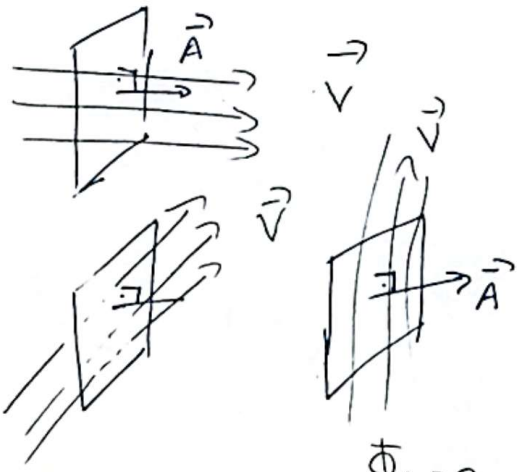
Lei de Gauss: Cap 23

1º) Vimos: $E \propto \frac{\#\ \text{linhas}}{\text{Área} \perp E} \Leftrightarrow E \propto \frac{1}{\text{distância entre linhas}}$

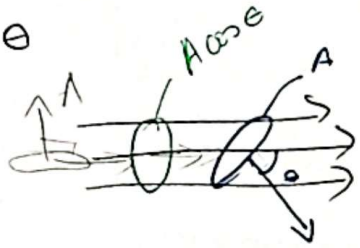
2º) Fluxo: $\Phi_A = \vec{V} \cdot \vec{A}$



$\Phi \propto$ # linhas que perpassam uma superfície

2.1) Fluxo elétrico:

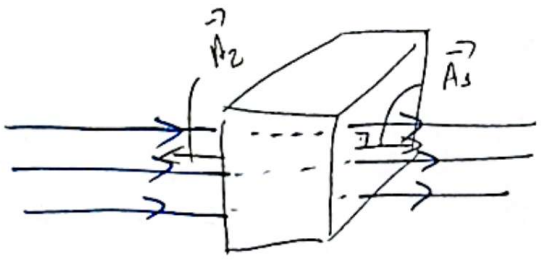
$$\Phi = \vec{E} \cdot \vec{A} = |\vec{E}| \cdot |\vec{A}| \cos\theta$$



- Se $\vec{E} \cdot \vec{A} > 0$
- Se $\vec{E} \cdot \vec{A} < 0$
- Se $\vec{E} \cdot \vec{A} = 0$ 90°

3º) Superfícies fechadas:

$\vec{A} = A\hat{n}$ ↳ sempre para fora da superfície



$\Phi_2 = \ominus E \cdot A \cos 180$

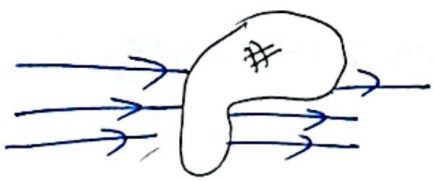
$\Phi_1 = E \cdot A \cos 0$

$\Phi_3 = \Phi_4 = \Phi_5 = \Phi_6 = 0$

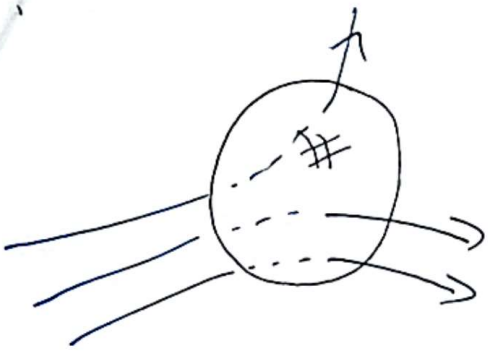
$$\Phi_T = \sum_{i=1}^{6} \Phi_i = 0$$

$$\Phi_T = \oint \vec{E} \cdot d\vec{A}$$

linhas que entra = # linhas que sai!

