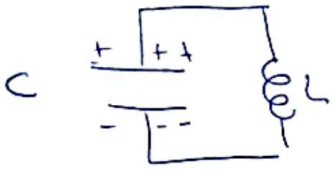
Cap 31: Oscilações Eletromagnéticas

• CL carga inicial q_0



$$\Delta V_C + \mathcal{E}_L = 0$$

$$\frac{q_C}{C} + \left(-L\frac{di}{dt}\right) = 0$$

$$i = \frac{dq_L}{dt} \qquad \frac{d}{dt}\left(q_C + q_L = q_T = q_0\right) \Rightarrow \frac{dq_C}{dt} = -\frac{dq_L}{dt}$$

$$\frac{q_C}{C} - L\frac{d}{dt}\left(-\frac{dq_L}{dt}\right) = 0$$

$$\frac{q_C}{C} + L\frac{d^2q_C}{dt^2} = 0 \quad (1) \qquad \frac{dx}{dt} + a\frac{d^2x}{dt^2} = 0 \Rightarrow x(t) = K\cos(\omega t + \varphi)$$

$K \to$? x_{max}; $\omega \to$? ω angular; $\varphi \to$? fase

Logo:

$$q_C(t) = K\cos(\omega t + \varphi)$$

$K \to q_0$, $\varphi \to 0$

$$q_C(t=0) = q_0 \qquad \Big\} \; q_C(t) = q_0\cos(\omega t)$$

Para determinar ω, retornamos em (1)

$$\frac{dq_C(t)}{dt} = -q_0\omega\,\mathrm{sen}(\omega t)$$

$$\frac{d^2}{dt^2}q_C = -q_0\omega^2\cos(\omega t) = -\omega^2 q_C(t) \quad (2)$$

(2) $\to$ (1)

$$\cancel{q_C} = -LC\left(-\omega^2 \cancel{q_C(t)}\right)$$

$$\omega^2 = \frac{1}{LC} \Rightarrow \omega = \pm\frac{1}{\sqrt{LC}}$$

(escolher um)

Logo

$$\boxed{q_C(t) = q_0\cos\left(\frac{1}{\sqrt{LC}}t\right)}$$

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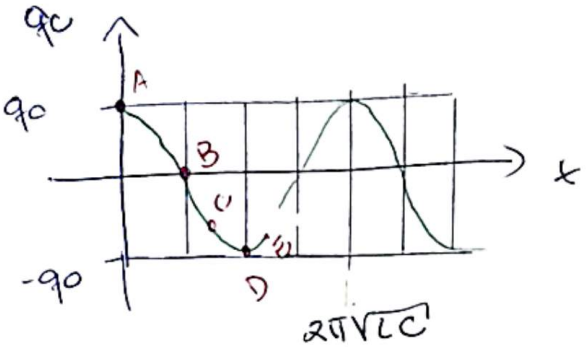
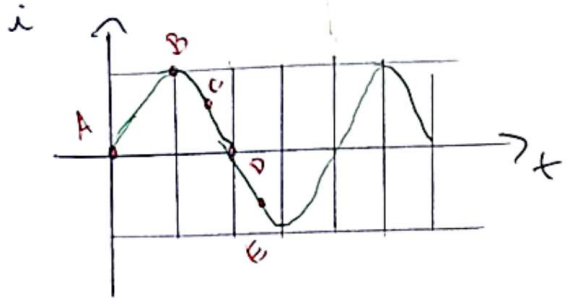
$$i = -\frac{dq_c}{dt} \Rightarrow i = \underbrace{+\frac{q_0}{\sqrt{LC}}}_{i_{max}} \cos\left(\frac{1}{\sqrt{LC}}\, t\right)$$



$$q_c(t) = q_0 \cos\left(\underbrace{\frac{1}{\sqrt{LC}}\, t}_{2\pi}\right)$$

$$\frac{1}{\sqrt{LC}}\, T = 2\pi \Rightarrow T = 2\pi\sqrt{LC}$$

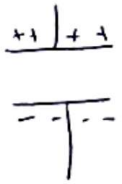
período.



$$i(t) = \frac{q_0}{\sqrt{LC}} \operatorname{sen}\left(\frac{1}{\sqrt{LC}}\, t\right)$$

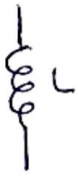
→ Em termos de energia: $(U = q \cdot \Delta V)$

