Field-line stochasticity in a Tokamak with an ergodic magnetic limiter

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Abstract. *We estimate the threshold value of the perturbation strength needed to cause magnetic field-line stochasticity in the peripheral region of a Tokamak with an ergodic magnetic limiter, accoriding to a Hamiltonian description for field lines. We model the limiter action as a periodic sequence of impulsive excitations. Two global stochasticity prescriptions are used: Chirikov's overlapping criterion and the renormalization scheme of Escande, Doveil and Benkadda.*

1 Introduction

There are various plasma magnetic confinement schemes devised for fusion applications, and the Tokamak is one of the most promising candidates to achieve this goal in the future. Tokamaks are basically toroidal pinches in which a plasma column is formed by ohmic heating produced by electric fields generated by transformer coils. The plasma torus is then confined by the superposition of two basic fields: a toroidal magnetic field produced by coils wound around the Tokamak, and a poloidal field generated by the plasma column itself. The combination of these fields yields helical magnetic field lines (Wesson, 1987).

Several drift as well as field curvature effects, however, lead to complicated particle motions in the plasma, even though it is supposed to be in a macroscopic equilibrium state. Plasma–wall interactions occur frequently as a result of collisions between the Tokamak inner metallic wall and particles which escape from the plasma and cross the vacuum region that surrounds it.

One of the main technological problems in the operation of Tokamaks is the control of these plasma–wall interactions. The quality of the confinement is affected by impurities released from the inner wall due to localized heat and particle loadings. Much effort has been devoted in recent years to reduce these interactions, in order to decrease the impurity content in the plasma core. The

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ergodic magnetic limiter (EML) is such a device, since its main purpose is to create a 'cold' boundary layer of stochastic magnetic field lines in the peripheral region of the Tokamak (Feneberg, 1977; Karger & Lackner, 1977). The words 'ergodic', 'stochastic' and 'chaotic' will be used here as synonyms, according to the current usage in the Tokamak literature.

Theoretical studies suggest that a stochastic field can enhance heat and particle diffusion in this region, so as to uniformize these loadings on the metallic wall (Engelhardt & Feneberg, 1978). A number of experiments involving EML action in various Tokamaks, such as TEXT (McCool *et al.*, 1989), CSTN-II (Takamura *et al.*, 1987) and TORE SUPRA (Grosman *et al.*, 1995), have shown a decrease of intrinsic impurity levels consistent with a lower edge electron temperature, while interior values are not significantly affected.

A recent design for EML consists of one or more grid-shaped coils wound around the torus, each of them with toroidally oriented wires conducting a current in opposite senses for adjacent segments (see Fig. 1(a)). The magnetic field generated by these currents falls down rapidly with the distance from the wall, and can interact with the equilibrium magnetic field in order to create chains of magnetic islands in the peripheral region of the torus. Magnetic islands are field-line structures of tubular shape that wind around the plasma. A cross-section of these islands reveals a phase portrait (actually a Poincaré map of the field lines) very similar to pendulum trajectories in phase space (Greene & Johnson, 1965).

Since EML action is a symmetry-breaking perturbation, these islands are expected to have a thin region of stochastic field lines in the neigbourhood of their separatrices (Lichtenberg & Lieberman, 1983). Large-scale stochasticity is achieved by means of the interaction between the outermost island chains. Due to the fast radial decrease of the perturbation field, only these 'external' islands have a significant width. So, the inner region of the plasma column is not supposed to be noticeably affected by the process. This is actually necessary since stochasticity would destroy the plasma core.

Some recent works have been devoted to describing theoretically EML action on Tokamaks from this point of view. Martin and Taylor (1984) have proposed a model in which magnetic field-line behaviour is described by a Poincaré mapping, by taking a given transversal surface of section. Their mapping is locally approxi-

