



Chimera states in networks under external periodic perturbations

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Abstract. Spatially extended system can display spatiotemporal pattern with coexisting coherent and incoherent domains, known as chimera state. Chimera states have been observed in experiments and mathematical models of networks. We build a network of coupled logistic maps and other composed of Rössler oscillators. In the literature, there are research studies demonstrating the existence of chimeras in both networks. In this work, we study the effects of an external periodic perturbation on the chimera states. We show that the existence of chimera depends on the amplitude and frequency of the perturbation. The chimera states can be not only created, but also suppressed by means of an external periodic perturbation in non-local networks.

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1. Introduction

Chimera state is a spatiotemporal pattern where coherence and incoherence coexist. In 1989, Umberger *et al.* [1] observed chimera states in a dispersively coupled chain of nonlinear oscillators. Kuramoto and Battogtokh [2] in 2002 reported the coexistence of coherence and incoherence in non-locally coupled phase oscillators. In 2004, the word chimera, a mythological creature composed of different animal parts, was used by Abrams and Strogatz [3] when they were studying arrays of identical oscillators. Wolfrum and Omel'chenko [4] in 2011 investigated the lifetime and the dependence of the collapse of chimera states on the network size and initial conditions.

Patterns of coexisting coherent and incoherent dynamics have been found in many dynamical systems [5, 6], for instance in coupled mechanical [7, 8] and chemical [9, 10] oscillators. Chimera states are reported in biological systems, such as neuronal [11, 12] and ecological [13] networks. Gambuzza *et al.* [14] provide experimental evidence of chimeras in coupled electronic circuits and Wojewoda *et al.* [15] in coupled pendula. Analytical approach and numerical calculation

were performed by Smirnov *et al.* [16] to develop a theory of chimera patterns.

We analyse the formation of chimera states in networks with non-local coupling. Firstly, we build a network composed of coupled logistic maps. Coupled map networks have been considered to model spatially extended dynamical systems [17]. Batista and Viana [18] characterised chimera in a network of logistic maps connected by means of a smoothed finite range coupling. Secondly, we consider a spatially extended system where the local dynamics is given by the Rössler oscillator [19]. The existence of chimera states has been identified in networks of coupled Rössler systems [20–22].

In this paper, we examine the effects of external periodic perturbations on the chimera states. External periodic perturbations have been used to lead chaos to periodicity and periodicity to chaos [23]. Hsu *et al.* [24] demonstrated conditions to control chaotic behaviour by means of weak periodic perturbation. In our simulations, we verify that the perturbation has a relevant impact on the chimera. We show that the existence and the suppression of chimera states depend on the amplitude and frequency of the periodic perturbation.

This paper is organised as follows. In section 2, we introduce the network model. Section 3 presents our results about the chimera states in a network under an external periodic perturbation. Finally, in the last section, we draw our conclusions.

2. Networks

We build two different types of networks, one is composed of coupled logistic maps and the other of coupled Rössler oscillators. The network of logistic maps is given by

$$x_{n+1}^{(i)} = f(x_n^{(i)}) + \frac{\varepsilon}{2P} \sum_{j=i-P}^{i+P} [f(x_n^{(j)}) - f(x_n^{(i)})] + d \cos(\omega n), \quad (1)$$

where $x_n^{(i)}$ is the state variable with $i = 1, 2, \dots, N$ and n is the discrete time. ε and P are the coupling strength and the number of coupled neighbours, respectively. In our simulations, we consider the chaotic logistic map $f(x) = 3.8x(1 - x)$ and an external periodic perturbation with amplitude d .

The network of coupled Rössler oscillators is given by

$$\dot{x}_i = -y_i - z_i + \frac{\varepsilon}{2P} \sum_{j=i-P}^{i+P} (x_j - x_i) + d \cos(\omega t), \quad (2)$$

$$\dot{y}_i = x_i + ay_i + \frac{\varepsilon}{2P} \sum_{j=i-P}^{i+P} (y_j - y_i), \quad (3)$$

$$\dot{z}_i = b + z_i(x_i - c) + \frac{\varepsilon}{2P} \sum_{j=i-P}^{i+P} (z_j - z_i). \quad (4)$$

For $a = 0.42$, $b = 2$, and $c = 4$ the Rössler oscillator exhibits chaotic behaviour.

