LISTA DE EQUAÇÕES ÚTEIS

$x = \rho\cos\theta$ $\qquad y = \rho\,\mathrm{sen}\,\theta$ $\qquad \hat{\boldsymbol{\rho}} = \cos\theta\,\hat{\mathbf{\imath}} + \mathrm{sen}\,\theta\,\hat{\mathbf{\jmath}}$

$x = r\,\mathrm{sen}\,\theta\cos\phi$ $\qquad y = r\,\mathrm{sen}\,\theta\,\mathrm{sen}\,\phi$ $\qquad z = r\cos\theta$

$$\nabla = \hat{\mathbf{r}}\frac{\partial}{\partial r} + \frac{\hat{\boldsymbol{\theta}}}{r}\frac{\partial}{\partial\theta} + \frac{\hat{\boldsymbol{\phi}}}{r\,\mathrm{sen}\,\theta}\frac{\partial}{\partial\phi} \qquad \nabla = \hat{\boldsymbol{\rho}}\frac{\partial}{\partial\rho} + \frac{\hat{\boldsymbol{\theta}}}{\rho}\frac{\partial}{\partial\theta} + \hat{\mathbf{k}}\frac{\partial}{\partial z}$$

$\hat{\mathbf{r}} = \mathrm{sen}\,\theta\cos\phi\,\hat{\mathbf{\imath}} + \mathrm{sen}\,\theta\,\mathrm{sen}\,\phi\,\hat{\mathbf{\jmath}} + \cos\theta\,\hat{\mathbf{k}}$

$$\nabla\cdot\vec{\mathscr{V}} = \frac{1}{\rho}\frac{\partial}{\partial\rho}(\rho\mathscr{V}_\rho) + \frac{1}{\rho}\frac{\partial\mathscr{V}_\theta}{\partial\theta} + \frac{\partial\mathscr{V}_z}{\partial z}$$

$$\nabla\cdot\vec{\mathscr{V}} = \frac{1}{r^2}\frac{\partial}{\partial r}(r^2\mathscr{V}_r) + \frac{1}{r\,\mathrm{sen}\,\theta}\frac{\partial}{\partial\theta}(\mathrm{sen}\,\theta\mathscr{V}_\theta) + \frac{1}{r\,\mathrm{sen}\,\theta}\frac{\partial\mathscr{V}_\phi}{\partial\phi}$$

$$\nabla\times\vec{\mathscr{V}} = \Big(\frac{1}{\rho}\frac{\partial\mathscr{V}_z}{\partial\theta} - \frac{\partial\mathscr{V}_\theta}{\partial z}\Big)\hat{\boldsymbol{\rho}} + \Big(\frac{\partial\mathscr{V}_\rho}{\partial z} - \frac{\partial\mathscr{V}_z}{\partial\rho}\Big)\hat{\boldsymbol{\theta}} + \Big(\frac{\partial}{\partial\rho}(\rho\mathscr{V}_\theta) - \frac{\partial\mathscr{V}_\rho}{\partial\theta}\Big)\frac{\hat{\mathbf{k}}}{\rho}$$

$$\nabla\times\vec{\mathscr{V}} = \Big[\frac{\partial}{\partial\theta}(\mathrm{sen}\,\theta\mathscr{V}_\phi) - \frac{\partial\mathscr{V}_\theta}{\partial\phi}\Big]\frac{\hat{\mathbf{r}}}{r\,\mathrm{sen}\,\theta} + \Big[\frac{1}{\mathrm{sen}\,\theta}\frac{\partial\mathscr{V}_r}{\partial\phi} - \frac{\partial(r\mathscr{V}_\phi)}{\partial r}\Big]\frac{\hat{\boldsymbol{\theta}}}{r} + \Big[\frac{\partial}{\partial r}(r\mathscr{V}_\theta) - \frac{\partial\mathscr{V}_r}{\partial\theta}\Big]\frac{\hat{\boldsymbol{\phi}}}{r}$$

$$\nabla^2 f = \frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2} + \frac{\partial^2 f}{\partial z^2}$$

$$\nabla^2 f = \frac{1}{\rho}\frac{\partial}{\partial\rho}\Big(\rho\frac{\partial f}{\partial\rho}\Big) + \frac{1}{\rho^2}\frac{\partial^2 f}{\partial\theta^2} + \frac{\partial^2 f}{\partial z^2}$$

$$\nabla^2 f = \frac{1}{r}\frac{\partial^2}{\partial r^2}(rf) + \frac{1}{r^2\,\mathrm{sen}\,\theta}\frac{\partial}{\partial\theta}\Big(\mathrm{sen}\,\theta\frac{\partial f}{\partial\theta}\Big) + \frac{1}{r^2\,\mathrm{sen}^2\,\theta}\frac{\partial^2 f}{\partial\phi^2}$$

