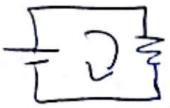
Cap 27: Circuitos

Vimos:
- A soma das variações de potencial em uma malha (fechada) é igual a zero (!) → Regra das malhas.



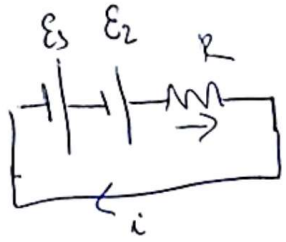
$-\Delta V_c + \Delta V_f = 0$



$-\Delta V_R + \Delta V_f = 0$

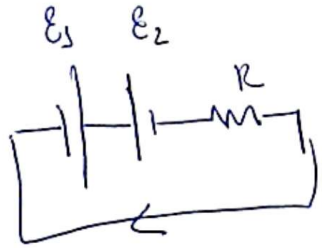
Valida em qualquer circuito.

- $P = i\,\Delta V$ → qualquer dispositivo que permite i



$\varepsilon_1 + \varepsilon_2 - iR = 0$

$+iR - \varepsilon_2 - \varepsilon_1 = 0$



$\varepsilon_1 - \varepsilon_2 - iR = 0$

$+iR + \varepsilon_2 - \varepsilon_1 = 0$

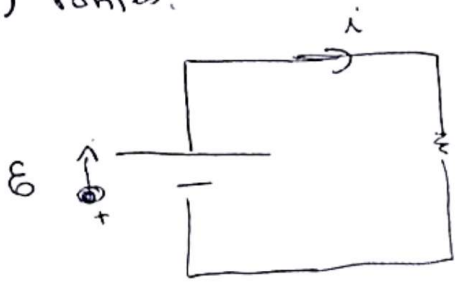
2, 4, 10, 12, 17, 23, 24, 28, 29, 30, 31, 32, 36, 45, 50, 51, 54

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Cap 27: Circuitos:

1) Fontes
2) Resistencia equivalente
3) Amperímetro Voltímetro e aterramento
4) Multiplas malhas
5) Circuito RC

1) Fontes:



É necessário um trabalho sobre os portadores de carga para que a corrente seja mantida (vá para o potencial maior)

• A Energia associada a este transporte pode ser escrita como

$\Delta U = q\,\Delta V$

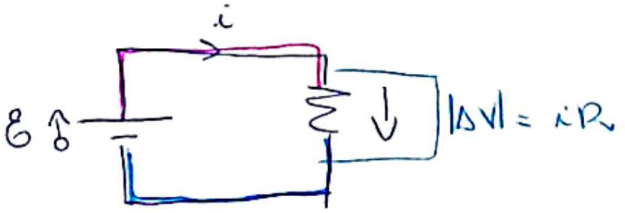
$E_{Transp} = q\,\underbrace{\Delta V}_{\mathcal{E}} = E_{Transp} = q\,\mathcal{E} = W_{trans.}$

↳ força eletromotriz

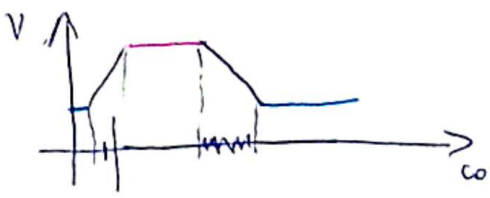
$P = \frac{W}{\Delta t} = \left(\frac{q}{\Delta t}\right)_{i}\mathcal{E} = i\,\underbrace{\mathcal{E}}_{\text{"}\Delta V\text{"}}$

