Cap 24: Potencial elétrico

F eletrostática (coulomb) é conservativa

$$W_F = -\Delta U = -(U_f - U_i)$$

↳ energia Potencial

⇒ não depende da trajetória, apenas dos pontos i e f (da distância entre eles)

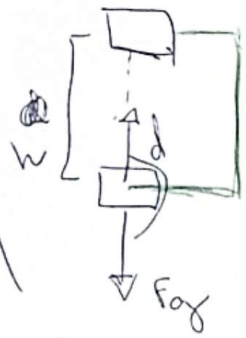
Lembrando: Força gravitacional

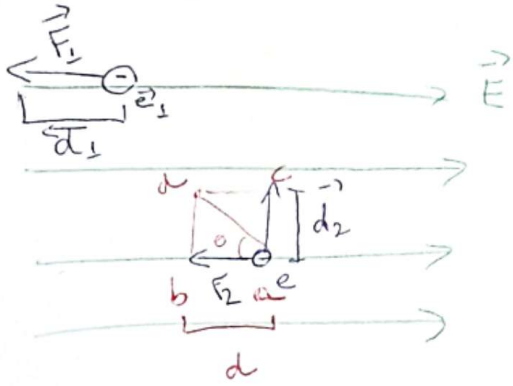
$F_g = mg$

$W_g = \vec{F} \cdot \vec{d} = \int \vec{F}\, d\vec{s}$

$W_g = -mgh = -\Delta U_g$

$\Rightarrow \Delta U_g = mgh$





O trabalho realizado pela força eletrostática:

$W_{F_1} = \vec{F}_1 \cdot \vec{d}_1 \cos\theta = F_1 d_1 \cos 0 = F_1 d_1$

$W_{F_2} = \vec{F}_2 \cdot \vec{d}_2 \cos\theta = F_2 d_2 \cos 90 = 0$

$W_{F_3} = |q\vec{E}| \cdot d_1 = qEd$

$W_{a \to d} = \vec{F} \cdot \vec{d}_d = F \cdot d_d \cos\theta = F \cdot d = W_{a-b}$

$\Delta U = -W_F = -qEd$

⇒ energia potencial depende da carga

$[U] = J$ Joule

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Potencial Elétrico

→ Uma medida que dependa apenas do campo $\vec{E}$

$$V = \frac{U}{q} \Rightarrow \Delta V = \frac{\Delta U}{q} \qquad [V] = \frac{J}{C} = V \text{ (volt)}$$

ΔV ↓ diferença de potencial

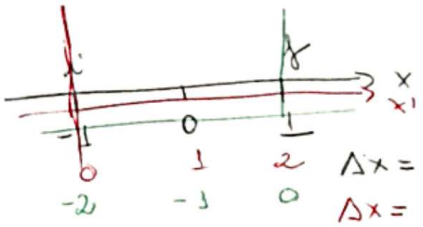
$$\Delta V = V_f - V_i = \frac{\Delta U}{q}$$