In both the networks we use $N = 500$, periodic boundary conditions, and the local order parameter R as diagnostic tool to identify chimera. The local order parameter is given by

$$R_i = \lim_{N \rightarrow \infty} \frac{1}{2\delta} \left| \sum_{j \in C} e^{i\psi_j} \right|, \quad (5)$$

where ψ_j is the phase [20],

$$C : \left| \frac{j}{N} - \frac{i}{N} \right| \leq \delta, \quad (6)$$

and $\delta \rightarrow 0$ for $N \rightarrow \infty$. The logistic maps or Rössler oscillators that belong to coherent domains have $R_i \approx 1$ and smaller values when they are in incoherent domains of chimera state.

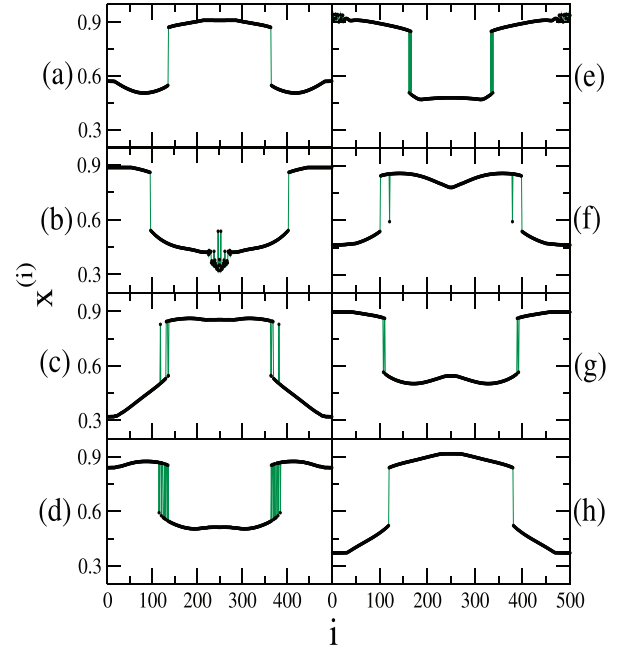


Figure 1. Snapshots of spatial pattern $x_i \times i$ for 500 logistic maps at particular time equal to 10^3 , $n = 10^4$, $\varepsilon = 0.32$, and $\omega = 0.01$. We consider (a) $d = 0$, (b) $d = 0.003$, (c) $d = 0.007$, (d) $d = 0.008$, (e) $d = 0.01$, (f) $d = 0.02$, (g) $d = 0.03$, and (h) $d = 0.04$.

3. Chimera states in networks under periodic perturbations

3.1 Coupled logistic maps

Networks of coupled logistic maps are simple models that have been used to analyse spatiotemporal patterns. We consider 500 logistic maps which are non-locally coupled with $P = 150$ and initial conditions given by a sine function. Figure 1 displays different spatial patterns for $\varepsilon = 0.32$ and $\omega = 0.01$. Increasing the perturbation amplitude d , we observe alterations in the patterns. Without external perturbation ($d = 0$) and for a small perturbation amplitude ($d = 0.003$), we identify regular behaviour (figure 1a) and chimera (figure 1b), respectively. Therefore, a periodic perturbation can induce chimera states in a regular spatiotemporal pattern. We also observe chimera states for $d = 0.007$ (figure 1c), $d = 0.008$ (figure 1e), and $d = 0.01$ (figure 1f). However, the network pattern becomes regular when d is increased to 0.03 (figure 1g) and 0.04 (figure 1h).

In figure 2, the coupling strength ε is varied for $d = 0.1$ and $\omega = 0.01$. The snapshot of the spatial pattern exhibits an irregular behaviour for $\varepsilon = 0.1$, as shown in figure 2a. For $\varepsilon = 0.15$ (figure 2b) and $\varepsilon = 0.17$ (figure 2c), we see chimera states with different structures. The coherent domains vanish for $\varepsilon = 0.2$ (figure 2d) and the network exhibits incoherent structures.

