~~Cap~~ 2 Campos elétricos (2 ou 3 aulas)

5

12/09 e 7/11 Prova

q $\qquad$ q_0 ↳ carga prova

$$\vec{F} = \frac{k|q||q_0|}{r^2}\hat{r}$$

$$\vec{E} = \frac{\vec{F}}{q_0} \quad , \quad q_0 > 0 \Rightarrow \vec{E} = \frac{k\,q}{r^2}\hat{r}$$

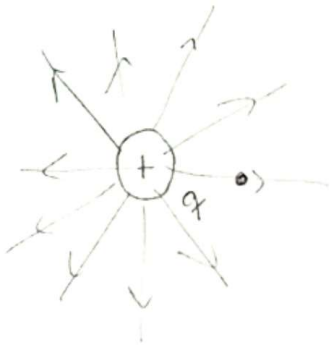
↳ radial p/ fora.

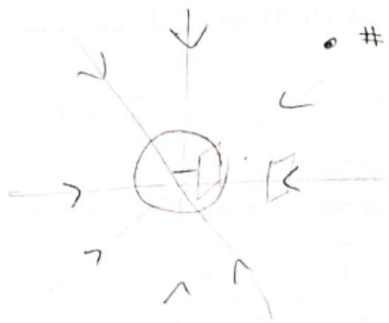
↳ Força depende de duas cargas

↳ campo elétrico depende apenas de 1 carga

→ Linhas de campo

- $\vec{E}$ // linha de campo
- # linhas/área $\perp E$ $\propto \vec{E}$

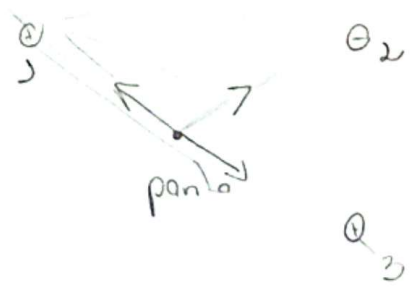




$$\vec{E} = \frac{kq}{r^2}\hat{r}$$ sendo $\hat{r}$, radial p/ fora

- Princípio da superposição

$$\vec{E}_{ponto,total} = \vec{E}_{ponto,1} + E_{ponto,2} \ldots$$



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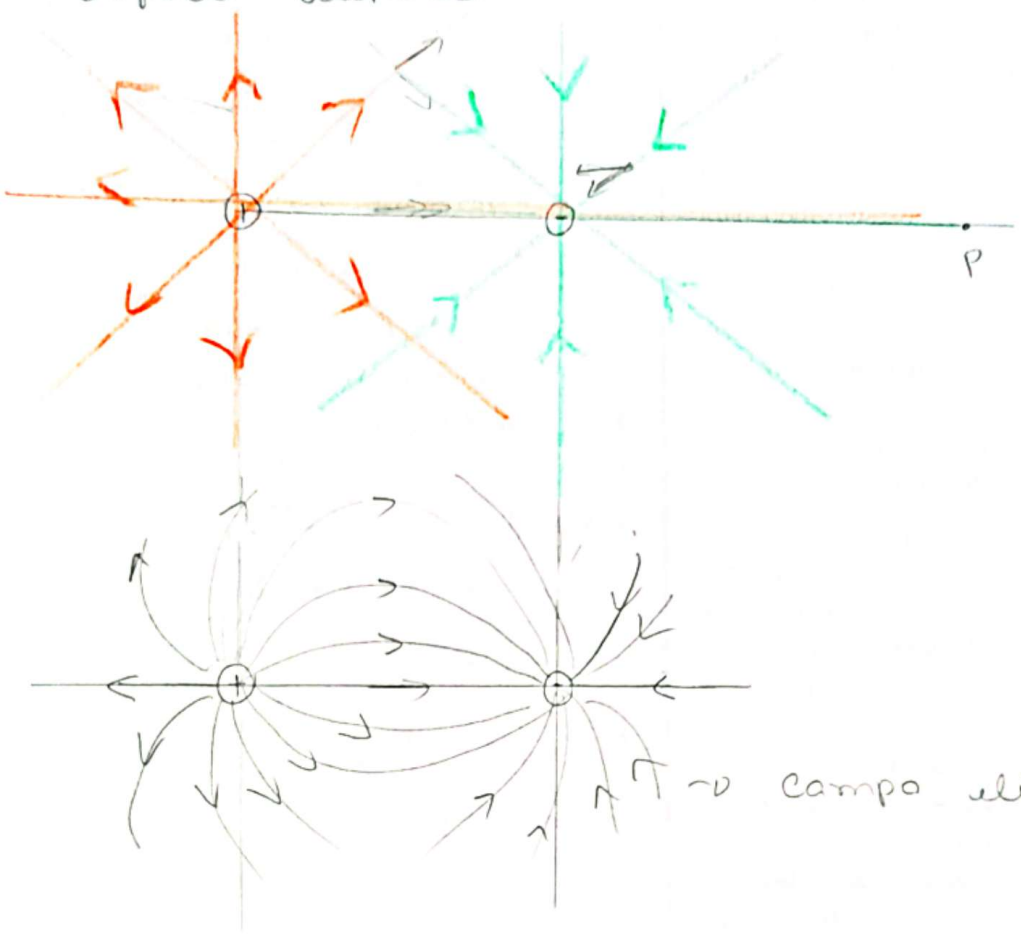
Dipolo elétrico