Potência

• Se a fonte é ideal $\Rightarrow \mathcal{E} = \Delta V$

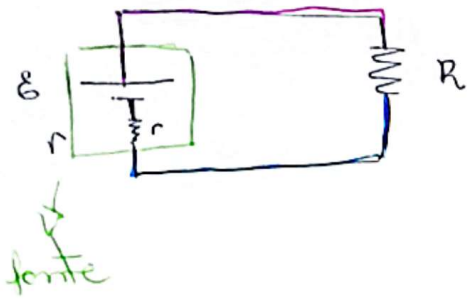


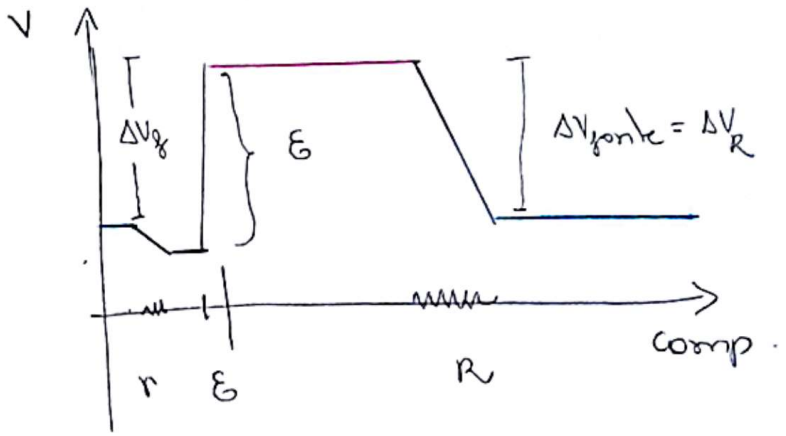
$+\mathcal{E} - iR = 0 \Rightarrow \mathcal{E} = iR.$



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• Se a fonte é real (não ideal)

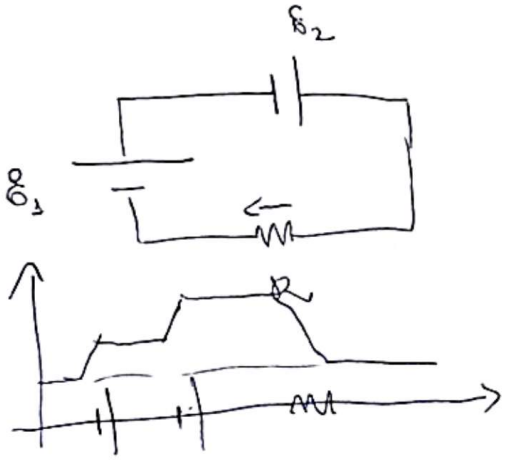




$$-|\Delta V_r| + \varepsilon + |\Delta V_R| = 0$$

$$-ir + \varepsilon - iR = 0$$

↝ Duas ou mais fontes:

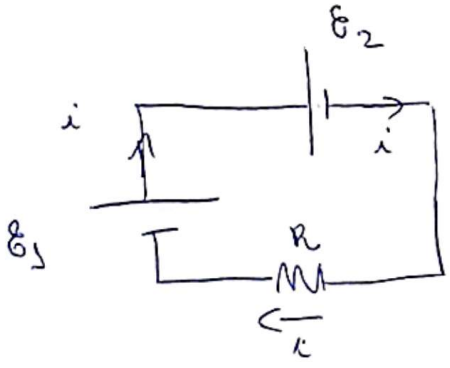


$$+|\varepsilon_1| + |\varepsilon_2| - |iR| = 0$$

As duas fontes doam energia para o sistema

($P_{doada} = P_{absorvida}$)

conservação de energia.



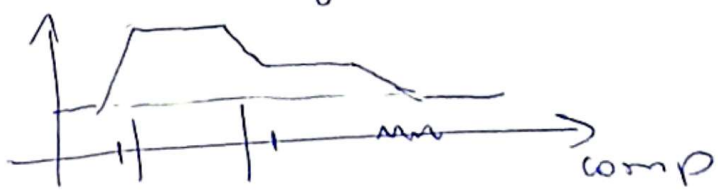
• $\varepsilon_1 > \varepsilon_2 \Rightarrow \varepsilon_1$ impõe o sentido da corrente.

$$|\varepsilon_1| - |\varepsilon_2| - |\Delta V_R| = 0$$

$$\varepsilon_1 - \varepsilon_2 - iR = 0$$

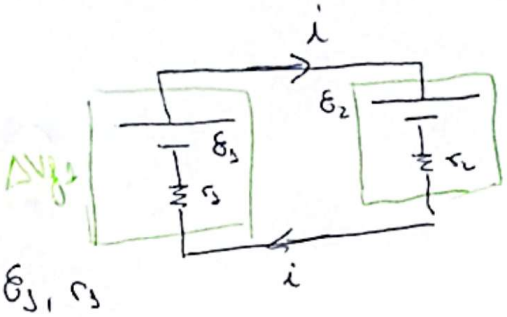
ε_1: doa; ε_2: absorve/retira; R: absorve/retira.

