Cap 29 → Campos magnéticos produzidos por corrente. (1)

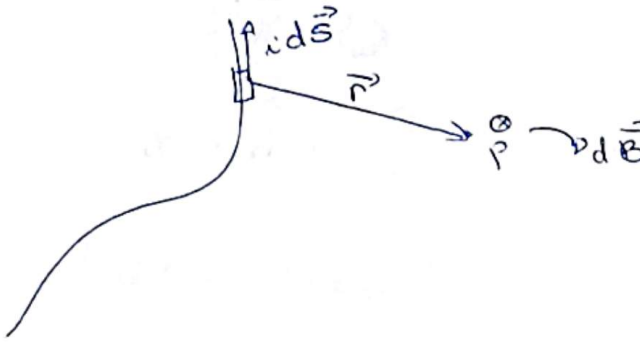
~1820 (Oersted)

- Lei de Biot-Savart

$$d\vec{B}_p = \frac{\mu_0}{4\pi} i \frac{d\vec{s} \times \hat{r}}{r^2}$$

μ_0 permeabilidade do vácuo.

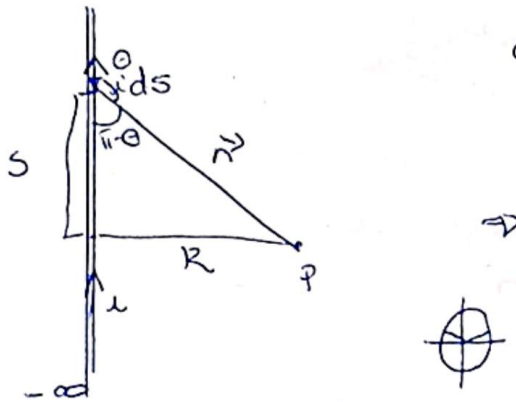
$\mu_0 = 4\pi \cdot 10^{-7}\ T \cdot m/A$



$\vec{r} = r\,\hat{r}$

↳ regra da mão direita

- Fio retilíneo: Fazer para L qualquer e depois fazer o limite



$$dB = \frac{\mu_0}{4\pi} i \frac{ds \sin\theta}{r^2}$$

$$\Rightarrow B = 2\int_0^{\infty} \frac{\mu_0 i}{4\pi} \frac{\sin\theta\, ds}{r^2}$$

$$\sin\theta = \sin(\pi - \theta) = \frac{R}{\sqrt{S^2 + R^2}}$$

$r = \sqrt{S^2 + R^2}$

$r^2 = S^2 + R^2$

$$\Rightarrow B = \frac{\mu_0 i}{2\pi} \int \frac{R}{\sqrt{S^2+R^2}} \cdot r^{-2}\, ds$$

$$B = \frac{\mu_0 i R}{2\pi} \int \frac{1}{(S^2+R^2)^{3/2}}\, ds$$

(Tabelada)

$$\int \frac{dx}{(x^2+a^2)^{3/2}} = \frac{x}{a^2(x^2+a^2)^{1/2}}$$

$$B = \frac{\mu_0 i R}{2\pi} \left[R^{-2} \frac{S}{(S^2+R^2)^{1/2}} \right]_0^{\infty} = \frac{\mu_0 i}{2\pi R} - 0$$

$$\left(\frac{L}{\left(\frac{L^2}{4}+R^2\right)^{1/2}}\right)^2 = \frac{L^2}{\frac{L^2}{4}+R^2}$$

$$\left(\frac{1}{\frac{L}{4}+R^2}\right)^{1/2} = \frac{1}{\left(\frac{1}{4}+\frac{R^2}{L^2}\right)}$$

$$B = \frac{\mu_0 i}{2\pi} \frac{S}{(S^2+R^2)^{1/2}} \Big|_0^{L/2} = \frac{\mu_0 i L}{4\pi} \left(\frac{L^2}{4}+R^2\right)^{-1/2}$$