→ campo elétrico resultante



$\vec{p}$ momento de dipolo elétrico

$$\vec{E}_p = \vec{E}_{p-} + \vec{E}_{p+}$$

$$|\vec{E}_p| = |\vec{E}_{p-}| - |\vec{E}_{p+}|$$

$$|\vec{E}_p| = \frac{kq}{(z - d/2)^2} - \frac{kq}{(z + d/2)^2}$$

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$$|\vec{E}_p| = kq\left[\frac{1}{z^2\left(\frac{z}{z}-\frac{d}{2z}\right)^2} - \frac{1}{z^2\left(\frac{z}{z}+\frac{d}{2z}\right)^2}\right]$$

$z \gg d$ queremos um valor aproximado.

$$\frac{d}{z} \Rightarrow 0$$

$$|\vec{E}_p| = \frac{kq}{z^2}\left[\frac{1}{\left(1-\frac{d}{2z}\right)^2} - \frac{1}{\left(1+\frac{d}{2z}\right)^2}\right]$$

$$\left(1 \pm \frac{d}{2z}\right)^2 = 1 \pm \frac{d}{z} + \frac{d^2}{4z^2}$$

$$\approx 1 \pm \frac{d}{z}$$ ignorando ordens de $\left(\frac{d}{z}\right)^2$

$$|\vec{E}_p| \approx \frac{kq}{z^2}\left[\frac{1}{\left(1-\frac{d}{z}\right)} - \frac{1}{\left(1+\frac{d}{z}\right)}\right]$$

$$\approx \frac{kq}{z^2}\left[\frac{1+\frac{d}{z} - 1+\frac{d}{z}}{1^2 - \left(\frac{d^2}{z^2}\right)}\right]$$

$\frac{d^2}{z^2} \to 0$

$$\approx \frac{2kqd}{z^3}$$ ▷ momento de dipolo p

$k = \frac{1}{4\pi\varepsilon_0}$

↳ permissividade do meio. Vácuo $8{,}85 \cdot 10^{-12}\ C^2/N\,m^2$

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22.3b

$$|\vec{E}_R| = \frac{kq}{z}\left[\frac{1}{(1-d/2z)^2} - \frac{1}{(1+d/2z)^2}\right]$$

se $z \gg d \Rightarrow \frac{d}{2z} \to 0$ $\quad |E_R| = 0$

obvio!
esperado.