$$\Rightarrow \Delta V = -\frac{W_{i \to f}}{q}$$

$$\Delta V = V_f - V_i$$

. Definir um referencial p/ $V = 0$





$\Delta x = 1 - (-1) = 2$

$\Delta x = 2$

$\Delta x = 0 - (-2) = 2$

~~$\Delta V = V_f - V_i$~~

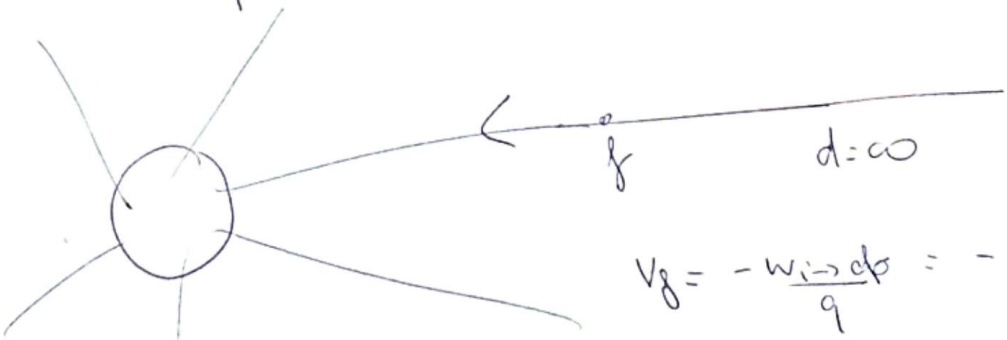
$V_i = 0$

$\Delta V = V_f - V_i$

$V_f = \Delta V + V_i$

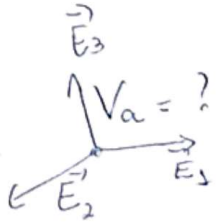
$\frac{\Delta V}{q} + 0$

$$\Rightarrow V_f = -\frac{W_{i \to f}}{q}$$



$$V_f = -\frac{W_{i \to df}}{q} = -\frac{W_{\infty}}{q}$$

Muitas cargas



V é escalar

$\Rightarrow V_{total} = V_1 + V_2 + V_3$

(soma algébrica)

1 +

2 −

3 +

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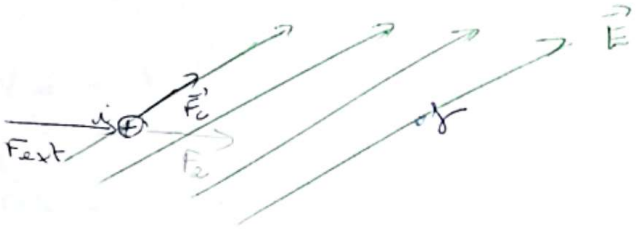
• Teorema Trabalho - Energia (depois)

$\Delta K = W_{total} = W_{F_1} + W_{F_2} + W_{F_3} + \dots$

↓

Trabalho realizado pelas forças

Considere:



$W_{total} = W_{F_1} + W_{F_2} + W_{F_3}$

Se $v_i = v_f = 0 \Rightarrow \Delta K = 0$

$W_{total} = W_{ext} + W_c$

$\Delta K = W_{ext} - q\Delta V$

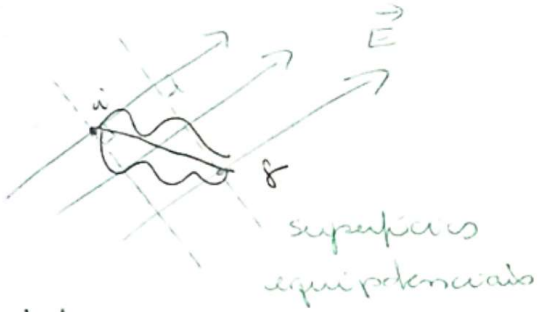
0

$\Rightarrow W_{ext} = q\Delta V$

$\Delta V = \frac{\Delta U}{q} = -\frac{W_c}{q}$

& falar junto com energia do sistema de partículas

• Superfícies equipotenciais



$W_{i \to f}$ não depende do caminho



$W_{a \to b} = W_{a \to d}$

$W_{b \to c} = 0$

$W_{a \to b \to c \to d} = W_{a \to d}$

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Potencial e campo

$$\Delta V = \frac{\Delta U}{q_0} = -\frac{W}{q_0} = -\frac{\vec{F}\cdot d\vec{s}}{q_0} = -q_0\frac{\vec{E}\cdot\vec{d}}{q_0} = \vec{E}\cdot\vec{d}$$

$$= -\int \frac{\vec{F}\,d\vec{s}}{q_0} = -\frac{q_0}{q_0}\int \vec{E}\,d\vec{s}$$

no limite $\Delta V \to 0$

$$dV = -\frac{\vec{F}\,d\vec{s}}{q} = -\vec{E}\,d\vec{s}$$

$$\int_i^f dV = -\int_i^f \vec{E}\,d\vec{s}$$

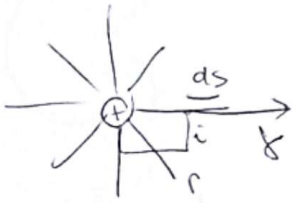
$$V_f - V_i = \Delta V = -\int_i^f \vec{E}\cdot d\vec{s}$$ (não)

$$\Delta V = \frac{\Delta U}{q_0} = -\frac{W}{q_0} = -\int \frac{\vec{F}\,d\vec{s}}{q_0}$$

$$= -\int \frac{q_0\vec{E}}{q_0}\,d\vec{s}$$

$$\Delta V = -\int \vec{E}\,d\vec{s}$$

Ex: Potencial produzido por uma carga pontual



$$V_f - V_i = -\int_i^f \vec{E}\cdot d\vec{s} \qquad ds = dr$$

$$= -\int_i^f E\,ds$$

$$= -\int \frac{kq}{r^2}\,dr$$

$$V_f - V_i = kq\int_i^f r^{-2}\,dr = -\underset{(-1)}{\frac{kq}{r}} = \frac{kq}{r}\Big|_i^f$$