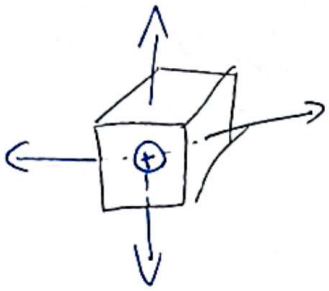


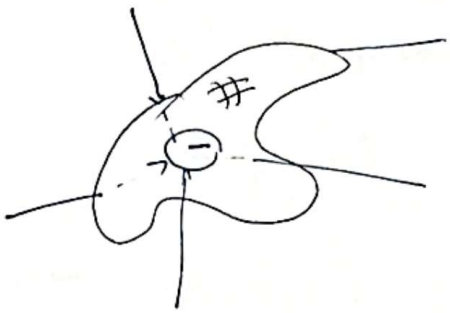
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$\Phi_T = 0$!

Qual a única forma de termos $\Phi_T \neq 0$?

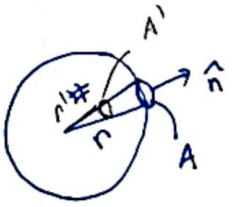




⇒ Deduzir a Lei de Gauss via Lei de Coulomb:
ou ir direto para o enunciado da Lei de Gauss.

Parênteses:

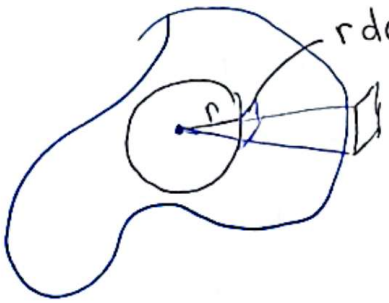
Ângulo sólido $d\Omega$



$$d\Omega = \frac{A_\perp}{r^2} = \frac{A'_\perp}{r'^2} \qquad d\Omega = \frac{dA_\perp}{r^2}$$

$$\int d\Omega = \frac{\text{Área total da esfera}}{r^2} = \frac{4\pi r^2}{r^2} = 4\pi$$

Isto é mais geral:



$r\,d\theta\,r\,d\varphi \Rightarrow \int \frac{r^2}{r^2} d\theta\, d\varphi =$

$\oint d\Omega = 4\pi$!

Fim do Parênteses

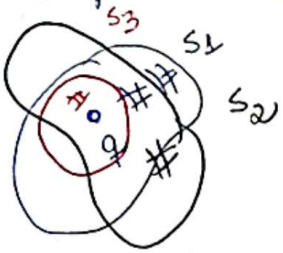
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Lei de Gauss

$$\oint \vec{E} \cdot d\vec{A} = \frac{q_{env}}{\varepsilon_0}$$

- $\Phi = \oint \vec{E} \cdot d\vec{A}$ ⇒ o fluxo de $\vec{E}$ em uma superfície fechada é igual a carga dentro desta superfície / ε_0

- O fluxo não depende do formato da superfície.





$$\Phi_1 = \Phi_2 = \Phi_3 = \frac{q_{env}}{\varepsilon_0}$$

⇒ Aplicando a Lei de Gauss para descobrir $\vec{E}_p$

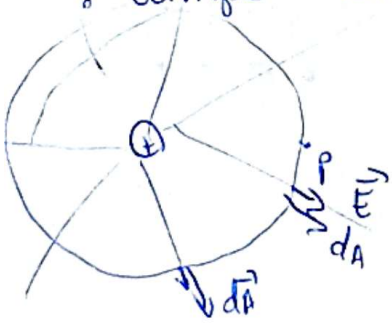
Método:

- Encontrar uma uma superfície que passa pelo ponto P e $\vec{E}$ é cte ao longo da superfície (ou $\vec{E} \cdot d\vec{A} = 0$ $\vec{E} \perp d\vec{A}$!)

- Deve-se saber as linhas de campo de antemão.