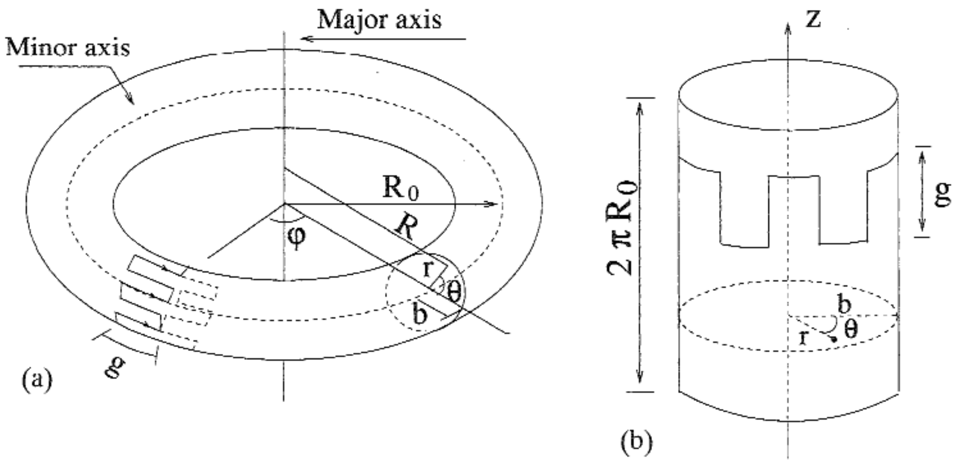

Fig. 1. Essential geometry of the Tokamak and EML: (a) toroidal geometry; (b) cylindrical geometry.

mated by the well-known Chirikov–Taylor standard map, and the threshold of stochasticity was estimated through a perturbative scheme. DeGrassie and co-workers (Yu & DeGrassie, 1986) have improved this mapping, adding torodicity effects and considering a more sophisicated design for EML rings.

The model of Martin and Taylor (1984) was revisited by Viana and Caldas (1992), by means of a flux-function approach to the magnetic island generation and destruction through global stochasticity. The stochasticity threshold was computed through Chirikov's overlapping criterion, which prescribes the touch of neighbouring separatrices as the critical condition to ergodize field lines between these islands (Chirikov, 1979).

A different theoretical framework for the magnetic field structure investigation is the Hamiltonian description for field-line flow. Even for a magnetostatic equilibrium, we can parameterize this flow by means of a spatial ignorable coordinate, i.e. when the equilibrium magnetic field exhibits symmetry with respect to it. This parameter plays the role of time in Hamilton's equations, the other variables being field-line coordinates as well. These techniques enable us to use the powerful methods of Hamiltonian dynamics, such as Kolmogorov–Arnold–Moser (KAM) theory, adiabatic invariance, perturbation theory and so on.

In this paper, we present a Hamiltonian description of field lines for a model very similar to that proposed by Martin and Taylor (1984). The field-line Hamiltonian will be split into two parts: one is the integrable part, physically interpreted as the equilibrium configuration; and the other is a non-integrable part, characterizing the effect of perturbing magnetic fields generated by EML on the equilibrium Tokamak field. The EML perturbation is supposed to be of an impulsive character, i.e. a sequence of periodic delta function pulses. The near-integrable Hamiltonian so obtained shows various resonant terms, from which we take the two main harmonics, in order to describe the two outermost magnetic islands which interact to generate stochasticity.

The resulting expression can be transformed into the so-called paradigm Hamiltonian, in such a way that two different criteria are applied to estimate the stochasticity threshold, namely the Chirikov and the Escande–Doveil–Benkadda (EDB) criteria (Escande, 1985). The EDB criterion gives an accurate result for the stochasticity threshold, since it analyzes the phase-space region between the two resonances in smaller scales, working as a renormalization scheme. The onset of global stochasticity is achieved when the last KAM torus (or magnetic surface) between the magnetic islands is destroyed. We have compared these two criteria and verify that both furnish almost the same result, provided Chirikov's prescription is empirically corrected (by means of the so-called 'two-thirds rule').

This paper is organized as follows. In Section 2, we outline the equilibrium and EML field models to be used. In Section 3 the Hamiltonian description for field lines is presented, with particular emphasis on the model concerned in this work. The following section discusses the global stochasticity criteria to be considered, and Section 5 presents some numerical results for a typical small Tokamak. The last section is devoted to our conclusions.

