Physica A 402 (2014) 8–13

Contents lists available at ScienceDirect

Physica A

journal homepage: www.elsevier.com/locate/physa

Evidence of determinism for intermittent convective transport in turbulence processes

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HIGHLIGHTS

- We present evidence that the intermittent transport has a strong signature of determinism.
- The loss of transversal instability of a manifold is used as the mechanism of bursts.
- A unique parabolic relation between skewness and kurtosis of the emissions is obtained.

ARTICLE INFO

Article history:
Received 19 January 2014
Available online 30 January 2014

Keywords:
Intermittent transport
On–off intermittency

ABSTRACT

In this letter we present evidence that intermittent transport observed in nature, *e.g.*, the scrape off layer of magnetically confined plasma devices, has a strong signature of determinism. We show that the universal distribution of density fluctuation as well as the unique parabolic relation between skewness and kurtosis observed in experimental data can be obtained by a superposition of stochastic and deterministic events. A well known deterministic effect, namely, the loss of transversal stability of periodic orbits embedded in an invariant (inertial) manifold is used to model the spiky nature of the emissions. The intermittent emissions are proposed to be due to local unstable transversal directions of the invariant manifold resulting in an ejection of particles and a consequent burst in the signal. We show that characteristics observed by the emissions namely an impulsive ejection followed by a slow recovery phase can be directly related to the deterministic mechanism proposed.

1. Introduction

Intermittent phenomena are ubiquitous in many different fields of science, from economics and biology, to meteorology and physics [1–4]. Simply put, it can be described as a seemly irregular shift between at least two different dynamical states. Even though many different types of intermittency have already been described and thoroughly scrutinized in the literature, there are still reports of intermittent phenomena lacking a better theoretical explanation. For example several recent works have reported the finding of a unique parabolic relation between the kurtosis and skewness of intermittent time series [1], in this paper we present numerical evidence that this statistical feature might be related with a very specific type of intermittency, namely *on–off intermittency* [5].

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http://dx.doi.org/10.1016/j.physa.2014.01.053

The third and fourth statistical moments become most important whenever a distribution deviates from gaussianity, as they measure its asymmetry and flatness, respectively. For a Gaussian distribution they are trivial quantifiers, but in a more general scenario they might be different from zero and usually independent. A relationship between them in a natural process denounces the existence of an underlying dynamical mechanism. Which is the case reported in works of meteorology [6–8], turbulence [9,10] and plasma physics [1,11].

Among these scenarios, one that has received much attention is that of plasma turbulence and transport, since this is an important limiting factor of plasma confinement in magnetically restrained devices. The control of such events is an active research field since many years [12,13]. In particular the turbulence of the scrape-off layer of plasma confined devices presents intermittent events characterized by a high velocity flux of particles in the radial direction [11], sometimes called avaloids [14]. Those are coherent structures that play a crucial role in the radial transport of the plasma and its appearance is intermittent in time. Such characteristics of the plasma emissions are observed in many devices, TORE SUPRA, MAST, ALCATOR C-MOD, TORPEX, TEXT-U, ADITYA, TJ-I, DIII-F and follow almost the same characteristics independently of the machine [1,14–16]. In general they are composed roughly by two components: an uncorrelated broad band emission (independent fluctuations) superposed by burst events that make the process to be non diffusive (non Gaussian).

A statistical behavior of the density fluctuation of plasma emissions has been experimentally investigated for long time [13]. Such statistical description of the intermittency displayed by the plasma turbulence seems to be useful to explain some underlying physical experiment [1]. Nevertheless within the frame of non-linear dynamics theory, the complete understanding of the mechanism seems far to be known. In fact a self-consistent dynamical description of how the process starts as well as its evolution occurs is not yet understood [1,17]. At the moment, there are only statistical descriptions [1], models based on random processes [18] or more accurate models based on coupled 2D (or 3D) partial differential equations [19], which are very complicated to deal with dynamically. Therefore, up-to-date no simple deterministic explanation has been provided.