Exemplos

- Campo elétrico de uma carga pontual



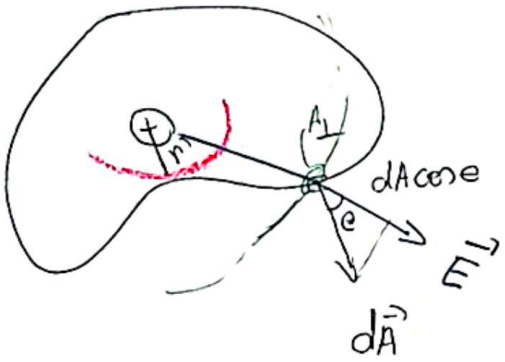
$\vec{E}_p$ Carga negativa

$$\oint \vec{E} \cdot d\vec{A} = \frac{q_{env}}{\varepsilon_0} \Rightarrow \oint E \cdot dA \cos 180 = -\frac{q_{env}}{\varepsilon_0}$$

$$E \underbrace{\oint dA}_{4\pi r^2} = \frac{q_{env}}{\varepsilon_0} \qquad \vec{E} = E_p = \frac{q}{4\pi\varepsilon_0 r^2}\hat{r}$$

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Fluxo elétrico gerado por uma carga pontual



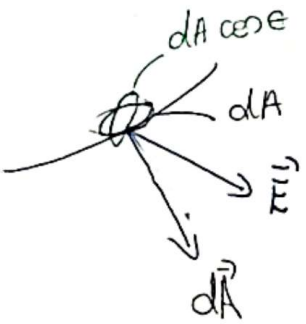
$$\Phi = \oint \vec{E} \cdot d\vec{A}$$

$$\Phi = \oint E \cdot \underbrace{dA \cos\theta}_{dA_\perp}$$

$= d\omega\, r^2$ (ângulo sólido)

$$\Phi = \oint \frac{kq}{r^2} r^2\, d\omega$$

↳ Lei de Coulomb



$$\Phi = kq \underbrace{\int d\omega}_{4\pi} \Rightarrow \Phi = \frac{1}{4\pi\varepsilon_0} q\, 4\pi$$

$$\Phi = \frac{q_{env}}{\varepsilon_0}$$

$k = \frac{1}{4\pi\varepsilon_0}$

→ Lei de Gauss

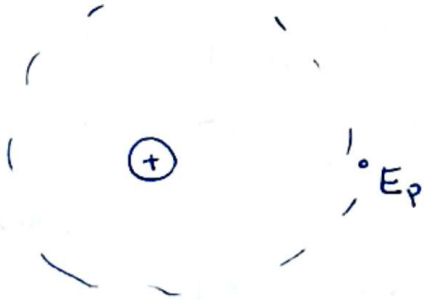
$$\Phi = \oint \vec{E} \cdot d\vec{A} = \frac{q_{env}}{\varepsilon_0}$$

($\vec{E}$: E sobre sup.)

• É crucial saber as linhas de campo de antemão.

• Dedução da Lei de Gauss via Lei de Coulomb

• Dedução da Lei de Coulomb via Lei de Gauss:



$$\frac{q_{env}}{\varepsilon_0} = \oint \vec{E} \cdot d\vec{A}$$

$$\frac{q_{env}}{\varepsilon_0} = \oint E \cdot \underbrace{dA \cos\epsilon}$$

$$\left(E = \frac{q_{env}}{4\pi r^2} \right) \quad //$$

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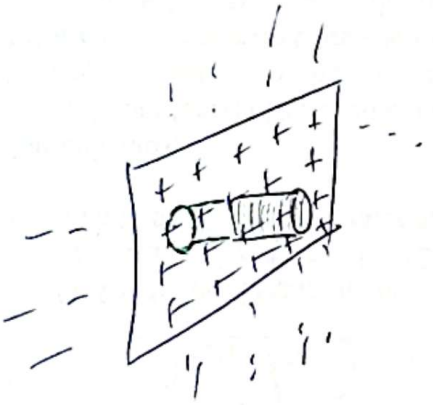
ricando Lei de Gauss para descobrir $\vec{E}$

Método:

. Encontrar uma superfície em que E é constante ao longo da superfície (ou zero)