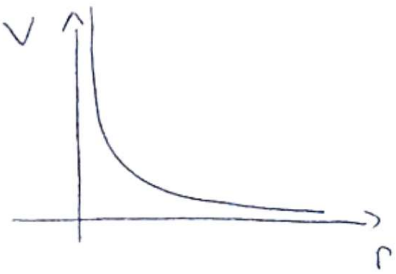
$$V_f - V_i = kq\left[\frac{1}{r_f} - \frac{1}{r_i}\right]$$

se $V(r=\infty) = 0$, vamos considerar $r_f = \infty$

$\Rightarrow V(r_f) = 0$

$$-V_i = kq\left[0 - \frac{1}{r}\right]$$

$$\Rightarrow \boxed{V_i = \frac{kq}{r}}$$



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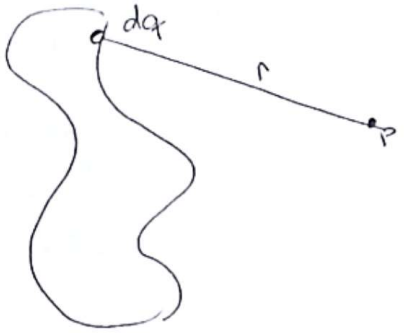
Potencial Produzido por uma distribuição de cargas.

Vimos que para uma carga pontual

- $V = \frac{kq}{r}$ $\left(\text{referencial } V_{\infty} = 0\right)$

- $V_T = V_1 + V_2 + V_3 \ldots$

Para um contínuo



$$dV = \frac{k}{r}\,dq$$

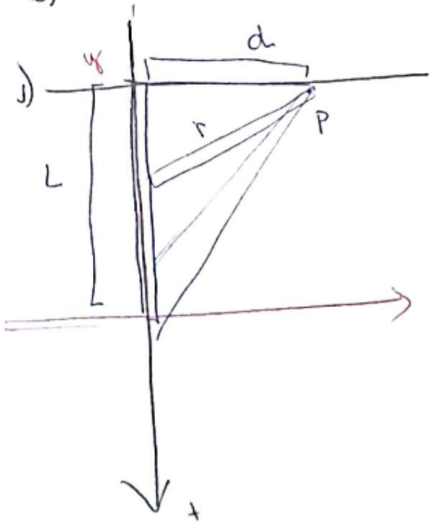
$$\int dV = k\int \frac{1}{r}\,dq \quad \text{(escalar!)}$$

Exemplos:

1) Haste carregada

2) Disco carregado

1)



d, L, λ

$$V = k\int \frac{1}{r}\,dq$$

$$\frac{q}{L} = \lambda = \frac{dq}{dy}$$

$$\Rightarrow dq = \lambda\,dy$$

$$r^2 = d^2 + (L-y)$$

$$r^2 = d^2 + y^2$$

$$r = (d^2 + y^2)^{1/2}$$

$$V = k\int \frac{1}{r}\lambda\,dy$$

$$= k\int_0^L \frac{1}{(d^2+y^2)^{1/2}}\,dy \quad \text{(tabulada)}$$

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$$\int \frac{dx}{(x^2+a^2)^{1/2}} = \ln\left(x+\sqrt{x^2+a^2}\right)$$

$\ln(a)+\ln b$

$\ln ab$

$$\Rightarrow V = k \ln\left(y+\sqrt{d^2+y^2}\right)\Big|_0^L$$

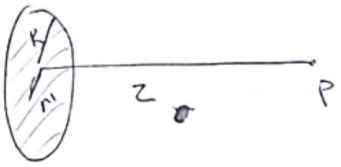
$$V = k\left[\ln\left(L+\sqrt{d^2+L^2}\right) - \ln\left(0+\sqrt{d^2}\right)\right]$$

$$= k\left(\ln\left(L+\sqrt{d^2+L^2}\right) - \ln d\right)$$

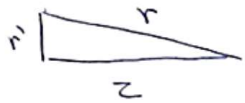
$$= k \ln\left(\frac{L+\sqrt{d^2+L^2}}{d}\right)$$