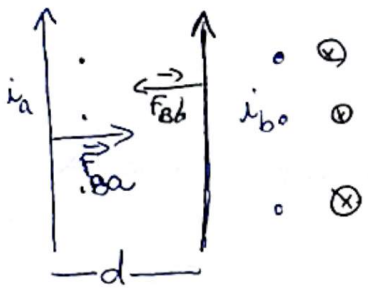
↳ Tentem calcular B no centro de uma circunferência

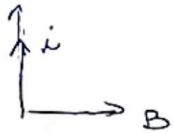
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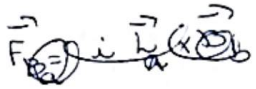
$$\vec{B}_R = \vec{B}_1 + \vec{B}_2 + \vec{B}_3$$



- Duas correntes paralelas







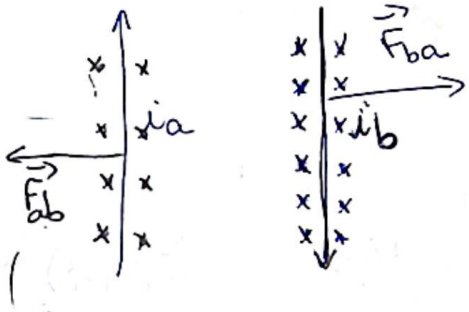
$$\vec{F}_{ba} = i\vec{L}_a \times \vec{B}_a$$

$$\vec{F}_{ab} = i_a\vec{L}_a \times \vec{B}_b$$

$$\vec{B}_b = \frac{\mu_0 i_b}{2\pi d}$$

$$F_{ab} = i_a L_a \frac{\mu i_b}{2\pi d} \Rightarrow F_{ab} = i_a i_b \frac{\mu_0}{2\pi d}$$

- Correntes paralelas se atraem



- Correntes antiparalelas se repelem.

Fem "a" devido à "b"

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Lei de Ampère

$\oint \vec{B} \cdot d\vec{A} = ?$

Vimos:

$$\oint \vec{E}\, d\vec{A} = \frac{q_{env}}{\varepsilon_0}$$

(Lei de Gauss)

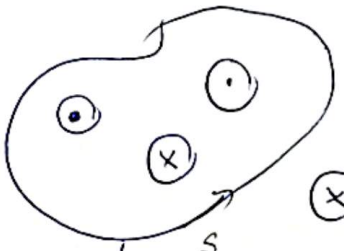


superfície gaussiana (fechada)

Lei de Ampère

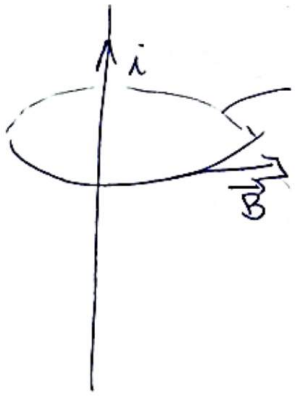
$$\oint \vec{B} \cdot d\vec{S} = \mu_0 i_{env}$$

$d\vec{S}$ → sentido da integração (tangente à curva)



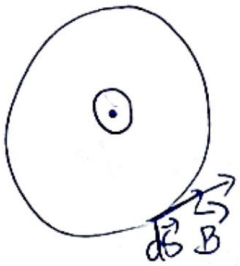
~~superfície~~ curva amperiana (fechada) caminho fechado