Vamos expandir o termo em serie de Taylor)

$$\left(1-\frac{d}{2z}\right)^2$$

$$x = \frac{d}{2z} \qquad x' \to 0 \text{ em torno.}$$

$$f(x) = \sum_{n=0}^{\infty} \frac{f^n(a)(x-a)^n}{n!}$$

$$= f(a) + \frac{f'(a)(x-a)}{1!} + \frac{f''(a)(x-a)^2}{2!}$$

$$(1-x)^2 = f(x)$$

$$f'(x) = -2(1-x)$$

$$(1-x)^2 = f(0) + \frac{f'(0)(x-0)}{1!}$$

$$\approx 1 + -2x$$

$$\left(1-\frac{d}{2z}\right)^2 \approx 1 - \frac{d}{z}$$

$$\left(1+d/2z\right)^2 \approx 1 + \frac{d}{z}$$

$d^2/z^2 \to 0$

logo: $E_R = \frac{kq}{z}\left[\frac{1}{1-\frac{d}{z}} - \frac{1}{1+\frac{d}{z}}\right]$

$$E_R = \frac{kq}{z}\left[\frac{1+d/z - 1 + d/z}{1^2 - (d^2/z^2)}\right]$$

$$E_R = 2\frac{kq}{z}\frac{d}{3}$$

ou ainda,

$$E_R \simeq \frac{2d}{4\pi\varepsilon_0 z^3} = \frac{d}{2\pi\varepsilon_0 z^3}$$

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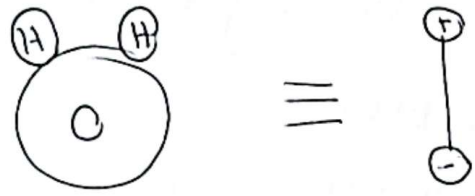
A molécula de água (H_2O)

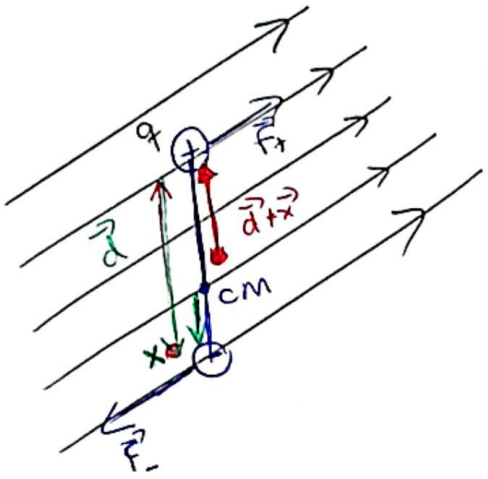


dipolo elétrico

$Z_O = 8$

$Z_H = 1$

Dipolo em um campo elétrico uniforme:



$$\vec{F}_- = -\vec{F}_+ = -q\vec{E}$$

- Torque

$$\vec{\tau} = \vec{r} \times \vec{F}$$

distância do eixo



$$|\vec{\tau}| = |\vec{r}||\vec{F}| \operatorname{sen}\theta$$

$$\vec{\tau}_R = \vec{\tau}_- + \vec{\tau}_+$$

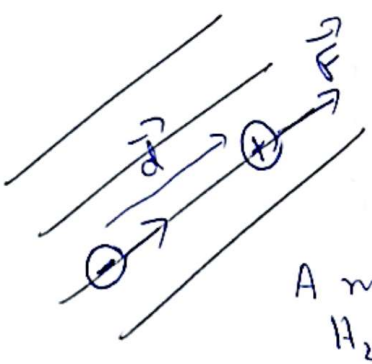
$$= -\vec{x} \times (q\vec{E}) + (\vec{d}+\vec{x}) \times (q\vec{E})$$

$$= -q\vec{x} \times \vec{E} + q\vec{d} \times \vec{E} + q\vec{x} \times \vec{E}$$