$C = \frac{q}{\Delta V}$



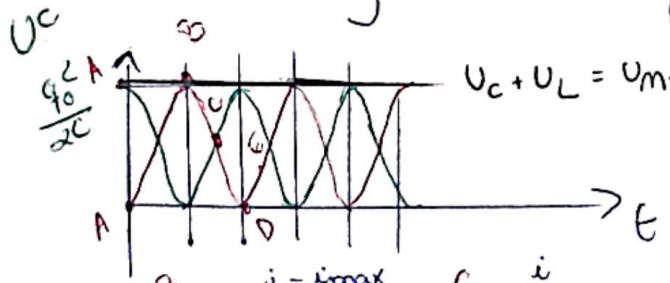
$$\int dU = \int dq\, \Delta V \Rightarrow U = \int \frac{q}{C}\, dq \Rightarrow \boxed{U_C = \frac{q^2}{2C}}$$

$\mathcal{E}_L = -L\frac{di}{dt}$



$$U_L = \int dq\, L\frac{di}{dt} = \lim_{\Delta \to 0} \sum_i \overbrace{\Delta q}^{i}\, L\frac{\Delta i}{\Delta t} = \int L\, i\, di$$

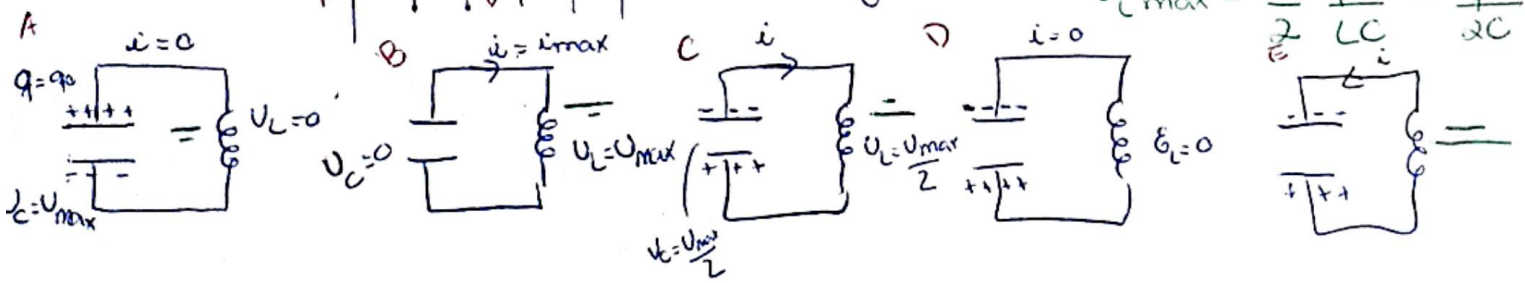
$$U_L = L\int i\, di \Rightarrow \boxed{U_L = \frac{L i^2}{2}}$$



$U_c + U_L = U_{mec}$

$$U^c_{max} = \frac{q_0^2}{2C}$$

$$U_{L\,max} = \frac{L}{2}\frac{q_0^2}{LC} = \frac{q_0^2}{2C}$$



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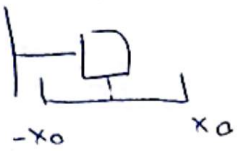
Lembrando OHS