- Disco carregado

σ, R



$$V = k\int \frac{1}{r}\,dq$$



$$\sigma = \frac{q}{2\pi R} = \frac{dq}{2\pi r' dr'}$$

$$dq = \sigma 2\pi r' dr'$$

$$r = \sqrt{r'^2+z^2}$$

$$\Rightarrow V = k\sigma 2\pi \int_0^R \frac{r'\,dr'}{\sqrt{r'^2+z^2}}$$

(tabelada)

$$\int \frac{y\,dx}{(y^2+a^2)^{1/2}} = \sqrt{y^2+a^2}$$

$$x = r'^2 + z^2$$

$$dx = 2r'dr' \Rightarrow r'dr' = \frac{1}{2}dx$$

$$x(r'=0) = z^2$$

$$x(r'=R) = R^2+z^2$$

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24.7

$$V = k\sigma 2\pi \frac{1}{2} \int_{z^2}^{R^2+z^2} \frac{1}{x^{1/2}} dx = k\sigma\pi 2 \sqrt{x} \Big|_{z^2}^{R^2+z^2}$$

$$V = \frac{1}{2k_0}\sigma \left(\sqrt{R^2+z^2} - z\right) //$$

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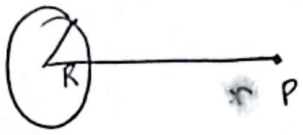
Potencial em uma casca esférica



Considerando $V_\infty = 0$

→ Fora

$$V_\infty - V_P = -\int_P^\infty \vec{E}\, d\vec{s} \qquad ds = dr$$

$$0 - V_P = -\int_{r_P}^{r_\infty} \frac{q}{4\pi\varepsilon_0 r^2}\, dr$$

$$V_P = \frac{-q}{4\pi\varepsilon_0}\left.\frac{1}{r}\right|_{r_P}^{r_\infty} = \frac{q}{4\pi\varepsilon_0}\left[\frac{1}{\infty} - \frac{1}{r_P}\right]$$

$$V_P = \frac{q}{4\pi\varepsilon_0 r_P}$$

para $r_P \to R \Rightarrow V_R = \frac{q}{4\pi\varepsilon_0 R}$

→ Dentro

$r_P < R$

$$V_\infty - V_P = -\int_P^\infty \vec{E}\cdot d\vec{s}$$

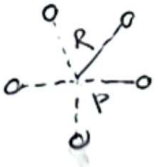
$$0 - V_P = \int_{r_P}^{R} \vec{E}\cdot d\vec{r} + \int_R^\infty \vec{E}\cdot d\vec{r}$$

($\int_{r_P}^{R} \vec{E}\cdot d\vec{r} = 0$; $\int_R^\infty \vec{E}\cdot d\vec{r}$ ou ainda $-(V_\infty - V_R)$)

$$= 0 - \frac{q}{4\pi\varepsilon_0 r}\Big|_R^\infty = -\frac{q}{4\pi\varepsilon_0}\left[\frac{1}{\infty} - \frac{1}{R}\right]$$

$$\boxed{V_P = \frac{q}{4\pi\varepsilon_0 R}} = \text{de!}$$

Observe: no anto:



$$V_P = V_{1P} + V_{2P} + V_{3P} + V_{4P} + V_{5P}$$

$$= \frac{1}{4\pi\varepsilon_0 R}(q_1 + q_2 + q_3 + q_4 + q_5) = \frac{q_T}{4\pi\varepsilon_0 R}$$

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Potencial em uma esfera maciça



Vimos que $|\vec{E}|_{dentro} = \frac{q}{4\pi R^3 \varepsilon_0} r$

$$V_f - V_i = -\int_i^f \vec{E} \cdot d\vec{s}$$

Se i e f estão dentro:

$$V_f - V_i = -\int_i^f \frac{q}{4\pi\varepsilon_0 R^3} r\, dr$$

$$= -\frac{q}{4\pi\varepsilon_0 R^3} \frac{r^2}{2}\Big|_{r_i}^{r_f} \qquad r_f = R$$

$$V_R - V_i = \frac{-q}{8\pi\varepsilon_0 R^3}\left(R^2 - r^2\right)$$

$$= \frac{q}{8\pi\varepsilon_0 R^3}$$

Fora

$$V_r = \frac{q}{4\pi\varepsilon_0 r}$$

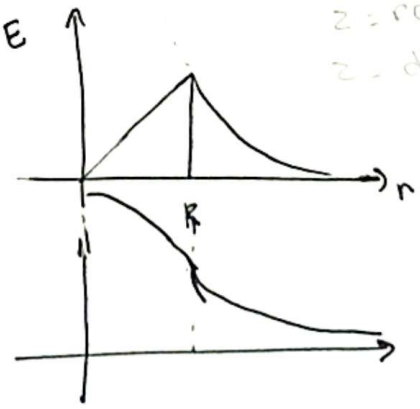
para $r = R \Rightarrow V_R = \frac{q}{4\pi\varepsilon_0 R}$