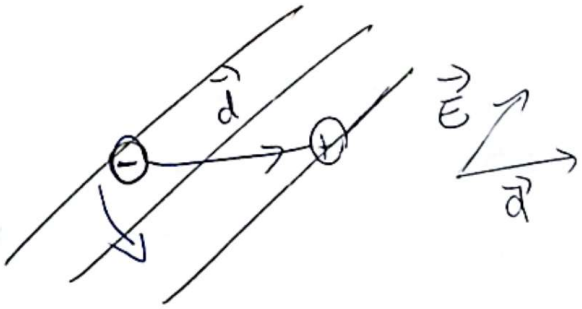
($-q\vec{x} \times \vec{E}$ e $q\vec{x} \times \vec{E}$ se cancelam)

$$\vec{\tau}_R = q\vec{d} \times \vec{E}$$

$\vec{p}$ ou $\vec{\mu}$ momento de dipolo



$\tau = 0$



A molécula de H_2O fica oscilando!

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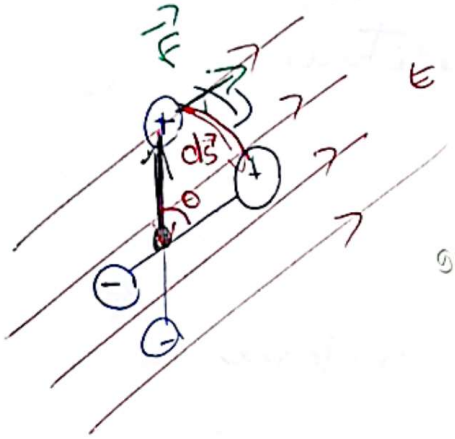
mas qual é a energia associada a esta oscilação?

Física I

$$\Delta U = -W = -\int \vec{F}\cdot d\vec{s}$$



$$|d\vec{s}| = r\,d\theta$$

$$W = \int |\vec{F}||d\vec{s}| \cos\alpha \, d\alpha$$

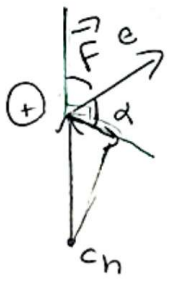
$90 + \theta + \alpha$

$d\alpha = -d\theta$

$\cos\alpha = \sin\theta$

$$W = -\int |\vec{F}||d\vec{s}| \sin\theta \, d\theta$$

$$W_{int} = -\int |\vec{\tau}| d\theta$$



$$\Delta U_{dipolo} = +\int |\vec{\tau}| d\theta$$

$$W_{ext} = \int \vec{F}\cdot d\vec{s} = \int |\vec{\tau}| d\theta$$

$$\Delta U_{dipolo} = -\Delta U_{ext}$$

$$\Delta U = +\int |\vec{p}\times\vec{E}| \, d\theta$$

$$= +\int |\vec{p}||\vec{E}| \sin\theta \, d\theta$$

$$= +|p||E| \cos\theta \Big|_{\theta_i}^{\theta_f}$$

$U_f - U_i$

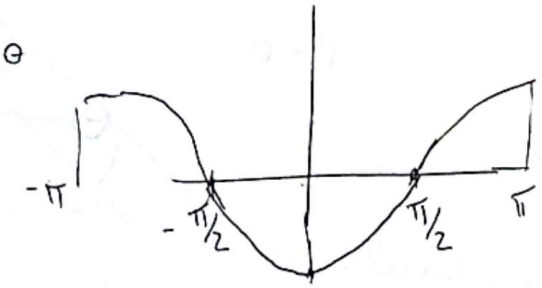
$$\Delta U = -|p||E|\cos\theta_f - |p||E|\cos\theta_i$$

U_i → potencial

$\theta_i = 90 \Rightarrow U_i = 0$

$$U(\theta) = -|p||E|\cos\theta$$

$$U(\theta) = -\vec{p}\cdot\vec{E}$$



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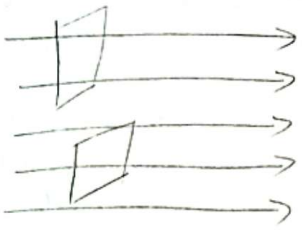
Linhas de campo