. Deve-se saber as linhas de força de antemão

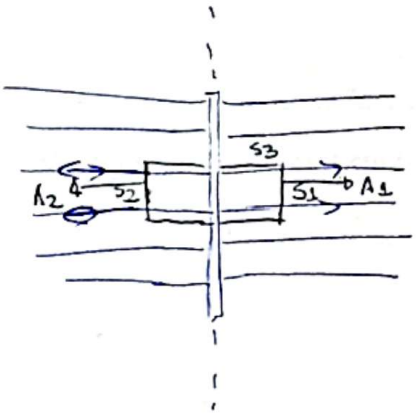
Placa infinita



Deduzimos anteriormente

$$E = \frac{\sigma}{2\varepsilon_0}$$

(Integração + limite)



$$\Phi = \Phi_{S1} + \Phi_{S2} + \Phi_{S3}$$

$$\Phi = EA + EA = \frac{q_{env}}{\varepsilon_0}$$

$$2EA = \frac{q_{env}}{\varepsilon_0} \qquad E = \frac{q_{env}}{2\varepsilon_0 A} \qquad \sigma$$

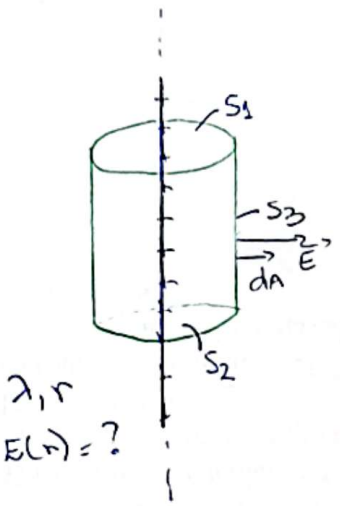
$$\boxed{E = \frac{\sigma}{2\varepsilon_0}}$$

Placa condutora infinita

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OBS: Q

Fio infinito (isolante)



$$\Phi = \Phi_{S_1} + \Phi_{S_2} + \Phi_{S_3}$$

$$\Phi = \int \vec{E} \cdot d\vec{A}$$

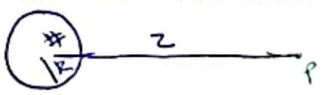
$$= E \int dA$$

$$= E\, 2\pi r h = \frac{q_{env}}{\varepsilon_0}$$

$$\Rightarrow E = \left(\frac{q}{h}\right)\frac{1}{\varepsilon_0 2\pi r} \qquad (\lambda = \frac{q}{h}) \qquad \Rightarrow \boxed{E = \frac{\lambda}{2\pi\varepsilon_0 r}}$$

- Simetria esférica

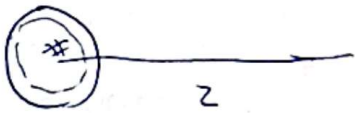
↳ Esfera (maciça) carregada q, R, $E(z) = ?$



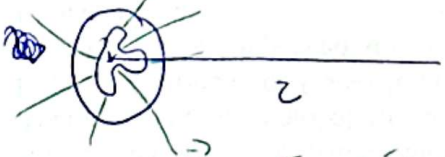
$$\Phi = \oint E\, dA = E \int A = E\, 4\pi z^2$$

$$\Rightarrow E = \frac{1}{4\pi\varepsilon_0}\frac{q}{z^2}$$

↳ casca esférica q, R, $E(z) = ?$



↳ Irregular



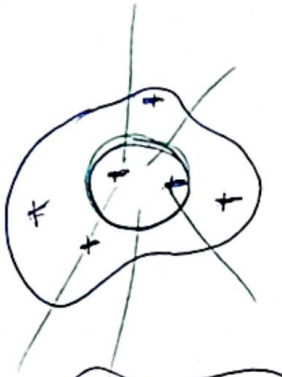
$q, E(z) = ?$ $\quad E \neq \frac{1}{4\pi\varepsilon_0}\frac{q}{z^2}$

$\vec{E}$ não é constante! Cuidado ao aplicar a lei de Gauss. Entenda porque

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Obs: Q

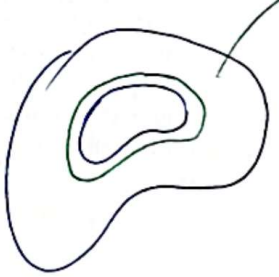
Lei de Gauss x Condutor carregado



condutor ⇒ elétrons em movimento

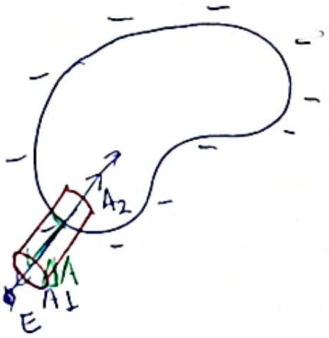


o campo elétrico dentro de um condutor é sempre nulo. mesmo que ele tenha uma cavidade.