2 Equilibrium and limiter fields

We suppose a Tokamak with major (minor) radius $R_0(b)$, and with a large aspect ratio ($b/R_0 \ll 1$), such that it can be considered in a first approximation a periodic cylinder of length $2\pi R_0$ (Figs 1(a) and (b)). According to the model

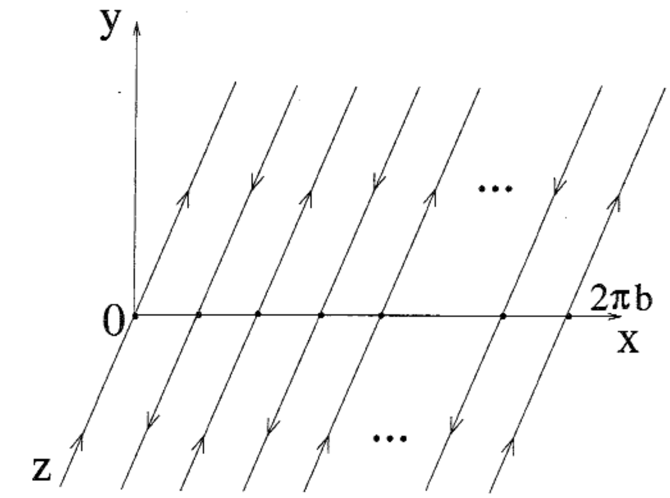

Fig. 2. Rectangular coordinates describing the edge of a Tokamak with an EML.

proposed by Martin and Taylor (1984), we use a particular coordinate system, devised to describe only the peripheral region of the toroidal chamber. Denoting by (r, θ) the polar coordinates on a surface of section $\varphi = \text{constant}$ (φ being the toroidal angle), coordinates in this 'slab geometry' are defined as

$$x = b\theta, \qquad y = b - r, \qquad z = R_0\varphi \tag{1}$$

in such a way that we neglect the poloidal and toroidal curvatures as well. The arc length measured on the Tokamak edge is represented by x, with $0 \leq x < 2\pi b$; while y is the radial distance measured from the inner wall. The internal Tokamak region is denoted by $y \geq 0$; z stands for the rectified toroidal distance on the magnetic axis, so that $0 \leq z < 2\pi R_0$. Obviously, the validity of this rectangular coordinate system is restricted to a region close to the wall ($|y| \ll b$).

The Tokamak equilibrium field is modelled by a cylindrical plasma column, conducting a total current I_P. The details of the current density profile will not be taken into account here, because we are intending to describe the peripheral region of the torus only. In this case, the field components are $\mathbf{B}^{(0)} = (B_x^{(0)}(r), 0, B_z)$, where $B_z = B_0 = \text{constant}$ is a uniform toroidal field. A simple application of Ampére's law gives for the poloidal field component $B_x^{(0)}(y) = \mu_0 I_P / 2\pi(b - y)$.

In this paper, we are considering only one EML ring, consisting of a unique coil, wound around the torus (Fig. 1(b)). The coil shape is designed to have two kinds of segments—toroidally oriented (TO) and poloidally oriented (PO). There are m pairs of TO segments, equally spaced along the toroidal direction. Adjacent TO pieces conduct a current I in opposite senses. In terms of our slab coordinates, they lay on the xy-plane, $\pi b/m$ apart from each other (Fig. 2). The role of PO pieces is to enhance or decrease locally the equilibrium toroidal field B_0 and, since this perturbation is very tiny, we can neglect the contribution of these PO segments and consider only the TO segments.

Assuming that these TO pieces are very long, the magnetic field so generated presents only x and y components, given by (Viana & Caldas, 1991)

$$\tilde{B}_x^{(1)}(x, y) = -\frac{\mu_0 m I}{\pi b} e^{-my/b} \cos\left(\frac{mx}{b}\right) \tag{2a}$$

$$\tilde{B}_y^{(1)}(x, y) = \frac{\mu_0 m I}{\pi b} e^{-my/b} \sin\left(\frac{mx}{b}\right) \tag{2b}$$

These expressions were derived by supposing that: (i) the penetration time of the metallic wall is sufficiently small (typically 100 μs for a plasma current of 9 ms); (ii) the dynamical plasma response is negligible (for low beta values), so that we can deal only with vacuum fields, for which $\tilde{\mathbf{B}}^{(1)} = \nabla\Phi^{(1)}$, where $\Phi^{(1)}$ is a scalar magnetic potential satisfying the Laplace equation $\nabla^2\Phi^{(1)}(x, y) = 0$ and proper boundary conditions at the interface $y = 0$. The solution is a superposition of harmonics, but for our purpose only the lowest-order mode, equation (2), is sufficient.