- $\vec{E}_p$ é tangente à linha no ponto p
- # linhas / Área ⊥ E $\propto E_p$

quanto mais proximo de ⊕ maior o # de linhas que "perfuram" a área ⇒ maior é o campo

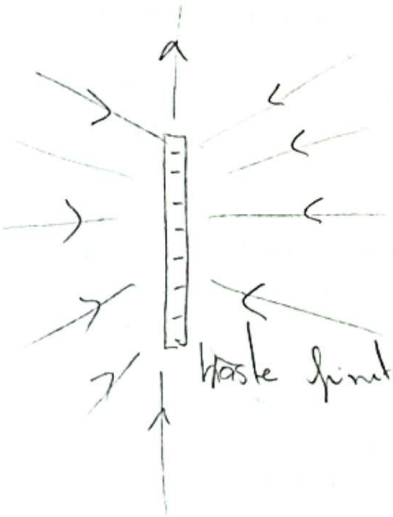
Campo uniforme



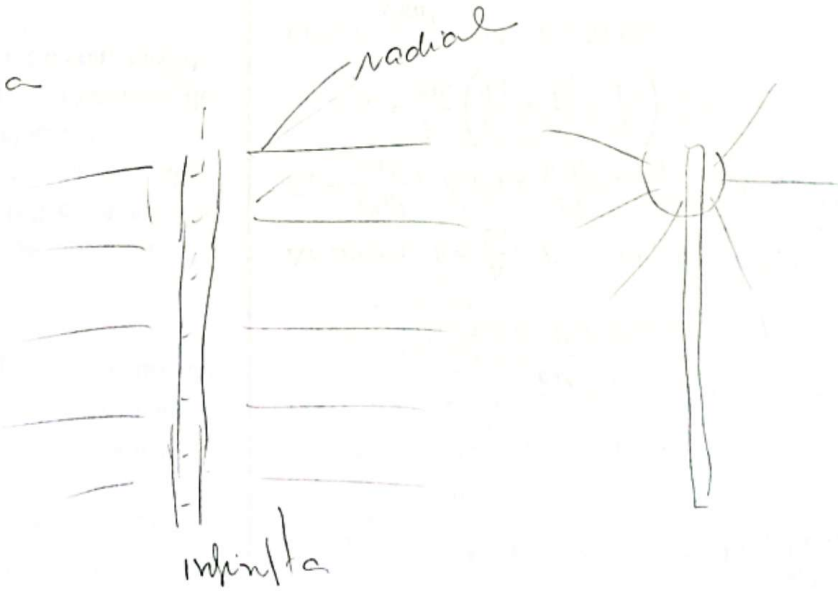
o # de linhas que passa □ igual em qualquer posição



↳ placa infinita



haste finita



infinita

• Falar

• Por que $\vec{E} = dc$ em uma placa infinita?

Antes disso:

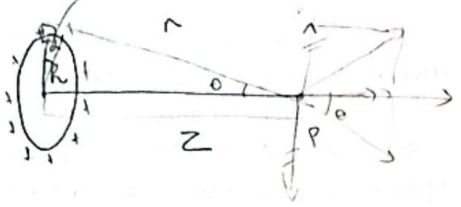
- $\vec{E}_{Anel}$
- $\vec{E}_{disco}$

* Falar de Densidade de carga

$$\vec{E}_{res} = \sum_i \vec{E}_i = \lim_{\Delta\vec{E}\to 0} \sum_i \Delta\vec{E}_i = \int d\vec{E}$$

$$\cos\theta = \frac{z}{r} = \frac{z}{\sqrt{R^2+z^2}}$$

Anel



por simetria

$$\vec{E}_P = E_P \hat{z}$$

• densidade de carga $\lambda = \frac{q}{L}$ $\quad [\lambda] = \frac{C}{m}$

conhecido, R, λ, z

$$\int dE = \int \frac{k\,dq\cos\theta}{r^2} \Rightarrow E = \frac{k\cos\theta}{r^2}\int dq$$