Voltando em R

$$V_R = \frac{q}{8\pi\varepsilon_0 R^3}(R^2 - r^2) + \frac{q}{4\pi\varepsilon_0 R}$$

$$V_r = \frac{q}{4\pi\varepsilon_0 R}\left(1 + 1 - \frac{r^2}{R^2}\right)$$

$$V_r = \frac{3q}{4\pi\varepsilon_0 R} - \frac{q r^2}{8\pi\varepsilon_0 R^3}$$



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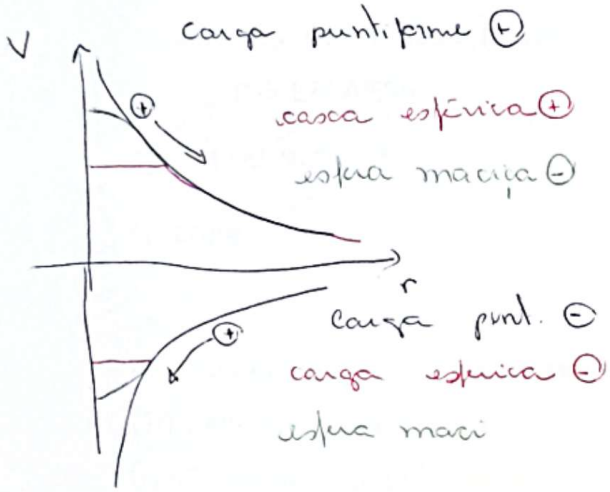
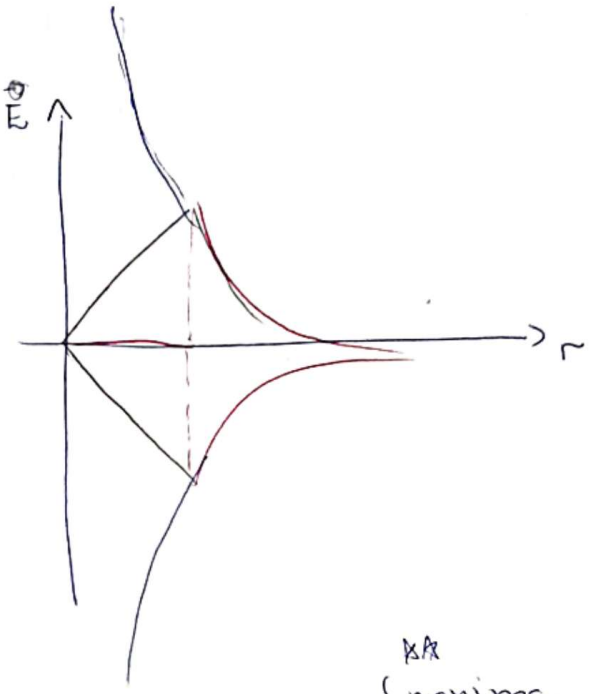
24.10

Vimos:





** (maxima folha)

Vimos:

Como calcular V?

* $V_f - V_i = -\int_i^f \vec{E} d\vec{s}$ (para qualquer superfície)

$V_p = \frac{kq}{r_p}$ (assumindo $V_\infty = 0$)

ou

$V_p = \int \frac{k dq}{r}$ ← * quando conhecemos $\vec{E}$

* Sobre energia



$W_{i \to f} = -\Delta U = -q\Delta V$

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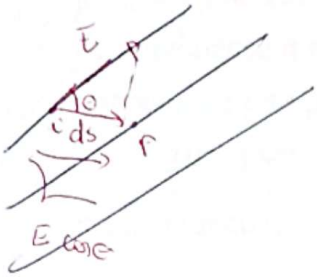
$dV = -E\,ds\cos\theta$

$\Rightarrow \dfrac{dV}{ds} = -E\cos\theta \quad \Rightarrow \quad \dfrac{dV}{ds} = -E_s$