O campo magnético em torno de um fio longo:



caminho fechado

$$\int \vec{B} \cdot d\vec{S} = \mu\, i_{env}$$



$$\int \vec{B} \cdot d\vec{S} = \mu_0 i$$

$$\int B \cdot ds \cos\theta = \mu_0 i$$

$$B\, 2\pi r = \mu_0 i \Rightarrow$$

$$B = \frac{\mu_0 i}{2\pi r}$$

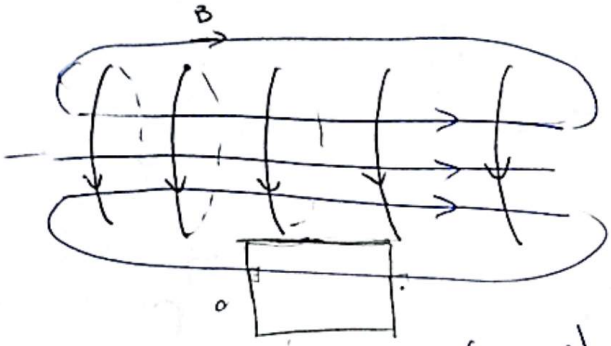
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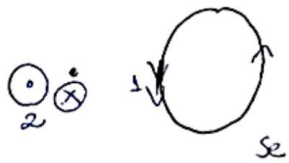
Solenoide

- espira
- bobina plana (N espiras)
- bobina longa (solenoide)



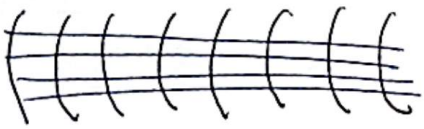
comprimento >> diâmetro





se $da \to 0$
$\Rightarrow B_1 \sim B_2$
$B_f \sim 0$

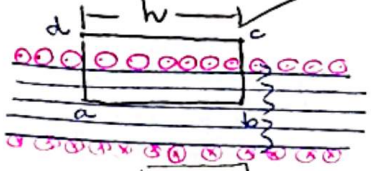
Solenoide ideal (muito longo)

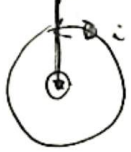


Campo interno apenas

Qual o campo dentro do solenoide?

→ Lei de Ampère — curva amperiana





De fora

resolver a integral só para justificar $B_{fora} = 0$

$$\oint \vec{B} \cdot d\vec{s} = \mu_0 i$$

$$\int_a^b \vec{B}\, d\vec{s}_1 + \int_b^c \vec{B}\, d\vec{s}_2 + \int_c^d \vec{B} \cdot d\vec{s}_3 + \int_c^a \vec{B}\, ds_4 = \mu_0 i$$

$$B \cdot h + 0 + 0 + 0 = \mu_0 i_{env}$$

$$i_{env} = \#\, i = n h i$$

n = # espiras / comprimento

$\Rightarrow$ # espiras $= n \cdot h$

$$B \not{h} = \mu_0 \not{n} \not{h} i$$

$$B = \mu_0 \cdot i \cdot n$$

$$B = \mu_0 \frac{N}{h} \cdot i$$

$$B \cdot h = \mu_0 i N$$

$$B = \mu_0 i \left(\frac{N}{h}\right)$$

espiras / comprimento

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Campo magnético no centro de uma espira

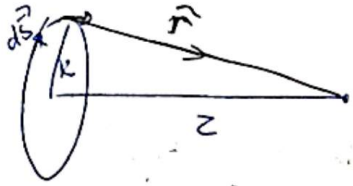


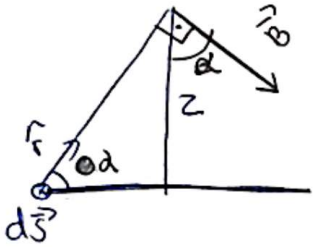
$$dB = \frac{\mu_0 i}{4\pi} \frac{d\vec{s} \times \hat{r}}{r^2}$$

$$dB = \frac{\mu_0 i}{4\pi} \frac{ds}{R^2}$$

$$B = \frac{\mu_0 i \, 2\pi R}{4\pi R^2} = \frac{\mu_0 i}{2R}$$

2) No eixo de simetria:





observe $|d\vec{s} \times \hat{r}| = ds \sin 90°$

$$dB = \frac{\mu_0 i \, ds}{4\pi r^2}$$

$$dB_{eixo} = dB \cos\alpha \qquad \cos\alpha = \frac{R}{r}$$

$$= \int \frac{\mu_0 i R \, ds}{4\pi r^3}$$

$$= \frac{\mu_0 i R}{4\pi r^3} 2\pi R = \frac{\mu_0 i R^2}{2 r^3}$$

mas $r = (z^2 + R^2)^{1/2}$

$$B_{eixo} = \frac{\mu_0 i R^2}{2(z^2 + R^2)^{3/2}}$$

Se $z = 0$

$$\Rightarrow B_{centro} = \frac{\mu_0 i R^2}{2(\quad R}$$

Para $z \gg R$

$$B = \frac{\mu_0 i R^2}{2 z^3}$$

$A = \pi R^2$

$\Rightarrow R^2 = \frac{A}{\pi}$

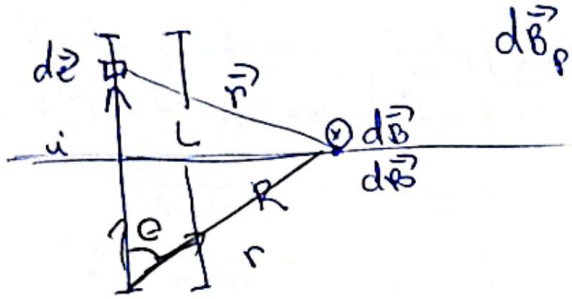
$$B = \frac{\mu_0 i A}{2\pi z^3}$$

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EXEMPLOS: Cap 29

Fio Retilíneo de comprimento L

Sabe-se i, R, L



$$d\vec{B}_p = \frac{\mu_0}{4\pi} i \frac{d\vec{\ell} \times \hat{r}}{r^2}$$



$$\sin\theta = \frac{R}{r} = \frac{L}{r}$$

$$B_p = \int_0^L \frac{\mu_0 i}{4\pi} \frac{d\ell}{r^2} \cdot \sin\theta$$

$$= 2\int_0^L \frac{\mu i}{4\pi} d\ell \cdot \frac{L}{r^3}$$

$$B_2 = 2B \int_0^{L/2}$$

$$r = \sqrt{\ell^2 + R^2}$$

$$B_p = \frac{\mu i R}{2\pi} \int \frac{1}{r^3} d\ell$$

$$B_p = \frac{\mu_0 i R}{2\pi} \int_0^{L/2} \frac{1}{(\ell^2 + R^2)^{3/2}} d\ell .$$

Tabulada.

$$\int \frac{dx}{(x^2+a^2)^{3/2}} = \frac{x}{a^2 (x^2+a^2)^{1/2}}$$

$$B_p = \frac{\mu_0 i R}{2\pi} \frac{\ell}{R^2 (\ell^2 + R^2)^{1/2}} \Bigg|_a^{L/2}$$

$$= \frac{\mu_0 i R}{2\pi} \frac{L}{2R^2 (L^2/4 + R^2)^{1/2}}$$

$$= \frac{\mu_0 i L^2 R}{4\pi R^2 (L^2/4 + R^2)^{1/2}}$$

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$$\left(\frac{L^2}{4} + R^2\right)^{1/2} = \left[L^2\left(\frac{1}{4} + \frac{R^2}{L^2}\right)\right]^{1/2}$$

$$\left(\frac{L^2}{4} + R^2\right) = L\left(\frac{1}{4} + \frac{R^2}{L^2}\right)^{1/2}$$

$$B_p = \frac{\mu_0 i R}{4\pi R^2\left(\frac{1}{4} + \frac{R^2}{L^2}\right)^{1/2}}$$

Se $L \to \infty$

Fio infinito: se $L \to \infty$

$$\Rightarrow B_p = \frac{\mu_0 i R}{\pi R^2} \Rightarrow \boxed{B_p = \frac{\mu_0 i}{\pi R}}$$

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