu = (23.028 \pm 0.069) \times 10^{-6} \text{ Pa s}, \quad u_r = 3 \times 10^{-3} \quad (73)$$

Relative difference = 5.3×10^{-4}

Viscosity of Xe at 25° C: theory vs. Experiment

Experiment: Berg & Burton, *Mol.Phys.* **111** (2013)

$$\mu = (23.0162 \pm 0.0040) \times 10^{-6} \text{ Pa s}, \quad u_r = 1.7 \times 10^{-4} \quad (72)$$

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Relative difference = 5.3×10^{-4}

Viscosity *ab initio*

are most complete and most reliable data.

Equivalent free path for mixtures

$$\ell = \frac{\mu v_m}{p}, \quad v_m = \sqrt{\frac{2k_B T}{\bar{m}}}, \quad \bar{m} = \sum_i x_i m_i \quad (74)$$

\bar{m} is mean molecular mass

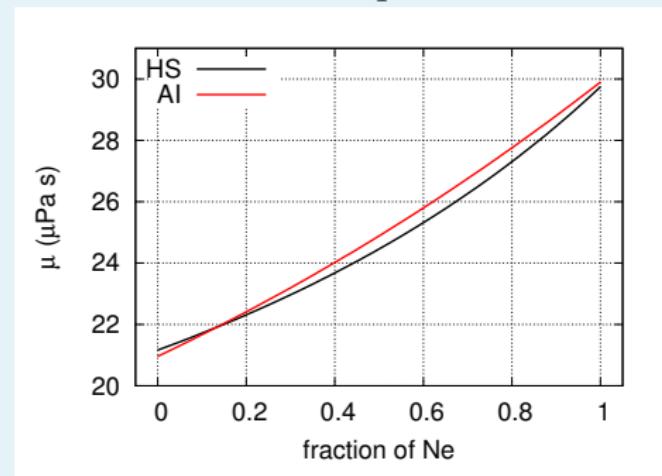
μ is viscosity of mixture calculated from system of Boltzmann equations

μ is a function of temperature T and chemical composition x_i

Viscosity of mixture Ne-Ar at $T = 273$ K, HS vs AI

HS: Tipton et al. *Eur. J. Mech. B-Fluids* **28**, 335 (2009).

AI: Sharipov & Benites *Fluid Phase Equilibria* **498**, 23-32 (2019).

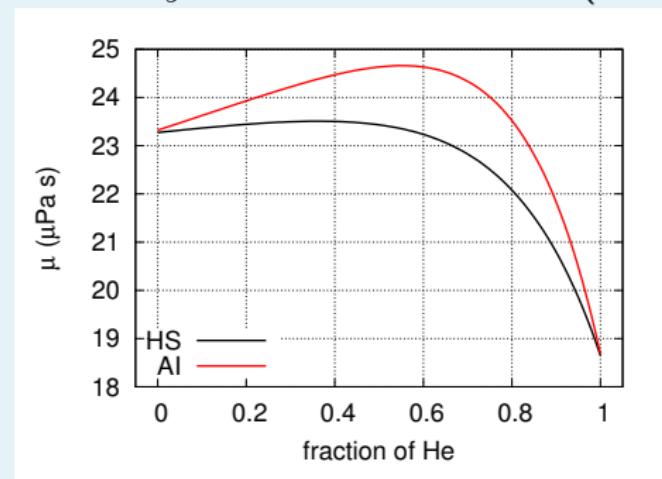


$$\frac{|\mu^{(HS)} - \mu^{(AI)}|}{\mu^{(AI)}} \times 100\% = 2.1\% \quad (75)$$

Viscosity of mixture He-Kr at $T = 273$ K, HS vs AI

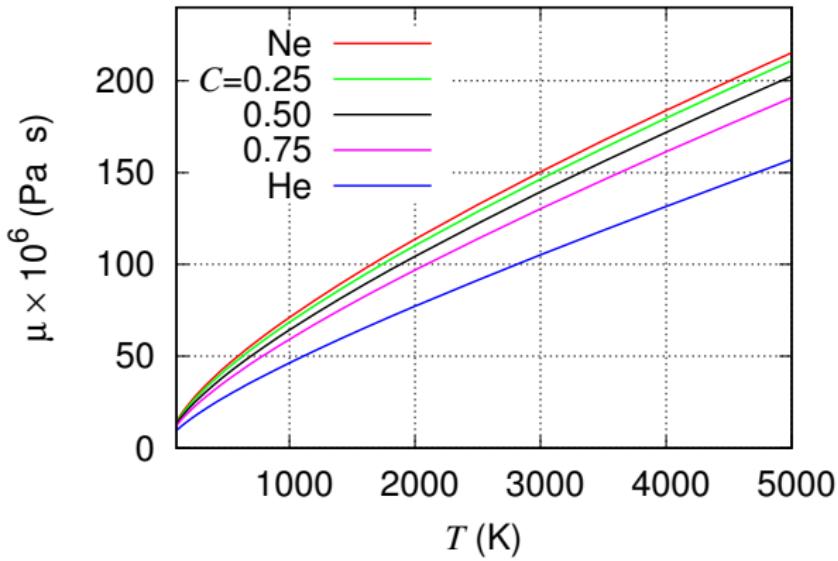
HS: Tipton et al. *Eur. J. Mech. B-Fluids* **28**, 335 (2009).