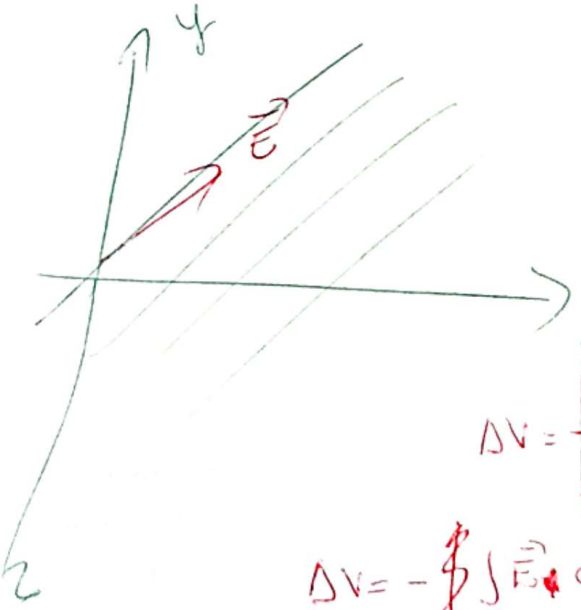
$\dfrac{dV}{dx} = -E_x \; ; \quad \dfrac{dV}{dy} = -E_y \; ; \quad \dfrac{dV}{dz} = -E_z$

$V_f - V_i = -\int E\,ds$

$\Delta V = -\int E\,ds$

$\dfrac{dV}{ds} = -E_s$

$\Delta V = -\int \vec{E}\cdot d\vec{s}$



$\Delta V = -\int \vec{E}\cdot d\vec{x}$

$\Delta V = -\left(\int \vec{E}_x\cdot d\vec{x} + \int E_y\,d\vec{y} + \int E_z\,dz\right)$

$\Delta V_x \qquad \Delta V_y \qquad \Delta V_z$

ΔV

$\Rightarrow E_x = -\dfrac{\partial V}{\partial x}\,\hat{x}$

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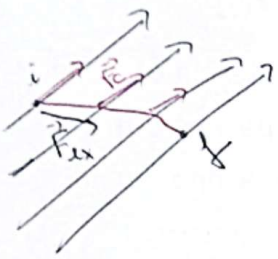
Teorema trabalho energia:

$\Delta K = W_{total} = -\Delta U$ ou seja: $K_f - K_i = U_i - U_f$

$K_f + U_f = K_i + U_i$

(conservação da Energia mecânica)



$K_f - K_i = W_C + W_{ext}$

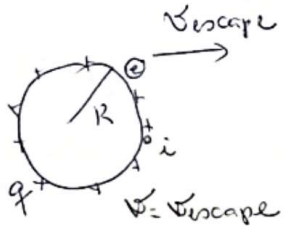
$K_f - K_i = -q\Delta V + W_{ext}$

W_{ext} depende do caminho.

Se $K_f = K_i \Rightarrow W_{ext} = +q\Delta V$

Se $W_{ext} = 0 \Rightarrow K_f - K_i = -q\Delta V$

Exp. 49)



f
(∞)
$v = 0$
$V_\infty = 0$

$V_R = \frac{kq}{R}$

$K_\infty - K_R = (0 - e\frac{(kq)}{R})$

$-K_R = -e\frac{kq}{R} \Rightarrow \frac{mv_{escape}^2}{2} = \frac{ekq}{R}$

$$v_{escape} = \frac{2ekq}{mR}$$

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Energia Potencial em um sistema de cargas

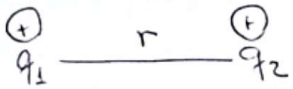
- Cargas pontuais fixas

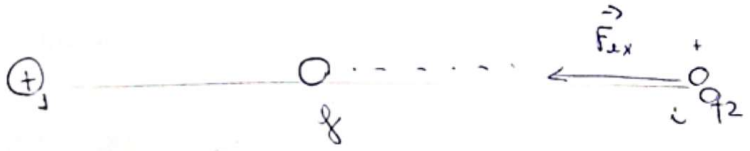
Definição

- $U_{sistema\ fixo} \equiv W_{ext}$ para montar o sistema com → partição

Exemplos:

- 2 cargas:





$W_{F_{ext}\ i \to f} = +q\,\Delta V$ - $q_2\,\Delta V_{gerado\ por\ 1} = q_2(V_{f} - V_{i})$