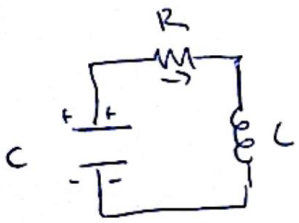
$$U_{Total} = U_k + U_{elast.}$$

$$= \frac{mv^2}{2} + \frac{1}{2} k x^2$$

$$x(t) = x_a \cos(\omega t + \varphi)$$

$$v = \frac{dx}{dt}$$

• Circuito RLC



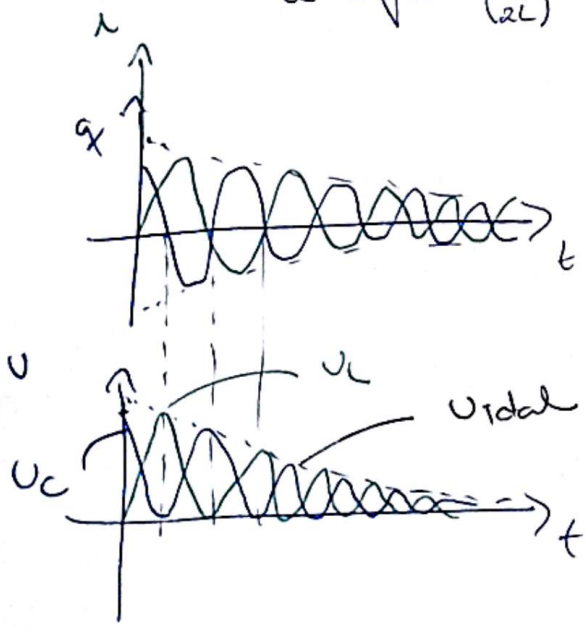
$$\Delta V_C - iR + \mathcal{E}_L = 0$$

$$\frac{q_c}{C} - \left(-\frac{dq_c}{dt} R\right) + L \frac{d^2 q_c}{dt^2} = 0$$

$$L \frac{d^2 q_c}{dt^2} + R \frac{dq_c}{dt} + \frac{q_c}{C} = 0$$

$$q_c = q_0 e^{-Rt/2L} \cos(\omega' t + \varphi)$$

$$\omega' = \sqrt{\omega^2 + \left(\frac{R}{2L}\right)^2} \qquad \omega = \frac{1}{\sqrt{LC}}$$



• Se a fonte fornecer energia suficiente para compensar a energia dissipada na resistência ⇒ as oscilações não são amortecidas.

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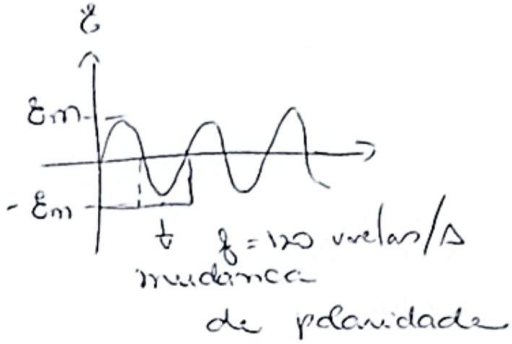
Corrente alternada



$$\mathcal{E}(t) = \mathcal{E}_m \operatorname{sen}(\omega t)$$



no Brasil

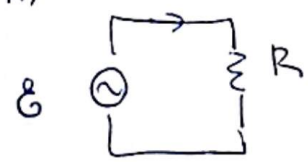
$f = 60\,Hz$ (mudança de polarização)

f

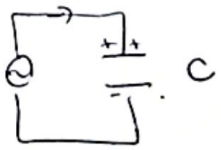
$$\cos\alpha = \operatorname{sen}(\alpha + 90^\circ)$$
$$-\cos\alpha = \operatorname{sen}(\alpha - 90^\circ)$$

• Alguns casos simples:

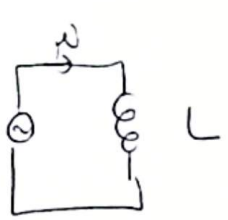
A)



B)

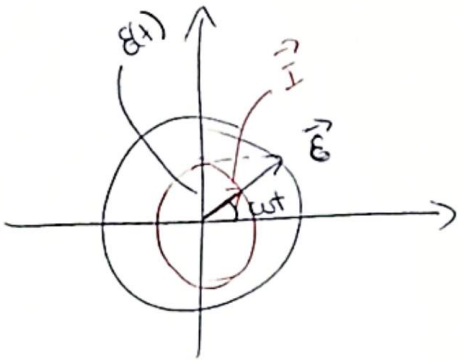


C)



$\mathcal{E}(t) = \mathcal{E}_m \operatorname{sen}(\omega t)$

$i(t) = ?$



A) $\mathcal{E} - iR = 0$

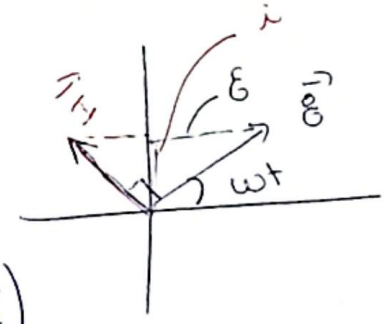
$$i(t) = \frac{\mathcal{E}_m}{R}\operatorname{sen}\omega t$$

B) $\mathcal{E} - \Delta V_C = 0$

$$\mathcal{E} - \frac{q_C}{C} = 0$$

$$q_C = C\mathcal{E}_m \operatorname{sen}\omega t$$

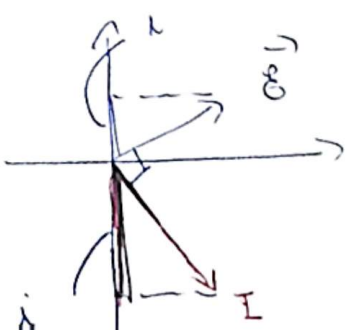
$$i = \frac{dq_C}{dt} = C\omega\mathcal{E}_m \cos\omega t = C\omega\mathcal{E}_m \operatorname{sen}\left(\omega t + \frac{\pi}{2}\right)$$