$$\lambda = \frac{q}{2\pi R} = \frac{dq}{R\,d\alpha} \Rightarrow dq = \lambda R\,d\alpha$$

$$E = \frac{k\,z}{(R^2+z^2)^{1/2}(R^2+z^2)^{1}} \lambda R \int_0^{2\pi} d\alpha \Rightarrow E = \frac{k\lambda z R}{(R^2+z^2)^{3/2}} \cdot 2\pi$$

$$E = \frac{1}{4\pi\varepsilon_0}\frac{\lambda z R}{(R^2+z^2)^{3/2}} \cdot 2\pi = \frac{1}{2\varepsilon_0}\frac{\lambda z R}{(R^2+z^2)^{3/2}}$$

$\cos\theta$

Para longas distancias $z \gg R$ $\quad \lambda = \frac{q}{2\pi R}$

$$E \approx \frac{1}{2}\frac{\lambda z R}{z^3} = \frac{1}{2\varepsilon_0}\frac{\lambda R}{z^2} = \frac{1}{4\pi\varepsilon_0}\frac{q}{z^2}$$

$$E \approx \frac{q}{4\pi\varepsilon_0 z^2} \rightsquigarrow \text{pontiforme.}$$

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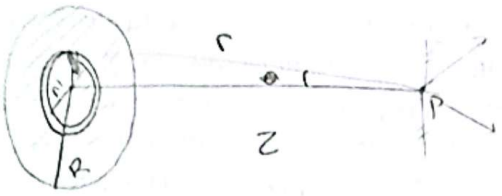
provável

Campo elétrico em um disco:



$$\cos\theta = \frac{z}{(r'^2+z^2)^{1/2}}$$

$$dE = k\frac{dQ}{r^2} \qquad E = k\int \frac{dQ}{r^2}\cos\theta$$

$$[\sigma] = \frac{C}{m^2} \qquad \sigma = \frac{q}{\pi R^2} = \frac{dq}{r'd\alpha dr'} \Rightarrow dq = \sigma r' d\alpha dr'$$

$$\sigma = \frac{q}{2\pi r' dr'}$$

mas $r^2 = r'^2 + z^2$

$$E = \iint k\sigma \frac{r'd\alpha dr'}{r^2}\frac{z}{(r'^2+z^2)^{1/2}}$$

$$E = \iint \frac{k\sigma z d\alpha r' dr'}{(r'^2+z^2)^{3/2}} = k\sigma z\int_0^{2\pi} d\alpha \int \frac{r'dr'}{(r'^2+z^2)^{3/2}}$$

$$x = r'^2 + z^2 \Rightarrow dx = 2r'dr' \Rightarrow r'dr' = \frac{dx}{2}$$

$$r' \in [0, R] \Rightarrow x \in [z^2, R^2+z^2]$$

$$\int \frac{r'dr'}{(r'^2+z^2)^{3/2}} = \frac{1}{2}\int^{-3/2} x^{-3/2}dx = \frac{1}{2}\frac{x^{-1/2}}{-1/2}\Bigg|_{z^2}^{R^2+z^2} = -x^{-1/2}\Big|_{z^2}^{R^2+z^2}$$

$$E = \ldots = -\frac{1}{\sqrt{R^2+z^2}} + \frac{1}{\sqrt{z^2}}$$

$$E = k\sigma z 2\pi\left[\frac{1}{z} - \frac{z}{z\sqrt{R^2+z^2}}\right] = k\sigma 2\pi\left[1 - \frac{z}{\sqrt{R^2+z^2}}\right]$$

$$= \frac{\sigma}{2\varepsilon_0}\left[1 - \frac{z}{\sqrt{R^2+z^2}}\right]$$

Placa infinita:

Se $R \to \infty \Rightarrow E = \frac{\sigma}{2\varepsilon_0}$

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