$\vec{E}$ dentro de um condutor é sempre nulo; ao contrário as cargas estariam se movimentando.

Campo elétrico na superfície do condutor:



$$\Phi = \Phi_{A1} + \Phi_{A2} + \Phi_{A3}$$

$$\frac{q}{\varepsilon_0} = E\Delta A_1 + E\Delta A_2^{\nearrow 0}$$

$$\frac{q}{\varepsilon_0} = E\Delta A_\perp \qquad \frac{q}{A} = \sigma$$

$$\Rightarrow E = \frac{q}{A\varepsilon_0} \Rightarrow E = \frac{\sigma}{\varepsilon_0}$$

na superfície condutora

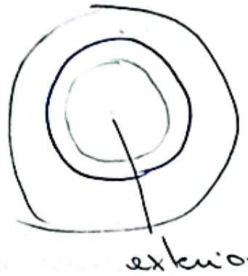
- Se a σ = de (casca esférica) ⇒



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(Voltando à) casca carregada:

Interior da casca



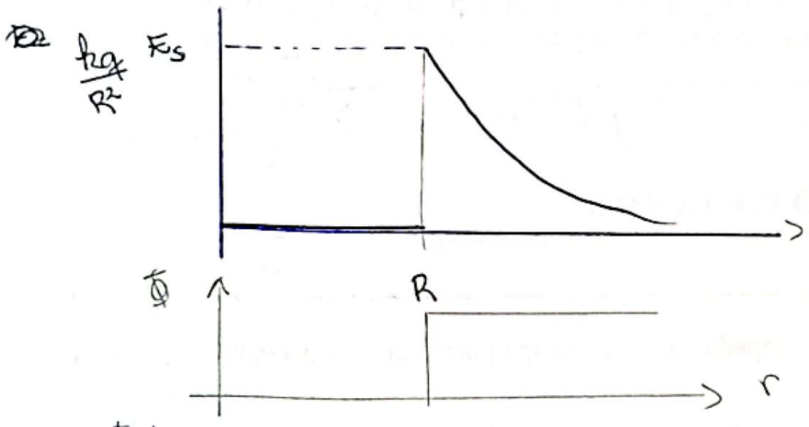
$$\Phi = \oint \vec{E}\, d\vec{A} = \frac{q_{env}}{\varepsilon_0} = 0$$

. as H linhas que entra = sai

. ou $\boxed{\vec{E} = 0}$

$$\Phi = \oint \vec{E}\, d\vec{A} = E \int dA \cos 0 = E\, 4\pi r^2 = \frac{q}{\varepsilon_0}$$

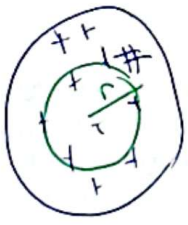
$$E = \frac{1}{4\pi\varepsilon_0}\,\frac{q}{r^2}$$



$$E_S = \frac{1}{4\pi\varepsilon_0}\,\frac{q}{R^2} = \frac{\sigma}{\varepsilon_0}$$