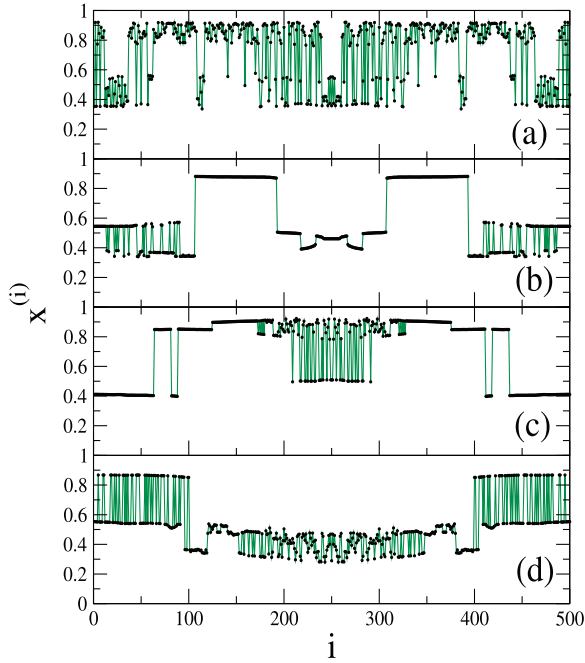


Figure 2. Snapshots of spatial pattern $x_i \times i$ for 500 logistic maps at particular time equal to 10^3 , $n = 10^4$, $d = 0.1$ and $\omega = 0.01$. We consider (a) $\varepsilon = 0.1$, (b) $\varepsilon = 0.15$, (c) $\varepsilon = 0.17$, and (d) $\varepsilon = 0.2$.

Our results show that the existence of chimera states depends on ε , d , and ω . With this in mind, we calculate the degree of coherence by varying these three parameters. The degree of coherence p is obtained by means of the local order parameter and is given by [18]

$$p = \frac{\tilde{N}}{N}, \quad (7)$$

where

$$\tilde{N} = \frac{1}{N_p} \sum_{l=1}^{N_p} N_l, \quad (8)$$

such that N_l is the length of the l th coherence domain ($R_i \approx 1$) and N_p is the total number of domains. For $p = 1$, the network has only one coherent domain, while there is no coherence when $p \rightarrow 0$ ($N \rightarrow \infty$).

The parameter space $\omega \times d$, shown in figure 3a, displays many regions where it is possible to identify chimera states. In addition, there are small regions in which completely incoherent snapshot patterns are observed. Figure 3b shows the existence of chimera states in various regions. The incoherent patterns appear in different regions but mainly for small values of d and ε , as well as for d greater than 0.08. In the parameter space $\omega \times \varepsilon$ (figure 3c), chimera states are identified in the range $0.1 < \varepsilon < 0.3$. The regions for $\varepsilon < 0.1$ and $\varepsilon > 0.3$ correspond to incoherent and coherent patterns. Furthermore, we see mixed regions with coherence, incoherence, and chimera in the range $0.1 < \varepsilon < 0.3$.

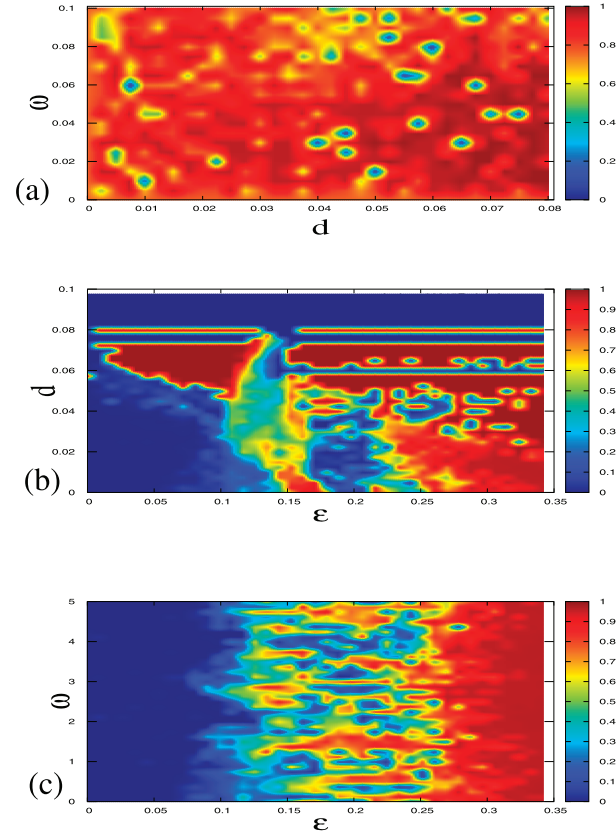


Figure 3. Parameter space for 500 logistic maps: (a) $\omega \times d$ for $\varepsilon = 0.27$, (b) $d \times \varepsilon$ for $\omega = 0.315$, and (c) $\omega \times \varepsilon$ for $d = 0.05$. The colour bar corresponds to the degree of coherence.