⇒ ε_2 funciona como um carregador sendo carregado.



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→ Duas fontes não ideais:



$|\Delta V_1| = |\Delta V_2|$

A direção da corrente será dada pela fonte com maior força eletromotriz:

. se $\varepsilon_1 > \varepsilon_2$:

$$\underbrace{-|\Delta V_{r_1}| + |\varepsilon_1|}_{\Delta V_1} \underbrace{- |\varepsilon_2| - |\Delta V_{r_2}|}_{\Delta V_2} = 0$$

$\Delta V_{g1} = \varepsilon_1 - i r_1$ (doada)

Atenção!

$\Delta V_{g2} = \varepsilon_2 + i r_2$ (retirada)

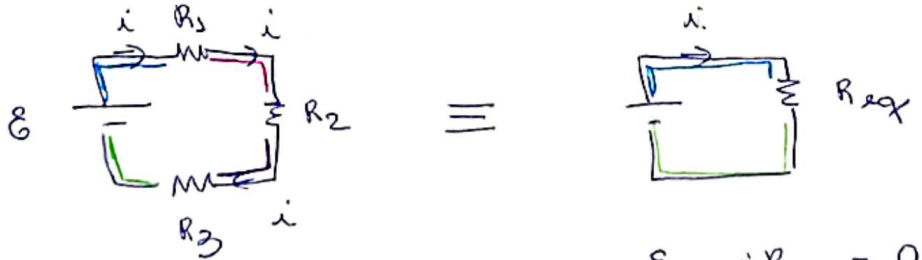
. se $\varepsilon_1 < \varepsilon_2 \Rightarrow$ a corrente é invertida

e $\Delta V_{g1} = \varepsilon_1 + ir$ (retirada)

$\Delta V_{g2} = \varepsilon_2 - ir$ (doada)

2) Resistência equivalente:

→ Resistências em série:



$i_{eq} = i_1 = i_2 = i_3$

$\Delta V_{eq} = \Delta V_1 + \Delta V_2 + \Delta V_3$

$\varepsilon - iR_1 - iR_2 - iR_3 = 0$ (1)

$\varepsilon - iR_{eq} = 0$ (2)

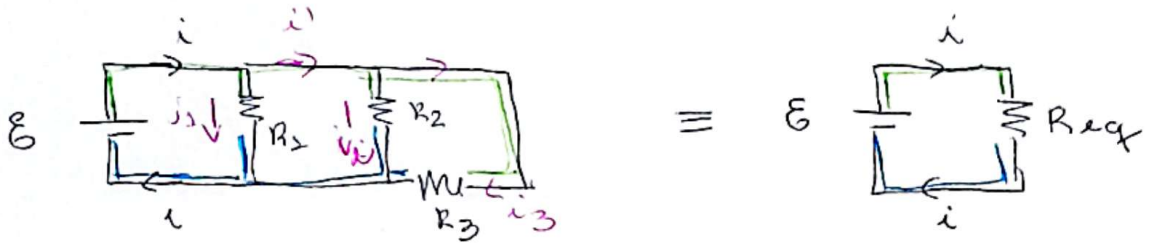
(2) → (1) $\Rightarrow$ $iR_{eq} = iR_1 + iR_2 + iR_3$

$$\Rightarrow \boxed{R_{eq} = R_1 + R_2 + R_3}$$

Generalizando: $R_{eq} = \sum_{i=1}^{n} R_n$

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N Resistências em paralelo



$i' = i_2 + i_3$

$i = i_1 + i'$

$\Rightarrow$

$i = i_1 + i_2 + i_3 \quad (1)$

$\varepsilon - iR_{eq} = 0$

$i = \frac{\varepsilon}{R_{eq}} \quad (2)$



$\Rightarrow |\Delta V_1| = |\Delta V_2| = |\Delta V_3|$

$i_1 R_1 = i_2 R_2 = i_3 R_3 = \varepsilon$

$i_1 = \frac{\varepsilon}{R_1} \quad i_2 = \frac{\varepsilon}{R_2} \quad i_3 = \frac{\varepsilon}{R_3} \quad (3)$