In the following calculations, we will describe the localized character of this perturbation by supposing a periodic sequence of delta function pulses, which modulates the magnetic field components in equation (2), in the form: $B_{x,y}^{(1)}(x, y, z) = \tilde{B}_{x,y}^{(1)}(x, y)\delta(z/2\pi R_0)$. A more realistic model would assume a square pulse waveform of length g, but a Fourier analysis shows similar results if $g \ll 2\pi R_0$, and the mode number n (to be defined later) is not too large (Caldas *et al.*, 1996).

3 Hamiltonian description

The structure of magnetic fields in a static and non-symmetric configuration can be studied by means of a Hamiltonian description for field lines. It is based on the similarity between the field-line equations, $\mathbf{B} \times \mathbf{dl} = \mathbf{0}$, and Hamilton's equations. This analogy was first described by Kerst (1962) and later applied to various problems involving plasma confinement schemes: Tokamaks (Hamzeh, 1974), Stellarators (Filonenko *et al.*, 1967), Levitrons (Freis *et al.*, 1973) and Compact Tori (Viana, 1995), among others.

Since the configurations to be studied are magnetostatic, the role of time is played by a given spatial coordinate. It is normally the ignorable coordinate, i.e. the system exhibits a certain symmetry with respect to it. So, the Hamilton's equations in this case are not actually dynamical, but rather describe the spatial (Lagrangian) structure of the magnetic field. A general formulation (in a curvilinear coordinate system) of this problem has been proposed by Whiteman (1977).

In the case of our rectangular coordinates (x, y, z) describing the edge region of the Tokamak, we call (x, y) the canonical coordinate and momentum respectively. In the equilibrium (symmetric) case, magnetic field components do not depend on z, thus z is our 'time' variable, because it parameterizes the field-line flow: given initial conditions $(x_0, y_0, z_0 = 0)$ the position of a field line at a later 'time' is characterized by the parametric equations $x = x(x_0, y_0\, z_0; z)$ and $y = y(x_0, y_0, z_0; z)$. The field-line equations $(\mathbf{B}^{(0)} + \mathbf{B}^{(1)}) \times \mathbf{dl} = \mathbf{0}$ are cast in a Hamiltonian form

$$\frac{dx}{dz} = \frac{\partial H}{\partial y} = \frac{B_x^{(0)} + B_x^{(1)}}{B_0} \tag{3a}$$

$$\frac{\mathrm{d}y}{\mathrm{d}z} = -\frac{\partial H}{\partial x} = \frac{B_y^{(1)}}{B_0} \tag{3b}$$

in which $H(x, y, z)$ is the field-line Hamiltonian. It describes (in the non-symmetric case) a non-autonomous, one-degree-of-freedom system, and thus is not generally integrable.

This case opens the possibility of many dynamical features like periodic, quasi-periodic and even chaotic behaviour. We emphasize that 'chaos' here is to be intended in the Lagrangian sense: two field lines, initially very close, will diverge exponentially in space, following the parameterization introduced by the ignorable coordinate. So, we are speaking of a spatial phenomenon, rather than a proper temporal evolution.

The equilibrium part of the magnetic field, however, does represent an integrable system, because it has the necessary symmetry with respect to z. This means that the corresponding Hamiltonian is a function of the momentum y only, and (x, y) are actually action-angle variables for the system. Supposing that the perturbation caused by the limiter is sufficiently weak, and using equations (2) and (3), we write the field-line Hamiltonian for this model in the standard form of a near-integrable system

$$\begin{aligned} H(x, y, z) &= H_0(y) + H_1(x, y, z) \\ &= \frac{1}{2\pi R_0}\left(\alpha y + \frac{\beta}{2} y^2\right) + \Gamma\, e^{-my/b} \cos\left(\frac{mx}{b}\right)\left[1 + 2\sum_{n=1}^{\infty} \cos\left(\frac{nz}{R_0}\right)\right] \end{aligned} \tag{4}$$

where

$$\alpha = \frac{\mu_0 I_P R_0}{B_0 b}, \qquad \beta = \frac{2\alpha}{b}, \qquad \Gamma = \frac{\mu_0 I}{\pi B_0} \tag{5}$$

and the term within brackets stems from the Fourier decomposition of the periodic delta function used to modulate the limiter field in the z-direction.