C) $\mathcal{E} + \mathcal{E}_L = 0$

$$\mathcal{E} - L\frac{di}{dt} = 0 \Rightarrow \int\frac{di}{dt} = \int\frac{\mathcal{E}_m}{L}\operatorname{sen}\omega t$$

$$i = -\frac{\mathcal{E}_m}{\omega L}\cos\omega t = \frac{\mathcal{E}_m}{\omega L}\operatorname{sen}(\omega t - 90)$$



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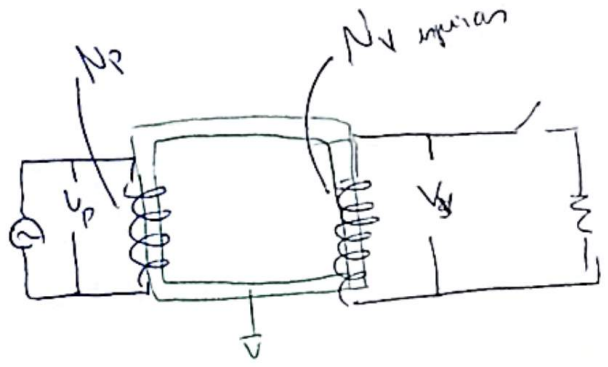
. O transformador ideal



$$\mathcal{E}_L = -L\frac{di}{dt} \qquad \mathcal{E}_L = N\frac{d\Phi_B}{dt}$$

$$\mathcal{E}_P = N_P\frac{d\Phi_B}{dt} \qquad \mathcal{E}_V = N_V\frac{d\Phi_B}{dt}$$

$$\frac{\mathcal{E}_P}{N_P} = \frac{\mathcal{E}_V}{N_V} \qquad \mathcal{E}_V = \frac{N_V}{N_P}\mathcal{E}_P$$

$N_V > N_P \Rightarrow$ eleva a tensão.

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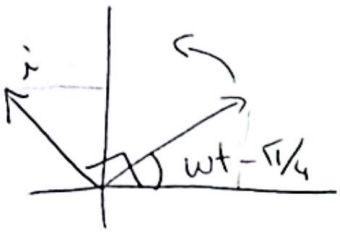
Exercícios Cap 31

33.

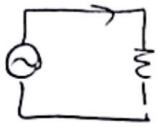
$$\mathcal{E} = \mathcal{E}_m \operatorname{sen}(\omega t - \pi/4)$$

$$i(t) = I \operatorname{sen}(\omega_d t - 3\pi/4)$$



$$\frac{\pi}{4} + \frac{\pi}{2} = \frac{3\pi}{4}$$

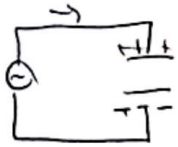
$$\cos(\omega t - \pi/4) = \operatorname{sen}(\omega t - \pi/4 + \pi/2)$$



$$\mathcal{E} - iR = 0$$

$$i = \frac{\mathcal{E}}{R}$$

(não se trata deste circuito)



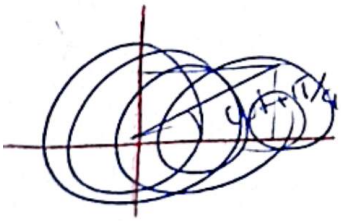
$$\mathcal{E} - \Delta V_C = 0 \qquad \Delta V_C = \frac{q}{C}$$

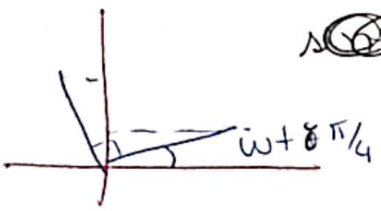
$$\mathcal{E} - \frac{q_C}{C} = 0$$

$$q_C = \mathcal{E}_m C \operatorname{sen}(\omega t + \pi/4)$$

$$i = \frac{dq}{dt} = \mathcal{E}_m C \omega \cos(\omega t + \pi/4)$$

$$\cos(\alpha) = \operatorname{sen}$$







$$\cos(\omega t \pm \pi/4) = \operatorname{sen}(\omega t - \pi/4 + \pi/2)$$

$$\mathcal{E} + \mathcal{E}_L = 0$$

$$\mathcal{E} - L\frac{di}{dt} = 0$$



$$\frac{di}{dt} = \frac{\mathcal{E}}{L} \Rightarrow i = \frac{\mathcal{E}_m}{L}\int \operatorname{sen}(\omega t \pm \pi/4)\,dt = -\frac{\mathcal{E}_m}{L\omega}\cos(\omega t - \pi/4)$$

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