(2) e (3) $\Rightarrow$ (1):

$$\frac{\varepsilon}{R_{eq}} = \frac{\varepsilon}{R_1} + \frac{\varepsilon}{R_2} + \frac{\varepsilon}{R_3} \Rightarrow \boxed{\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}}$$

Generalizando:

$$\frac{1}{R_{eq}} = \sum_{i=1}^{N} \frac{1}{R_i}$$

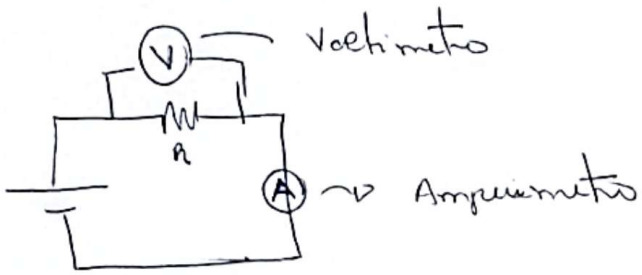
- $\Delta V_1 = \Delta V_i$
- $i = \sum_{i=1}^{N} i_i$

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3) Amperímetro + Voltímetro - aterramento

↓ medidor de corrente

↓ medidor de ddp



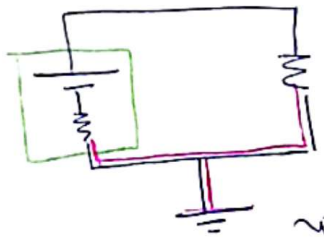
⇒ Para que o Amperímetro e o Voltímetro não interfira no sistema

é necessário que

$r_{Amp} \to 0$ $\quad (r_{Amp} \ll R)$

$r_{Voltim} \to \infty$ $\quad (r_{Vol} \gg R)$

Aterramento:



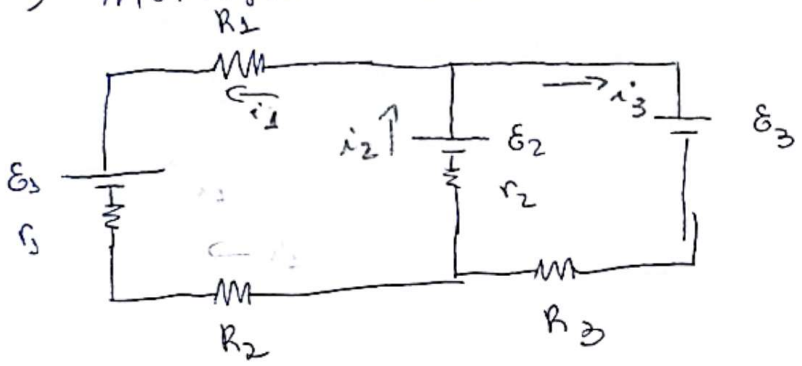
↳ Aterramento

$V = V_{Terra} = 0$ (referência)

Funciona como um referencial.

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4) Multiplas Malhas



Considere

$R_1 = R_2 = R_3 = 2\Omega$

$r_1 = r_2 = 1\Omega$

$\mathcal{E}_1 = 12V$; $\mathcal{E}_2 = 8V$; $\mathcal{E}_3 = 4V$

$i_1 = 5A$

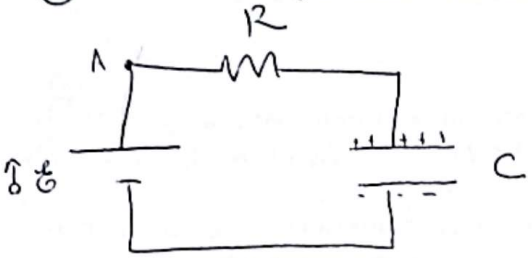
$i_2 = 2A$

$i_3 = 3A$.

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Circuito RC

① Capacitor descarregado + fonte + Resistência



Regra das malhas:

$$-iR - V_C + \mathcal{E} = 0$$

$$C = \frac{q_C}{V_C} \Rightarrow V_C = \frac{q_C}{C}$$

$$-iR - \frac{q}{C} + \mathcal{E} = 0$$

$q = q_A = q_R = q_C$

mas $i = \frac{dq}{dt} = \frac{dq_R}{dt}$

$$\Rightarrow R\frac{dq}{dt} + \frac{q}{C} = \mathcal{E} \quad (\text{eq. dif. 1ª ordem})$$