As the limiter field falls down exponentially with y, we are interested only in the two outermost resonances presented by equation (4), since they generate chains of magnetic islands in the peripheral region of the Tokamak, which will be considered for stochasticity analysis. These resonances are characterized by mode numbers (m, n_0) and $(m, n_0 + 1)$ respectively. Picking up only those resonances in equation (4), we get to the truncated two-mode Hamiltonian

$$H^{(1)}(x,y,z) = \frac{1}{2\pi R_0}\left(\alpha y + \frac{\beta}{2} y^2\right) + \Gamma\, e^{-my/b}\left[\cos\left(\frac{mx}{b} - \frac{n_0 z}{R_0}\right) + \cos\left(\frac{mx}{b} - \frac{(n_0+1)z}{R_0}\right)\right] \tag{6}$$

It is possible to obtain an analytical estimate of n_0 by considering that a (m, n_0) resonance generates an island centred at a magnetic (KAM) surface with a rational rotation number. In the plasma physics literature (Greene & Johnson, 1965), it is customary to work with a safety factor $q = \mathrm{d}\varphi/\mathrm{d}\theta$ so that $q = m/n_0$ for a rational magnetic surface. Using equations (3a) and (5), and taking only the equilibrium magnetic field, one finds that n_0 may be the nearest integer to the expression

$$\frac{\beta m}{4\pi}\left(1 - \frac{y}{b}\right)^{-1}$$

where y stands for the magnetic surface radial location.

Now, making some canonical transformations (see Appendix A for details) as well as rescalings (to non-dimensionalize quantities), we obtain the following Hamiltonian

$$H_p(X, Y, t) = \frac{1}{2} Y^2 + M \cos X + P \cos (X - t) \tag{7}$$

where the coefficients of the cosines were evaluated at the resonance locations, namely $Y_0^\star = 0$ (for M) and $Y_1^\star = 1$ (for P), giving (see equations (A7) and (A8))

$$M \equiv \frac{2m^2 R_0 \mu_0 I}{\beta b^2 B_0} \exp\left(\frac{m\hat{\alpha}}{b\beta}\right), \qquad P \equiv \frac{M}{e} \tag{8}$$

where we have defined

$$\hat{\alpha} \equiv \alpha - \frac{2\pi b n_0}{m} \tag{9}$$

The rescaled coordinates (X, Y, t) are related to the original variables (x, y, z) through the following relations

$$X = \frac{mx}{b} - \frac{n_0 z}{R_0}, \qquad Y = \frac{my}{b} + \frac{\hat{\alpha}}{\beta}, \qquad t = \frac{z}{R_0} \tag{10}$$

where we have used equation (A5).

4 Global stochasticity

The Hamiltonian (7) is usually called 'paradigm', since it is the simplest non-integrable system to show local as well as global stochasticity. It can also be derived, for instance, through the analysis of the dynamics of a charged particle under the potential of two electrostatic waves with different frequencies and amplitudes (Escande, 1985).

If $M \neq 0$, $P = 0$, (7) reduces to the pendulum Hamiltonian, and the corresponding resonance half-width (located at $Y_0^\star = 0$) is

$$\Delta Y_0 = 2\sqrt{M} \tag{11}$$

If $M = 0$, $P \neq 0$, equation (7) still reduces to an integrable system, although it is time dependent, corresponding to a displaced pendulum centred at $Y_1^\star = 1$, and with half-width given by

$$\Delta Y_1 = 2\sqrt{P} \tag{12}$$

The case where both M and P are non-vanishing is far more complex. Integrability no longer exists, but if M and P are small enough, the phase-space structure comprises the coexistence of both resonances, separated by a distance $\delta Y = Y_1^\star - Y_0^\star = 1$. But these islands are no longer well defined, because their separatrices become a thin layer of chaotic field-line behaviour. Nevertheless, if the perturbation is not sufficiently strong, there are still a large number of KAM tori between the primary resonances. These tori, whose interceptions with a Poincaré surface of section are curves, act as dikes, preventing large-scale excursion of the stochastic trajectories lying in the local separatrix layers.