In this way, the opportunity to give a self-consistent description of the mechanism behind such signals and still presenting their statistical properties turn to be an important goal of the research on the onset of turbulence field [20,21]. The purpose of this letter is to present evidence that the bursty behavior displayed in some non-Gaussian processes shares several important statistical features with a deterministic process, namely the transversal instability of unstable periodic orbits embedded in an invariant manifold. It is based on the fact that, excluding transient times, in general for dissipative systems the dynamics take place in an invariant manifold, sometimes called the inertial manifold [22]. A prototypic model is presented and the results are corroborated by the behavior of a partial differential equation, namely the nonlinear Schrödinger equation (NLSE), a well known model that describes many phenomena in plasmas, from the coupled dynamics of the electric-field amplitude and the low-frequency density fluctuations of ions [23] to high intensity laser beam propagation [24]. The NLSE has also applications in other areas of science as Bose–Einstein condensation [25], fluid wave theory [26] and nonlinear optics [27]. Although we have used only two examples, the results obtained by both models are rather general and characteristic of the mechanism of instability dealt with here.

2. A simple model

In order to make clear the basic mechanism proposed for the bursting observed in several different time series, we consider first a simple model based on the following 2-dimensional dynamical system [28]

$$x_{n+1} = (1-\epsilon) r x_n (1-x_n) + \epsilon \mathcal{W}([0,1]), \tag{1}$$

$$\phi_{n+1} = \frac{1}{2\pi} p x_n \sin(2\pi\phi_n). \tag{2}$$

Here x displays a chaotic dynamics (for $\epsilon = 0$) and works as a driver for the ϕ dynamics. The scheme is a driver–slave since the driver is not forced by the ϕ dynamics. For $p \gtrapprox p_c \approx 1.725$ the symmetric (invariant) subspace $\phi = 0$ loses stability because of a bifurcation and an on–off intermittency phenomenon starts [28]. Here, we set the parameters $r = 3.8$, $p = 1.73$ fixed, such that the *on–off regime* is analyzed. The stochastic amplitude parameter ϵ is defined in the interval $[0, 1]$, and $\mathcal{W}(I)$ is a white noise in the interval I.

As we will see later on, under the influence of the driver x, ϕ displays a bursty dynamics amid periods of quiescence. Since the time series we are interested present an almost stochastic behavior in the inter-burst period, we have added a Gaussian noise to the ϕ dynamics, with average $\mu = 0.0$ and standard deviation $\sigma = 1.0 \times 10^{-2}$. The purpose of this additive noise is to mimic the noisy background of typical experimental time series, it does not play any role in ϕ dynamics.

In Fig. 1 we plot a time series for the ϕ dynamics described by Eq. (2), for $\epsilon = 0.0$. In panel (a) an overview of the emissions is displayed, it is evident the dual characteristic of the signal, namely a stochastic signal superposed by intermittent bursts. It is possible to observe the great variability in the time between bursts as well as the amplitude of the bursts. The intermittent spikes are the result of the transversal instability of unstable orbits embedded in the invariant manifold ($\phi = 0$). In this scenario, when an orbit visits the vicinity of the manifold and under the influence of transversally unstable directions of periodic orbits embedded on it, it will be ejected from that neighborhood exponentially. The time the divergence occurs and how fast will be the exponential rate depend on how tangent to the stable longitudinal direction the orbit is injected.

As a result of that mechanism, the amplitude of the burst will vary. After the ejection, the *global* stability of the manifold ($\phi = 0$) makes the orbit start to approximate again to it. It is the recovery phase. The global stability is an average over

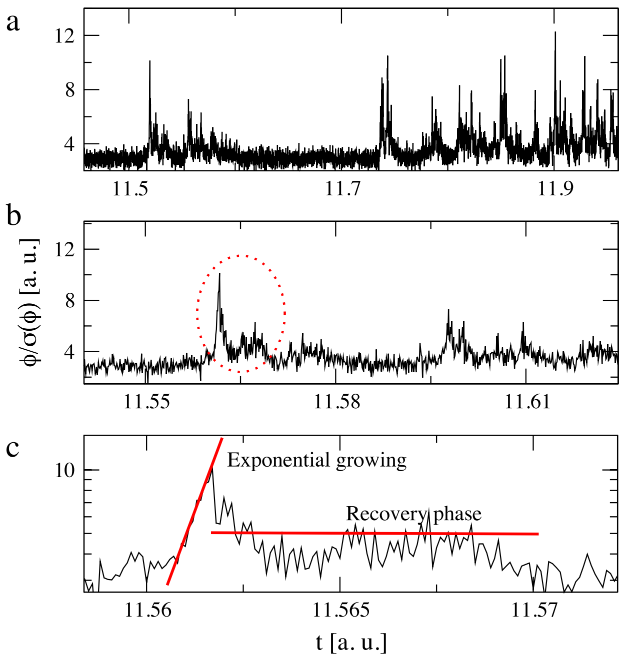

Fig. 1. (Color online) Time series for the ϕ signal normalized by its standard deviation $\sigma(\phi)$ (rescaled time). In panel (a) a long time sampling is shown. The intermittent nature of the spikes and their similarity to experimental signals of the ion saturation current reported in Ref. [15] are clear. In panel (b) and (c) two magnifications are displayed. The fast growing and slow recovery phases are a signature of the dynamical mechanism presented here.