3.2 Coupled Rössler oscillators

The Rössler oscillator is given by a set of ordinary differential equations and, depending on the values of the parameters, can exhibit chaotic behaviour. Networks of coupled Rössler oscillators have been used to simulate and study nonlinear high dimensional systems.

We construct a network with 500 Rössler oscillators coupled through non-local links with $P = 150$ and initial conditions given by a sine function. Figure 4 shows the snapshots of spatial pattern $x_i \times i$ for $\varepsilon = 0.23$ and $\omega = 0.1$. In figure 4a, we see chimera for the case without perturbation ($d = 0$). For $d = 0.1, 0.5, 1, 2, 4, 6$, and 9 , we observe alterations in the structures of the domains, as shown in figures 4b–4h. The coherent structures for $d > 0$ are smaller than for $d = 0$. Considering an external periodic perturbation with $d = 0.1$ and $\omega = 0.1$, we identify a coherent pattern for $\varepsilon = 0.25$ (figure 5a). Coexisting coherent and incoherent domains are found for $\varepsilon = 0.271$ (figure 5b) and $\varepsilon = 0.3$ (figure 5c). Figure 5d displays that the coexistence disappears for ε equal to 0.35 and there is a unique coherent pattern.

We also compute the degree of coherence for the coupled Rössler oscillators. The parameter space $\omega \times d$ (figure 6a) exhibits coherence pattern in the red region.

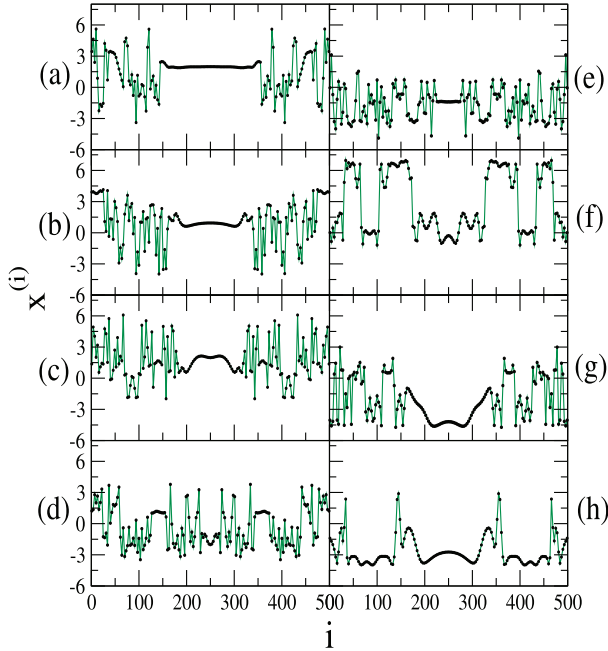


Figure 4. Snapshots of spatial pattern $x_i \times i$ for 500 Rössler oscillators at particular time equal to 23×10^3 , $t = 2.3 \times 10^4$, $\varepsilon = 0.23$, and $\omega = 0.1$. We consider (a) $d = 0$, (b) $d = 0.1$, (c) $d = 0.5$, (d) $d = 1$, (e) $d = 2$, (f) $d = 4$, (g) $d = 6$, and (h) $d = 9$.

Above this region, our simulations show coexistence of coherent and incoherent domains. We verify, below the red region, regions with coexistence and incoherent patterns, as well as small regions with coherent patterns.