AI: Sharipov & Benites *Phys. Fluids.* **32**, 077104 (2020)



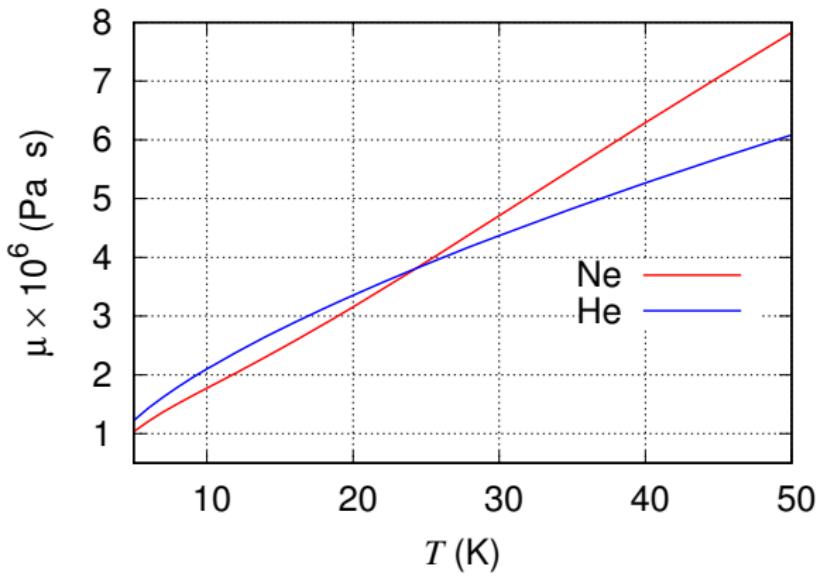
$$\frac{|\mu^{(HS)} - \mu^{(AI)}|}{\mu^{(AI)}} \times 100\% = 7.8\% \quad (76)$$

Viscosity of He-Ne mixture vs. temperature.
Ab initio. Quantum approach.



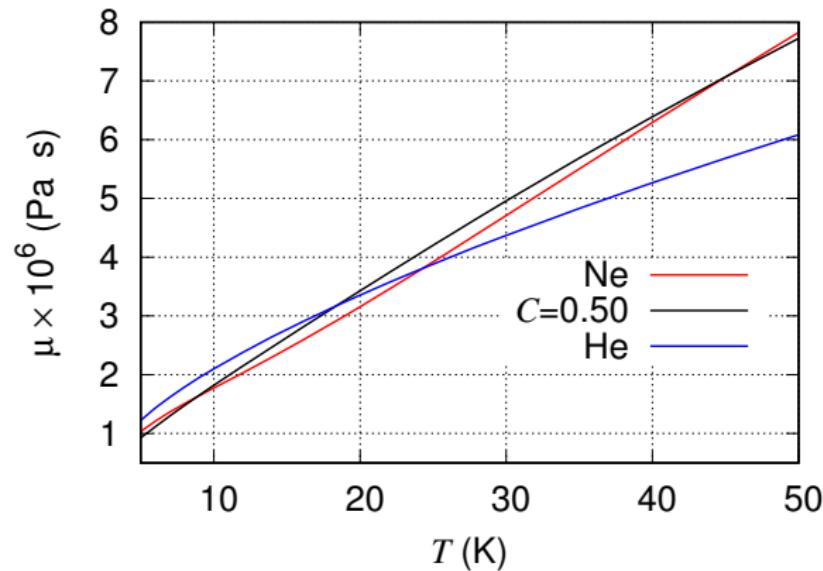
Sharipov & Benites *J. Chem. Phys.* **147**, 224302 (2017).

Viscosity of He-Ne mixture vs. temperature.
Ab initio. Quantum approach.



Sharipov & Benites *J. Chem. Phys.* **147**, 224302 (2017).

Viscosity of He-Ne mixture vs. temperature.
Ab initio. Quantum approach.



Sharipov & Benites *J. Chem. Phys.* **147**, 224302 (2017).