the transversal local stability of all the periodic orbits embedded in the manifold. Nevertheless the recovery phase will be much slower than the ejection since once apart from the vicinities of the manifold the stable and unstable directions of a particular periodic orbit embedded in the manifold are not distinguished anymore. In this case just the average convergence to the manifold affects the orbit. Since we are near a bifurcation point where the manifold is losing its global stability it is expected a slow recovery phase since the rate of convergence is almost null.

In panel (b) and (c) of Fig. 1 two magnifications of smaller time stamps are shown. In panel (b) a special burst event is highlighted and is displayed in panel (c) (ϕ in log-scale) where we can identify the almost exponential ejection and the slow recovery phase as explained above. The behavior discussed here can be compared to the shape of an avaloid [14,15] where authors point out the bursts of the experimental signal present a fast growing phase and a *nonexponential decay*. Such behavior is observed to occur in many plasmas devices [29,30]. Sometimes the recovery phase is reported as resulting from some "*relaxation phenomena*" and their physical mechanism is still unknown [15]. So the fast (exponential) growing and the slow recovery phases displayed in experimental signals show similar features to the phenomena described in our model by the ϕ dynamics.

The probability density function (PDF) for the ϕ signal superposed with the Gaussian noise is depicted in Fig. 2(a). The effect of the invariant manifold ejection observed in the PDF is a large tail, resulting from rare events (the spikes observed in Fig. 1). The PDF is very similar to other ones obtained from the experimental data of magnetically confined plasma devices [13, Fig. 3], [11, Fig. 3(b)], [15, Fig. 3] and [14, Fig. 5]. In Fig. 2(b) we plot the waiting time distribution between successive bursts. Observe that the waiting time is not centered around any specific τ value but instead have a wide distribution. This fact is also reported by experimental works [31] and associated to nonperiodical processes.

Other important features obtained from the experimental signal provided by several plasma devices are the time between two bursts as well as their amplitudes [15]. The analysis of the relation between the inter-bursts time (τ) and the amplitude of the bursts shows no relation between them. As we observe in Fig. 3 there is no trivial relation between the inter burst time and the next maximum amplitude of ϕ. Such behavior of the signal once again can be understood considering the on–off mechanism of ejection described in this letter. Since the ejections are caused by local unstable directions, a large burst occurs when the orbit is injected close to a stable manifold of a particular periodic orbit embedded in the invariant manifold ($\phi = 0$). In such cases the ejection occurs almost tangent to the unstable manifold leaving the orbit to suffer the ejection for a long period of time. Such tangent injection is not related to the time the orbit stays near the manifold (τ) and consequently the next ejection will be unrelated to the time the orbit spends in the vicinities of the manifold ($\phi = 0$). Fig. 3 shows that the amplitude of the bursts and the inter burst time do not follow any simple rule and can be easily compared with the experimental data [15].

Another feature of the model can be described when we consider the skewness and kurtosis of data emissions fluctuations