$\frac{dx}{dt} + ax = b \; , \; a \neq 0$

$x(t) = \frac{b}{a} + Ke^{-at}$

$$\frac{dq}{dt} + \frac{q}{RC} = \frac{\mathcal{E}}{R} \quad (1)$$

$a = \frac{1}{RC}$

$\frac{\mathcal{E}}{R} \cdot RC$

$\mathcal{E}C$

solução: $q(t) = q_P + Ke^{-at}$ (2)

$C = \frac{q}{\Delta V}$

- solução particular: q_P: quando o capacitor carregar, teremos que: $\frac{dq}{dt} = 0$

$\Rightarrow$ voltando em (1)

$$0 + \frac{q_P}{RC} = \frac{\mathcal{E}}{R} \Rightarrow \boxed{q_P = \mathcal{E}C} \quad (3)$$

- Condição inicial: $q(t=0) = 0$ (para descobrir K)

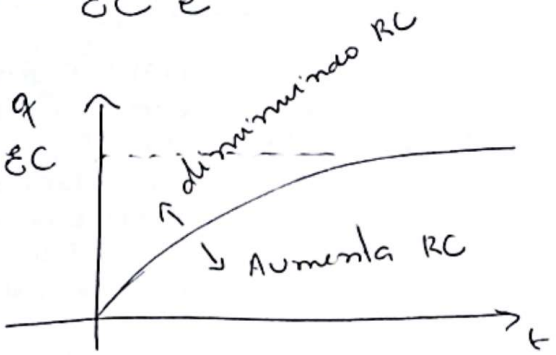
De (2)

$$0 = \mathcal{E}C + K \Rightarrow \boxed{K = -\mathcal{E}C} \quad (4)$$

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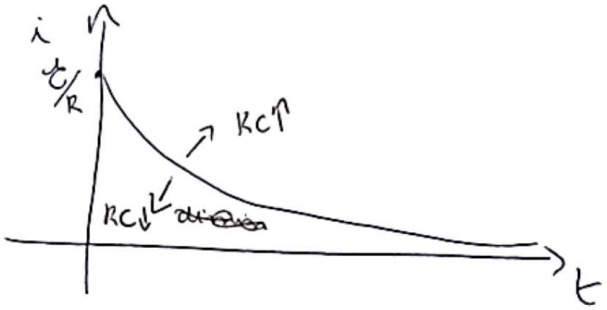
substituindo (3) e (4) em (2):

$$q(t) = \mathcal{E}C - \mathcal{E}C\, e^{-\frac{t}{RC}}$$



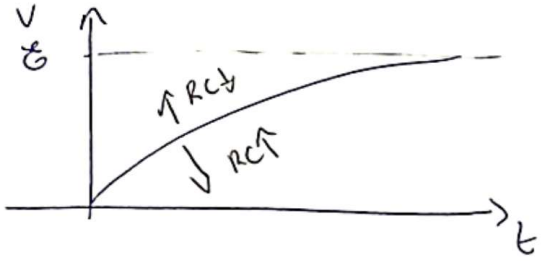
Quanto menor o valor de RC + rapido a curva chega no valor do ∞

$$i = \frac{dq}{dt} = +\frac{\mathcal{E}C}{RC}\, e^{-t/RC} = \frac{\mathcal{E}}{R}\, e^{-t/RC}$$



O Potencial:

$$C = \frac{q}{V_C} \Rightarrow V_C = \frac{q}{C} = \mathcal{E} - \mathcal{E}e^{-t/RC}$$



RC → constante de tempo

$[RC] = [t]$

Se $t = RC \Rightarrow q = \underbrace{\mathcal{E}C}_{q_{max}}(1 - e^{-1})$

$= \mathcal{E}C\left(1 - \frac{1}{2{,}72}\right) \approx 0{,}63\,\mathcal{E}C$

$\sim 0{,}37$

63% da carga máxima.