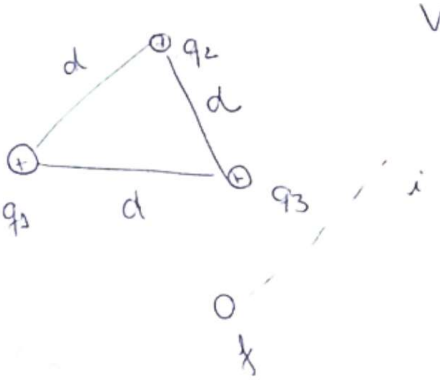
$= q_2 \dfrac{k q_1}{R}$

- 3 ou mais cargas

$U_{sistema} = W_{12} + W$

$U_{sist} = W_{12} + W_{F_{ext_3}\ i \to f}$

$W_{F_{ext_3}\ i \to f} = q_3 \underbrace{\Delta V} = q_3(\Delta V_1 + \Delta V_2)$



$W_{21} = q_2 \dfrac{k q_1}{d}$

$W_{3(1+2)} = W_{31} + W_{32}$

$= \dfrac{k q_3 q_1}{d} + \dfrac{k q_2 q_3}{d}$

$U_{sistema} = U_{12} + U_{13} + U_{23}$

$W_{12} + W_{13} + W_{23}$

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→ Relembrar

V carga pontual ⊕ e ⊖

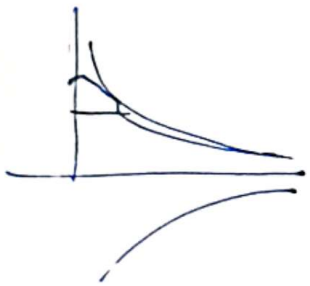
V esfera condutora ⊕ e ⊖

→ V para uma esfera maciça

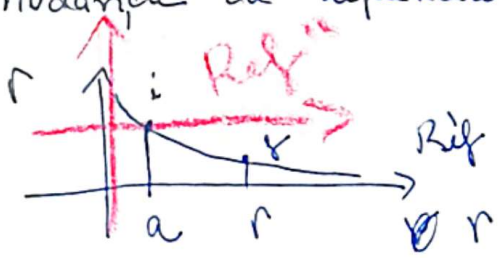
⊕ e ⊖

→ Discussão de potencial

• Graficos



• Mudança de referencial



referencial

$$\Delta V' = \Delta V''$$

$$V'_f - V'_i = V''_f - V''_i$$

$$\frac{kQ}{r} - \frac{kQ}{a} = V''_f(r) - 0$$

Ref' ~ $V'_\infty = 0$

$V'_0 = \frac{kQ}{r}$

Ref'' $V''(r=a) = 0$

$V''(r) = ?$

$$V''(r) = \frac{kQ}{r} - \frac{kQ}{a}$$

$$V'(r=a) = \frac{kQ}{r}$$

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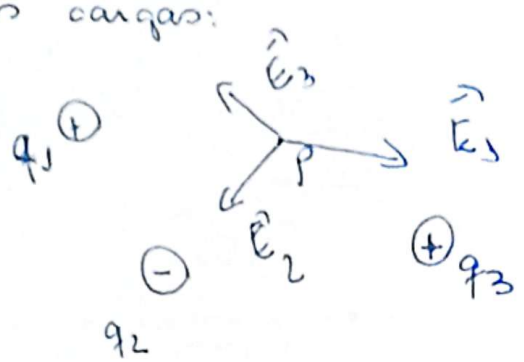
Muitas cargas:



$$\Delta V = V_f - V_i = -\int \vec{E} \cdot d\vec{s}$$

$$= -\int (\vec{E}_1 + \vec{E}_2 + \vec{E}_3) \cdot d\vec{s}$$

$$\Delta V = \underbrace{-\int_i^f \vec{E}_1 \, d\vec{s}}_{\Delta V_1} \underbrace{-\int_i^f \vec{E}_2 \, d\vec{s}}_{\Delta V_2} \underbrace{-\int_i^f \vec{E}_3 \cdot d\vec{s}}_{\Delta V_3}$$