$$S(x) = \langle x^3 \rangle / \langle x^2 \rangle^{3/2}, \tag{3}$$

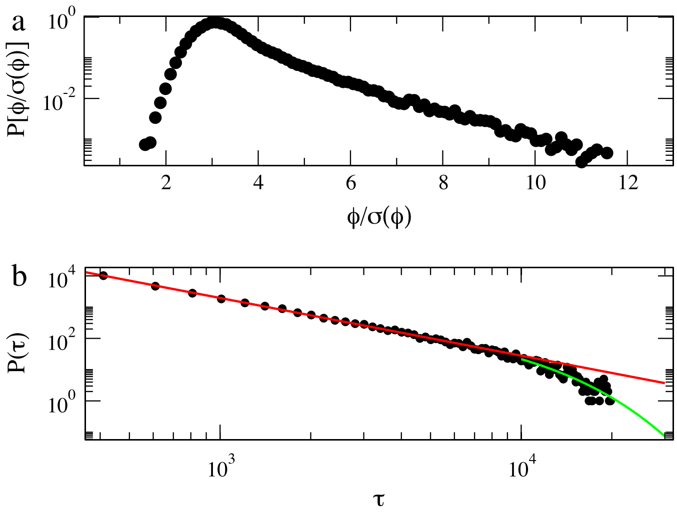

Fig. 2. (Color online) (a) Semilogarithmic plot of the PDF obtained from the ϕ signal. The long tail observed in the curve results from the intermittent bursts caused by the ejection of the invariant manifold ($\phi = 0$). (b) PDF of the inter bursts time. A signature of the physical mechanism behind the emission – the transversal instability of a inertial manifold can be observed in the power law fitting (red (gray) curve) for small inter burst time followed by a heavy tail (exponential fitting – green (light gray) curve) for large time interval. Such behavior is expected since large time intervals are inhibited by the presence of noise in the system.

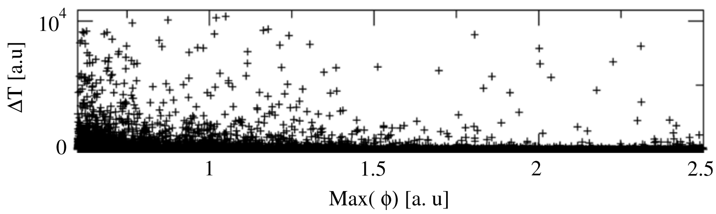

Fig. 3. Interval of time ΔT between two bursts that have reached a given threshold as a function of the burst amplitude.

$$K(x) = \langle x^4 \rangle / \langle x^2 \rangle^2. \tag{4}$$

In many physical situations ranging from pollutant diffused and convected by the turbulent atmosphere [32] to density fluctuation of plasma devices [11] the relation between K and S presents a well defined quadratic relation $K = \alpha S^2$. Here we claim that such observation is a clear signature of deterministic processes occurring in the physical system. In order to support this we plot in Fig. 4 the kurtosis as a function of the skewness for the signal ϕ described by Eqs. (1)–(2). For the black dots the dynamics is driven by a deterministic chaotic signal, the x dynamics for $\epsilon_1 = 0$. In that case the data spread quite well along a quadratic relation between kurtosis and skewness. On the other hand the blue (dark gray) dots present the same situation when the ϕ dynamics is driven by an almost stochastic driver. For this case we set $\epsilon_1 = 0.4$ such that the driver is not deterministic anymore. Here the result is similar to the one expected from a completely diffusive process, the skewness turns to be almost zero while the kurtosis converges to the value 3. As an intermediate example, the green (light gray) curve represents the case where we have used $\epsilon_1 = 0.1$, such that the driver can be characterized as partially deterministic (some noise level added). Clearly, the data spread along the parabolic fitting as the ϵ_1 parameter is varied. The red (gray) line is the quadratic fitting $K = 3.49 + 0.04S + 1.32S^2$ of the black dots curve.

In order to study how such phenomena behave in a large degree of freedom problem we consider a second example. The dynamics displayed by the forced and linearly damped NLSE (FD-NLSE) [33,34]. The NLSE is a universal nonlinear model for many physical systems. The equation can be applied to hydrodynamics, nonlinear optics, quantum condensates, and various other nonlinear instability phenomena and is described by a partial differential equation given in the form.

$$\psi_t - \psi_{xx} - (g|\psi|^2 - \Omega^2)\psi = \Gamma - i\nu\psi \tag{5}$$

where Γ and ν are forcing and damping rates. $\Omega^2 = 0.45$ and $g = 2.0$ are kept constant. In its conservative form ($\Gamma = \nu = 0$) the Eq. (5) admits envelope soliton solutions and has many applications in nonlinear optics and plasma physics. In that regime it preserves some quantities, including the mass balance

$$M(t) = \frac{1}{L} \int_{-L/2}^{L/2} |\psi(x, t)|^2 dx. \tag{6}$$

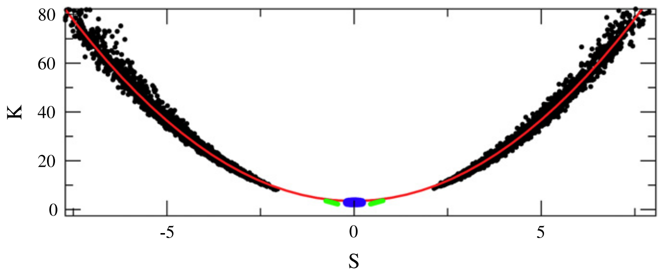

Fig. 4. (Color online) $K(S)$ curve for the dynamical system (2)–(2). For the black circles the ϕ dynamics is driven by a deterministic chaotic signal. The red (gray) line is a quadratic polynomial fitting for the black curve. The blue (dark gray) dots arise from the ϕ dynamics driven by an almost random signal. For this case the well known result $S = 0$, $K = 3$ expected for a diffusive process is observed. The green (light gray) curve is representative of an intermediate case, when the driver is partially deterministic and partially Gaussian noise. As can be observed the data spreads along the quadratic fitting.