$d\vec{\ell} = dx\,\hat{\mathbf{\imath}} + dy\,\hat{\mathbf{\jmath}} + dz\,\hat{\mathbf{k}}$	$d\vec{\ell} = d\rho\,\hat{\boldsymbol{\rho}} + \rho\,d\theta\,\hat{\boldsymbol{\theta}} + dz\,\hat{\mathbf{k}}$	$d\vec{\ell} = dr\,\hat{\mathbf{r}} + r\,d\theta\,\hat{\boldsymbol{\theta}} + r\,\mathrm{sen}\,\theta\,d\phi\,\hat{\boldsymbol{\phi}}$
$dv = dxdydz$	$dv = \rho\,d\rho d\theta dz$	$dv = r^2\,\mathrm{sen}\,\theta\,drd\theta d\phi$
$dA_z = dxdy$	$dA_y = dxdz$	$dA_x = dydz$
$dA_z = \rho\,d\rho d\theta$	$dA_\theta = d\rho dz$	$dA_\rho = \rho\,d\theta dz$
$dA_\phi = r\,drd\theta$	$dA_\theta = r\,\mathrm{sen}\,\theta\,drd\phi$	$dA_r = r^2\,\mathrm{sen}\,\theta\,d\theta d\phi$

$$\oint_C M\,dx + N\,dy = \int_S \Big(\frac{\partial N}{\partial x} - \frac{\partial M}{\partial y}\Big)dS \qquad A = \frac{1}{2}\oint_C x\,dy - y\,dx$$

$$\int_V \nabla\cdot\vec{\mathscr{V}}\,dv = \oint_S \vec{\mathscr{V}}\cdot\hat{n}\,dS \qquad \int_V \nabla\times\vec{\mathscr{V}}\,dv = \oint_S \hat{n}\times\vec{\mathscr{V}}\,dS$$

$$\int_V \nabla f\,dv = \oint_S f\hat{n}\,dS \qquad \int_S \nabla\times\vec{\mathscr{V}}\cdot\hat{n}\,dS = \oint_C \vec{\mathscr{V}}\cdot d\vec{\ell}$$

$$\int_S (\hat{n}\times\nabla)\times\vec{\mathscr{V}}\,dS = \oint_C d\vec{\ell}\times\vec{\mathscr{V}} \qquad \int_S \hat{n}\times\nabla f\,dS = \oint_C f\,d\vec{\ell}$$

$$\nabla(\phi+\psi) = \nabla\phi + \nabla\psi \tag{1a}$$
$$\nabla\cdot(\vec{\mathscr{V}}+\vec{\mathscr{U}}) = \nabla\cdot\vec{\mathscr{V}} + \nabla\cdot\vec{\mathscr{U}} \tag{1b}$$
$$\nabla\times(\vec{\mathscr{V}}+\vec{\mathscr{U}}) = \nabla\times\vec{\mathscr{V}} + \nabla\times\vec{\mathscr{U}} \tag{1c}$$
$$\nabla\cdot(\phi\vec{\mathscr{V}}) = \nabla\phi\cdot\vec{\mathscr{V}} + \phi\nabla\cdot\vec{\mathscr{V}} \tag{1d}$$
$$\nabla\times(\phi\vec{\mathscr{V}}) = \nabla\phi\times\vec{\mathscr{V}} + \phi\nabla\times\vec{\mathscr{V}} \tag{1e}$$
$$\nabla\cdot(\vec{\mathscr{V}}\times\vec{\mathscr{U}}) = \vec{\mathscr{U}}\cdot(\nabla\times\vec{\mathscr{V}}) - \vec{\mathscr{V}}\cdot(\nabla\times\vec{\mathscr{U}}) \tag{1f}$$
$$\nabla\times(\vec{\mathscr{V}}\times\vec{\mathscr{U}}) = (\vec{\mathscr{U}}\cdot\nabla)\vec{\mathscr{V}} - \vec{\mathscr{U}}(\nabla\cdot\vec{\mathscr{V}}) - (\vec{\mathscr{V}}\cdot\nabla)\vec{\mathscr{U}} + \vec{\mathscr{V}}(\nabla\cdot\vec{\mathscr{U}}) \tag{1g}$$
$$\nabla(\vec{\mathscr{V}}\cdot\vec{\mathscr{U}}) = (\vec{\mathscr{U}}\cdot\nabla)\vec{\mathscr{V}} + \vec{\mathscr{U}}\times(\nabla\times\vec{\mathscr{V}}) + (\vec{\mathscr{V}}\cdot\nabla)\vec{\mathscr{U}} + \vec{\mathscr{V}}\times(\nabla\times\vec{\mathscr{U}}) \tag{1h}$$
$$\nabla\times\nabla\phi = 0 \tag{1i}$$
$$\nabla\cdot(\nabla\times\vec{\mathscr{V}}) = 0 \tag{1j}$$
$$\nabla^2 = \nabla\cdot\nabla \tag{1k}$$
$$\nabla\times(\nabla\times\vec{\mathscr{V}}) = \nabla(\nabla\cdot\vec{\mathscr{V}}) - \nabla^2\vec{\mathscr{V}} \tag{1l}$$

$$\nabla\cdot\frac{\vec{r}-\vec{r}\,'}{|\vec{r}-\vec{r}\,'|^3} = 4\pi\delta(\vec{r}-\vec{r}\,') \qquad \nabla^2\frac{1}{|\vec{r}-\vec{r}\,'|} = -4\pi\delta(\vec{r}-\vec{r}\,') \qquad \nabla\frac{1}{|\vec{r}-\vec{r}\,'|} = -\frac{\vec{r}-\vec{r}\,'}{|\vec{r}-\vec{r}\,'|^3}$$

$$\int_a^b \delta(x-x_0)\,dx = \begin{cases} 1\,, & x_0\in[a,b] \\ 0\,, & x_0\notin[a,b] \end{cases} \qquad \int_a^b f(x)\delta(x-x_0)\,dx = \begin{cases} f(x_0)\,, & x_0\in[a,b] \\ 0\,, & x_0\notin[a,b] \end{cases}$$

$$\delta(x-x') = \delta(x'-x)$$