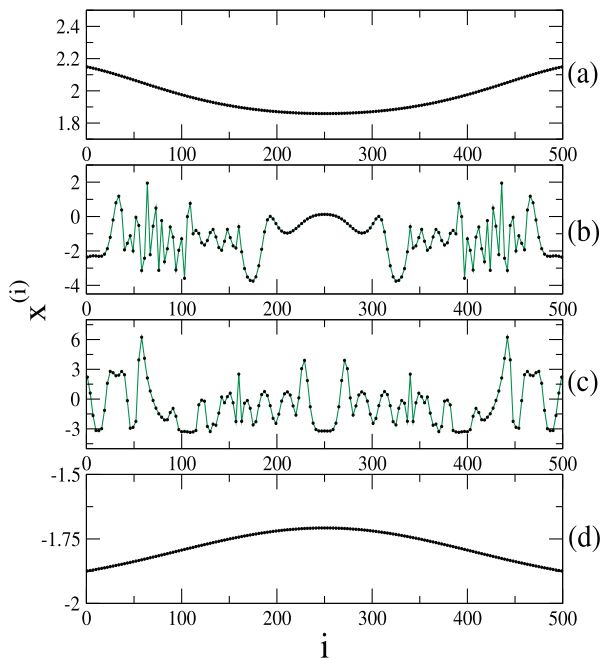


Figure 5. Snapshots of spatial pattern $x_i \times i$ for 500 Rössler oscillators at particular time equal to 23×10^3 , $t = 2.3 \times 10^4$, $d = 0.1$ and $\omega = 0.1$. We consider (a) $\varepsilon = 0.25$, (b) $\varepsilon = 0.271$, (c) $\varepsilon = 0.3$, and (d) $\varepsilon = 0.35$.

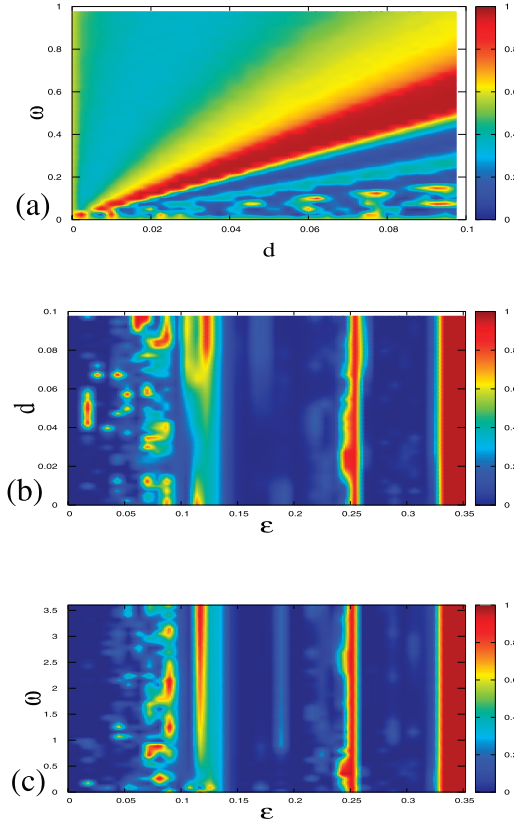


Figure 6. Parameter space for 500 Rössler oscillators: (a) $\omega \times d$ for $\varepsilon = 0.3$, (b) $d \times \varepsilon$ for $\omega = 0.1$, and (c) $\omega \times \varepsilon$ for $d = 0.1$. The colour bar corresponds to the degree of coherence.

For the parameter spaces $d \times \varepsilon$ (figure 6b) and $\omega \times \varepsilon$ (figure 6c), we see large regions (blue) that are related to incoherent patterns. The coherent patterns (red) appear for ε about 0.25 and greater than 0.33. The chimera states occur in small regions and the frontiers between regions with coherent and incoherent behaviour.

4. Conclusions

Spatiotemporal pattern with coexisting coherent and incoherent domains, known as chimera state, has been observed in various types of dynamical systems. The chimera states were found in networks of logistic maps and coupled Rössler oscillators. Due to this fact, we build networks of logistic maps and Rössler oscillators to study some conditions in which the coexistence can appear.

Many studies have demonstrated that external perturbation can induce abundant behaviour. In this work, we consider networks with non-local coupling and under external periodic perturbation. As a diagnostic tool to identify coherence, incoherence, and chimera, we compute the degree of coherence by means of the local order parameter.

We verify coherent and incoherent patterns, as well as coexisting coherent and incoherent domains in the networks composed of logistic maps and Rössler oscillators. For the coupled logistic maps and based on our time simulation, we find chimera states with long lifetimes. Furthermore, the state variables go to infinity for large perturbation amplitude [25]. With regard to the coupled Rössler oscillators, the chimeras are transient because we consider a finite number of oscillators. The chimera states exhibit symmetrical patterns because we use periodic initial conditions in our simulations.

In this work, we focus on the effects of external periodic perturbations on spatiotemporal patterns. We show that the appearance of chimera depends not only on the coupling strength, but also on the amplitude and frequency of the perturbation. Therefore, an external periodic perturbation plays an important role in inducing or suppressing chimera states.

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