$$V_f - V_i = V_{1f} - V_{1i} - (V_f^2 - V_i^2) - (V_f^3 - V_i^3)$$

Se $V = 0$ existe algum ponto (i ou f) que $V = 0$

$$\Rightarrow V_P = V_{1P} + V_{2P} + V_{3P}$$

$V_f = V_1(f) + V_2(f) + V_3(f)$ //

$$V_P = \sum_i V_i$$

Se $V_\infty = 0$

$$V_{Pi} = \frac{k q_i}{r_i} \Rightarrow V_P = \sum_i \frac{k q_i}{r_i}$$

Para corpo extenso:

$$V_P = \int dV = \int \frac{k \, dq}{r}$$

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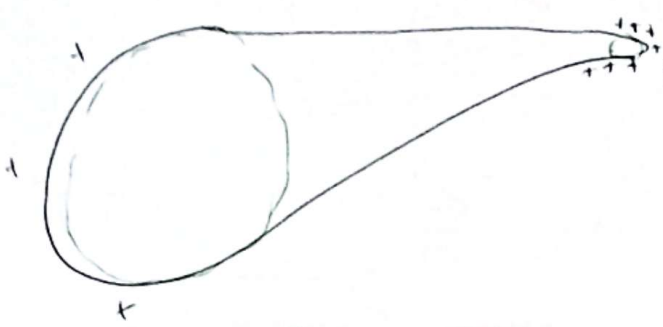
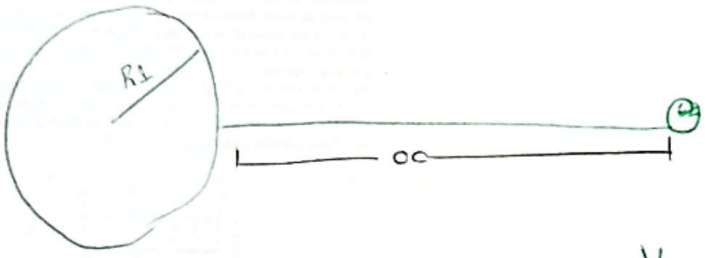
O poder das pontas



σ é mais alta nas pontas

Simplificação:



$$V_1 = \frac{k q_1}{R_1} \qquad V_2 = \frac{k q_2}{R_2}$$

mas, se esta é uma conf. estável $\Rightarrow V_1 = V_2$

$$V_1 = V_2 \Rightarrow \frac{k q_1}{R_1} = \frac{k q_2}{R_2}$$

$$\Rightarrow q_2 = \frac{R_2}{R_1} q_1$$

se ... !

mas $R_2 < R_1 \Rightarrow q_2 < q_1$.

Devemos olhar p/ σ

$$\sigma_1 = \frac{q_1}{4\pi R_1^2} \qquad \sigma_2 = \frac{q_2}{4\pi R_2}$$

$$q_1 = 4\pi R_1^2 \sigma_1 \qquad q_2 = 4\pi R_2^2 \sigma_2$$

$$\Rightarrow 4\pi R_2^2 \sigma_2 = \frac{R_2}{R_1} 4\pi R_1^2 \sigma_1$$

$$\sigma_2 = \frac{R_1}{R_2} \sigma_1 \Rightarrow \sigma_2 > \sigma_1 !$$

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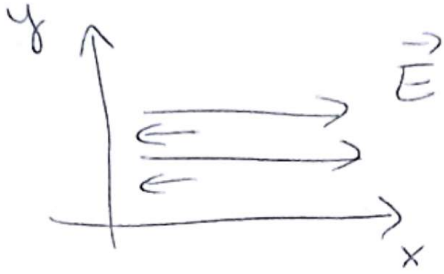
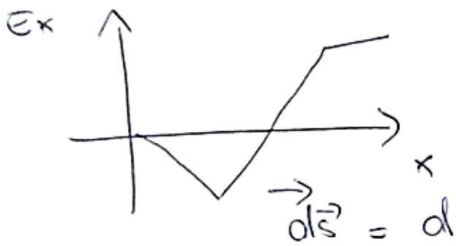
Vimos

$$V_f - V_i = -\int_i^f \vec{E} \cdot d\vec{s}$$



$\vec{E} \Rightarrow \vec{E} = E_x \hat{i}$

8)



$$V_f - V_i = -\int E_x \overbrace{\hat{i}\, dx\, \hat{i}}^{1}$$

$\vec{ds} = dx\,\hat{i}$

$$V_f - V_i = -\underbrace{\int_i^f E_x\, dx}_{\text{Área!}}$$

$i \longrightarrow f$

Cálculo de $\vec{E}$ a partir de V

ex. 35 e 37

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