No general expression of $\mu = F(\mu_1, \mu_2, \dots)$

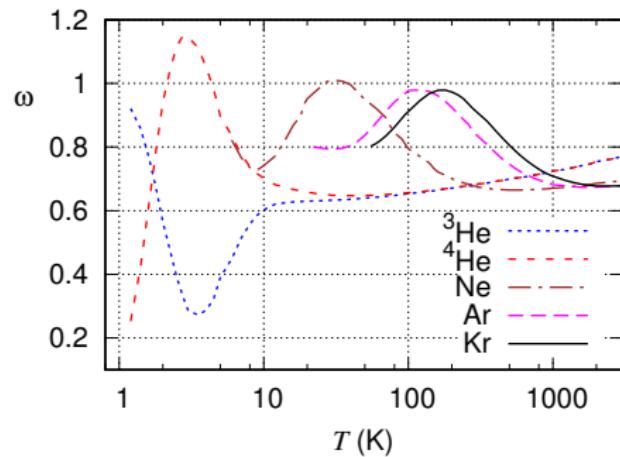
Viscous as function of temperature

$$\mu = \mu_{ref} \left(\frac{T}{T_{ref}} \right)^\omega, \quad \omega = \text{const} \quad (77)$$

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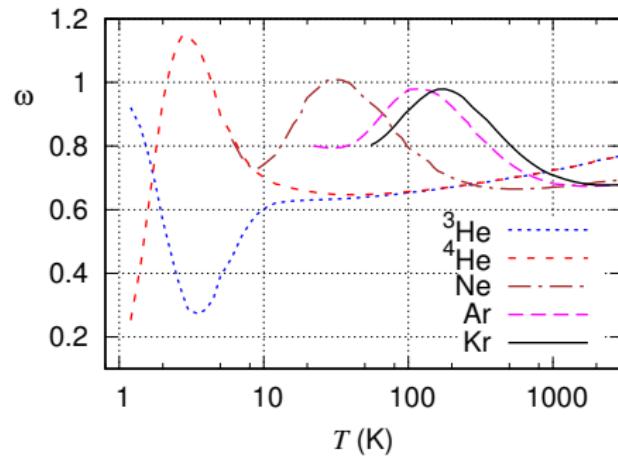
However, ω is not constant



Viscous as function of temperature

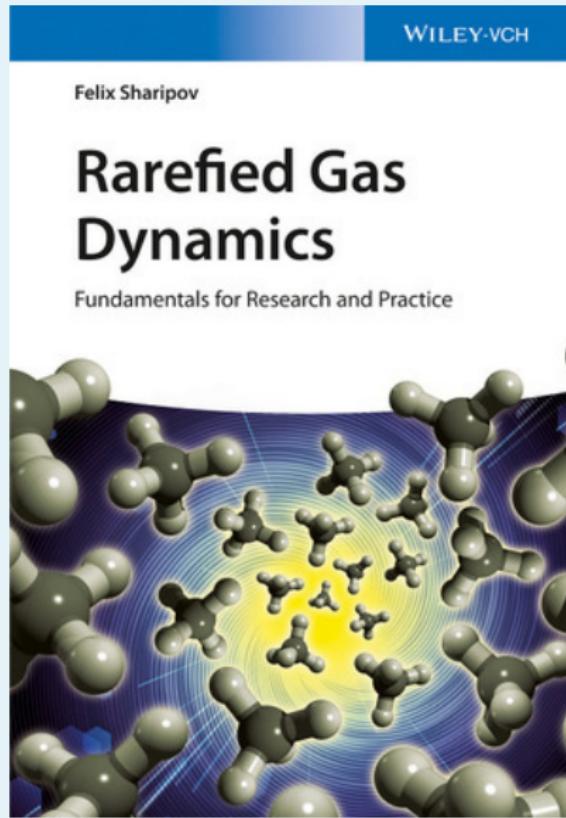
$$\mu = \mu_{ref} \left(\frac{T}{T_{ref}} \right)^\omega, \quad \omega = \text{const} \quad (77)$$

However, ω is not constant



Interpolation (??) is used for $T \approx T_{ref}$.

Book



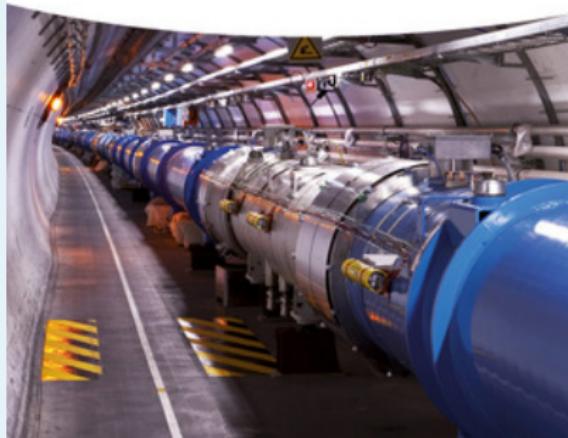
Handbook

WILEY-VCH

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Karl Jousten

Handbook of Vacuum Technology

Second Edition



Ab initio potentials

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THE END of Lecture 1

Thank you

for your attention

<http://fisica.ufpr.br/sharipov>