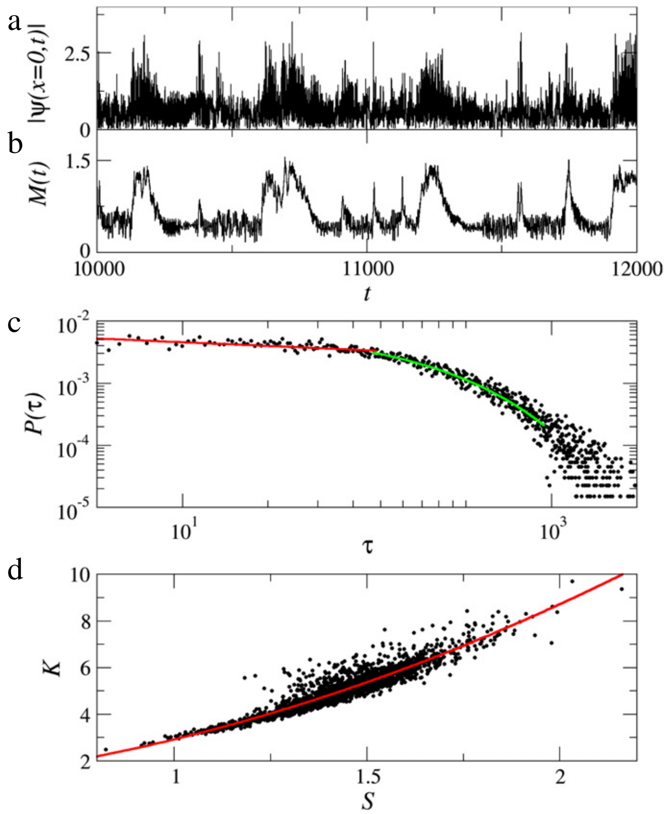

Fig. 5. (Color online) (a) Time series for the ψ dynamics considering a fixed spatial position. (b) Time series for the mass balance M displayed by the FD-NLS. An intermittent regime displaying a fast ejection from the diffusive-like regime and a slow recovery phase is observed as a result for the on–off intermittency regime. (c) The probability distribution of the inter spiky time for the M signal. The red (gray) and green (light gray) curves are, respectively power law and exponential fittings. (d) K as a function of S for the mass balance signal of the FD-NLS. The red (gray) curve is a quadratic fitting $K = 0.7 + 0.4S + 1.8S^2$.

Here we integrate the FD-NLSE considering periodic ($0 < x < L$ such that $2\pi/L = 0.9$) boundary condition. For the forced and damped regime with $\Gamma = 0.3$ and $\nu = 0.01$ the FD-NLSE displays intermittent oscillations. As we will see later on in this paper, the global behavior of the NLSE can be summarized by the following characteristics: the signal has a chaotic dynamics interrupted by impulsive bursts. Here we interpret the chaotic background as a result of the dynamics embedded in a particular subspace of the dynamical system that loses transversal stability. As a result of such an instability the signal presents intermittent bursts. This lower dimensional subspace can be related to the inertial manifold in our previous example ($\phi = 0$ in Eq. (2)). As an example of the behavior presented by the model, a time series for a fixed spatial position for the ψ-emission is displayed in Fig. 5(a). In order to clarify the intermittent character of the emission, we plot in Fig. 5(b) the M time series. Two levels of oscillations are observed in the signal. The lower branch is characterized as a diffusive regime and it is interrupted intermittently by bursts. The intervals between bursts and the time the system stays in the upper regime do not follow a regular pattern.