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② Capacitor carregado + ~~Bateria~~ + Resistência



$+iR - V_C = 0$

$\frac{dq_R}{dt} R - \frac{q_C}{C} = 0$

$C = \frac{q}{V_C}$

mas $q_R = q_0 - q_C$

$\frac{dq_R}{dt} - \frac{q_0}{RC} = 0$ $\Rightarrow \boxed{\frac{dq_R}{dt} = -\frac{dq_C}{dt}}$

$\rightarrow \frac{dq}{dt} + \frac{dq_C}{RC} = 0$

(antes tinhamos:

$\frac{dq}{dt} + \frac{q}{RC} = \frac{\mathcal{E}}{R}$)

$q(t) = q_P + K e^{-at}$ $\qquad a = -\frac{1}{RC}$

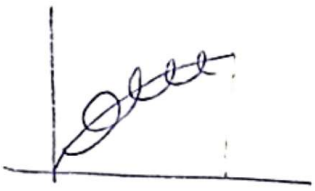
solução particular: $\frac{dq}{dt} = 0$ $\qquad q_P = 0$

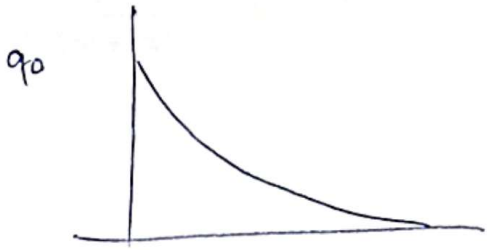
$q(t) = K e^{-\frac{t}{RC}}$

cond. inicial: $q(t=0) = q_0$

$\Rightarrow q(t=0) = K = q_0$

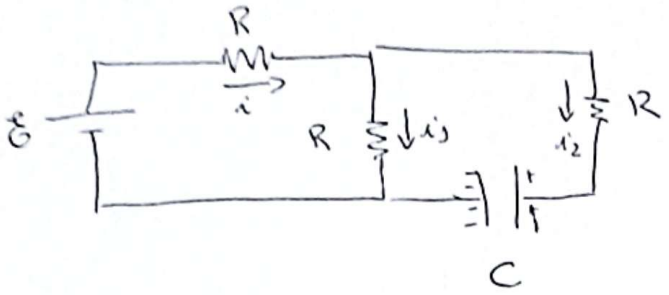
$q(t) = q_0 e^{-t/RC}$





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Exemplo, sala de aula: $q_C(1) = ?$



$i = i_1 + i_2$ (1)

$-iR - i_1R + \mathcal{E} = 0$ (2)

$-iR - i_2R - V_C + \mathcal{E} = 0$ (3)

De (1) $i_1 = i - i_2$ (4)

(4) em (3):

$-iR - (i - i_2)R + \mathcal{E} = 0$

$-2iR + i_2R + \mathcal{E} = 0 \Rightarrow +iR = \dfrac{i_2R + \mathcal{E}}{2}$ (5)

(5) em (3)

$-\dfrac{i_2R}{2} - \dfrac{\mathcal{E}}{2} - i_2R - V_C + \mathcal{E} = 0$

$-\dfrac{3}{2}i_2R + \dfrac{\mathcal{E}}{2} - V_C = 0 \Rightarrow i_2\dfrac{3}{2}R + \dfrac{q}{C} = \dfrac{\mathcal{E}}{2}$

$i_2 + \dfrac{2}{3RC}q = \dfrac{\mathcal{E}}{3R}$

$\underbrace{\dfrac{dq}{dt} + \dfrac{2}{3RC}q}_{\text{parte Homogênea}} = \dfrac{\mathcal{E}}{3R}$ (6)

$\dfrac{dq'}{dt} + \dfrac{2}{3RC}q' = 0$

$q' = ke^{-\frac{2t}{3RC}} \Rightarrow q(t) = q_P + ke^{-2t/3RC}$ (7)

A) $t \to \infty \quad \dfrac{dq}{dt} = 0 \quad q = q_{max}$

B) $t = 0 \quad q = 0$

Usando A: $\dfrac{2}{3RC}q_{max} = \dfrac{\mathcal{E}}{3R} \Rightarrow q_{max} = \dfrac{\mathcal{E}C}{2}$

de (7) $q_{max} = q_P$

(VIDE)

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Impondo B: $q(t=0) = 0 \Rightarrow K = -\frac{\mathcal{E}C}{2}$

$\Rightarrow$ Finalmente:

$$q(t) = \frac{\mathcal{E}C}{2} - \frac{\mathcal{E}C}{2} e^{-t/RC}$$

$$\frac{dx}{dt} + ax = b$$

$$x(t) = \frac{b}{a} + Ke^{-ta}$$

Discutir exercício 65

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