Vimos que E

. Esfera maciça q densidade volumetrica ρ constante



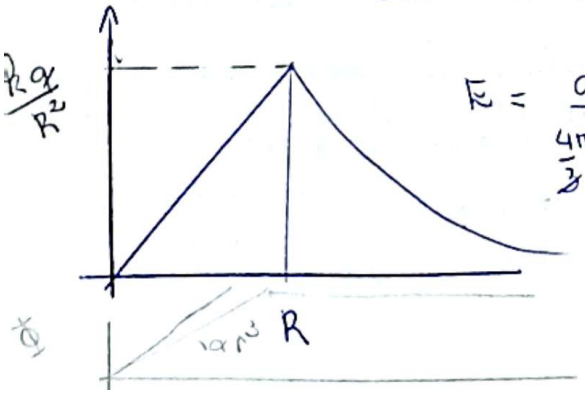
$$E = \frac{1}{4\pi\varepsilon_0}\,\frac{q'}{r'^2}$$

$$\rho = \frac{q}{V} = \frac{q}{\frac{4}{3}\pi R^3} = \frac{q'}{\frac{4}{3}\pi r'^3}$$

$$\rho = 3\,\frac{q'}{4\pi r'^2}\,\frac{1}{r'}$$

$$E = \left(\frac{1}{\varepsilon_0}\rho\right)\frac{r'}{3}$$

ou ainda



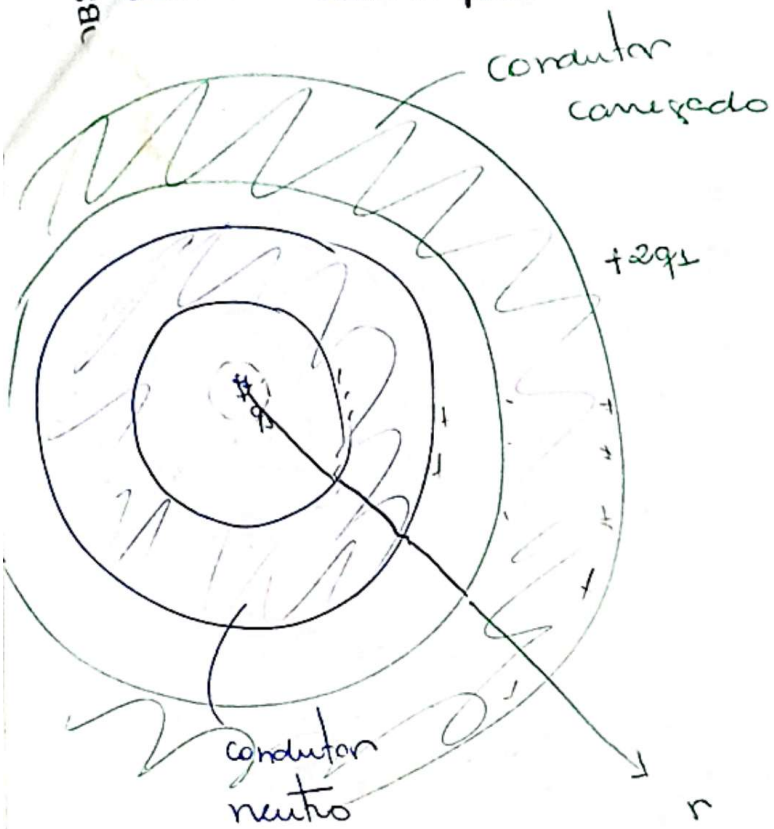
$$E = \frac{q}{\frac{4}{3}\pi R^3 \varepsilon_0}\,\frac{r'}{3}$$

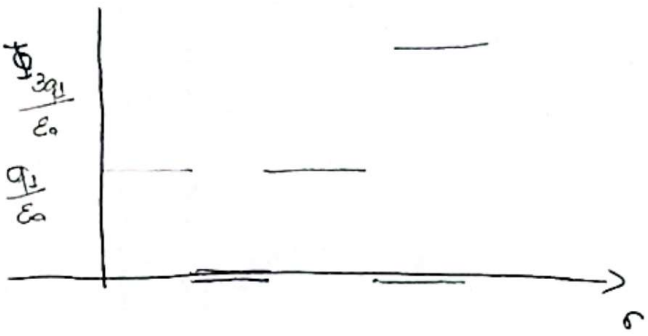
$$E = \left(\frac{q}{4\pi R^3 \varepsilon_0}\right) r'$$

$$\sigma = \frac{q}{\frac{4}{3}\pi r^3} \Rightarrow q_{env}(r) = \sigma\left(\frac{4}{3}\pi r^3\right)$$

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OBS: Outro exemplo







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