Generally the ejection from the lower state is faster than the injection on it. As explained before in this letter, such behavior is due to the mechanism of ejection, related to the on–off intermittency phenomenology. To reinforce the on–off scenario we plot in Fig. 5(c) the inter-burst distribution time for the M time series. We observe that it follows a power law fitting ($P(\tau) \sim \tau^{-\alpha}$ with $\alpha \approx 0.2$) for small intervals of time but displays a exponential fitting for larger time intervals. The exponential fitting for large time intervals is due to the unavoidable noise level induced in the dynamics by the numerical simulations. On the other hand in an experimental setup such behavior is also observed since some noise level is always present in real situations. Finally in 5(d) we plot the $K \times S$ curve for the FD-NLSE. Again, the nonstationarity of the time series results in a unique parabolic fitting for the emissions.

In conclusion we have shown that some recent observations of spiky emissions in the scrape off layer of plasmas device can be related to a known deterministic phenomena, namely the loss of transversal instability of an invariant manifold embedded into the dynamics. We show that in such a scenario we can obtain good agreement between the experimental data and numerical simulations. In particular we show that the unique parabolic relation between kurtosis and skewness is observed if we consider a deterministic mechanism of ejection of the invariant manifold.

Acknowledgments

This work is partially supported by CNPq, CAPES and Fundação Araucária.

References

[1] I. Sandberg, et al., Phys. Rev. Lett. 103 (2009) 165001.
[2] M.-K. Yum, J.-H. Kim, Pediatr. Res. 53 (2003) 915–919.
[3] G.G. Katul, et al., Phys. Fluids 6 (1994) 2480.
[4] A.C.L. Chian, et al., Chaos Solitons Fractals 29 (2006) 1194–1218.
[5] N. Platt, et al., Phys. Rev. Lett. 70 (1993) 279–282.
[6] S. Alberghi, et al., J. Appl. Meteorol. 41 (2002) 885–889.
[7] T.P. Schopflocher, P.J. Sullivan, Bound.-Layer Meteorol. 115 (2005) 341–358.
[8] P. Sura, et al., J. Phys. Oceanogr. 38 (2008) 639.
[9] A. Maurizi, Nonlinear Process. Geophys. 13 (2006) 119–123.
[10] K.R. Sreenivasan, R.A. Antonia, Annu. Rev. Fluid Mech. 29 (1997) 435.
[11] B. Labit, et al., Phys. Rev. Lett. 98 (2007) 255002.
[12] W. Horton, Phys. Rep. 192 (1990) 1.
[13] R. Jha, et al., Phys. Rev. Lett. 69 (1992) 1375.
[14] G.Y. Antar, et al., Phys. Rev. Lett. 87 (2001) 065001.
[15] G.Y. Antar, et al., Phys. Plasmas 10 (2003) 419.
[16] J.A. Krommes, Phys. Plasmas 15 (2008) 030703.
[17] O.E. Garcia, et al., Phys. Rev. Lett. 92 (2004) 165003.
[18] O.E. Garcia, Phys. Rev. Lett. 108 (2012) 265001.
[19] O.E. Garcia, et al., Phys. Plasmas 12 (2005) 062309.
[20] P.P. Galuzio, S.R. Lopes, R.L. Viana, Phys. Rev. Lett. 105 (2010) 055001.
[21] P.P. Galuzio, S.R. Lopes, R.L. Viana, Phys. Rev. E 84 (2011) 056211.
[22] Y. Hong-liu, G. Radons, Phys. Rev. Lett. 108 (2012) 154101.
[23] V.E. Zakharov, A.B. Shabat, Sov. Phys.—JETP 34 (1972) 62.
[24] M.V. Goldman, Rev. Modern Phys. 56 (1984) 709.
[25] Kevin E. Strecker, et al., Nature 417 (2002) 150.
[26] C. Kharif, E. Pelinovsky, Eur. J. Mech. B/Fluids 22 (2003) 603.
[27] K. Tai, A. Hasegawa, A. Tomita, Phys. Rev. Lett. 56 (1986) 135.
[28] Y.C. Lai, Phys. Rev. E 53 (1996) R4267.
[29] M.V. Umansky, et al., Phys. Plasmas 5 (1998) 3373.
[30] P. Devynck, et al., Plasma Phys. Control. Fusion 42 (2000) 327.
[31] Y.H. Xu, et al., Plasma. Phys. Control. Fusion 47 (2005) 1841.
[32] P.C. Chatwin, D.M. Lewis, N. Mole, Adv. Comput. Math. 6 (1996) 227.
[33] E. Shlizerman, V. Rom-Kedar, Phys. Rev. Lett. 96 (2006) 024104.
[34] I. Barashenkov, E. Zemlyanaya, Phys. Rev. Lett. 83 (1999) 2568.