As M and P grow together, the intermediate tori are progressively destroyed, according to KAM theory, and the local stochastic layers become wider. But as long as a critical value of the perturbation is not achieved, these layers are somewhat disjoint, and no global stochasticity is attained.

By global stochasticity, we mean large-scale excursion of trajectories, over the entire portion of the phase-space containing the two resonances as well as the intermediate region between them. It is the expected situation for EML operation. So, we are interested in computing the critical strength of perturbation (which in our case is related to the limiter current I, see equation (8)) necessary to achieve this regime.

A theoretical approach to give a reliable estimate of this threshold perturbation strength has been proposed by Chirikov (1979). Let us define a stochasticity parameter as

$$S=\frac{\Delta Y_0+\Delta Y_1}{\delta Y}=2(\sqrt{M}+\sqrt{P}) \tag{13}$$

The original version of Chirikov's criterion prescribes the touching of separatrices in order to generate global stochasticity, that is $S \geq S_c = 1$. But this value is actually overestimated, because higher-order islands (which are not being calculated here) do interact before the primary islands touch themselves. Another effect that might be taken into account is the width of the separatrix stochastic layer. Both effects were considered in order to improve the criterion (Chirikov, 1979).

On the other hand, numerical simulations made with the help of the Chirikov–Taylor standard map indicate that the critical value for S must be somewhat lower than 1 (Greene, 1979). In fact, a value of $S_c=\frac{2}{3}\approx 0.67$ has been proposed in some recent works (Lichtenberg, 1984), known as the 'two-thirds rule'; but it must be emphasized that it consists of an empirical correction to the Chirikov criterion, rather than a different criterion itself. The validity of this rule was first discussed in Escande (1985). As $P=M/e$, the criterion (13) jointly with this correction gives $M_c \approx 0.043$ as the critical value for the resonance amplitude.

Another approach is provided by the EDB renormalization scheme (Escande *et al.*, 1984; Lichtenberg & Lieberman, 1983). We will present here only a brief sketch of this method, more detailed explanations being found in the original papers. For a review, see Escande (1985). The starting point of this scheme is the paradigm Hamiltonian (7), which we rewrite here in a slightly different form

$$H_i(\mathcal{J}_i,\Theta_i,t_i)=\frac{1}{2}\mathcal{J}_i^2-M_i\cos\Theta_i-P_i\cos[k_i(\Theta_i-t_i)] \tag{14}$$

where $(\mathcal{J},\Theta)$ are action-angle variables and k is a 'wavenumber' of the perturbation. As we have seen, the two primary resonances of (14) are located at $\mathcal{J}_i=0$ and $\mathcal{J}_i=1$, with half-widths given by $\Delta\mathcal{J}_{M_i}=2\sqrt{M_i}$ and $\Delta\mathcal{J}_{P_i}=2\sqrt{P_i}$ respectively. We focus our attention on a specific KAM torus between these resonances, namely with $\mathcal{J}_i=u_i$.

Inserting, by means of a canonical transformation, the pendulum action-angle variables into equation (14) and Fourier analyzing the perturbing term, we recover the Hamiltonian (14), but with its parameters rescaled.

$$H_{i+1}(\mathcal{J}_{i+1},\Theta_{i+1},t_{i+1})=\frac{1}{2}\mathcal{J}_{i+1}^2-M_{i+1}\cos\Theta_{i+1}-P_{i+1}\cos[k_{i+1}(\Theta_{i+1}-t_{i+1})] \tag{15}$$

where the 'new' parameters $(M_{i+1}, P_{i+1}, k_{i+1}, u_{i+1})$ are expressed in terms of the 'old' parameters (M_i, P_i, k_i, u_i) by a transformation. This rescaling acts as a 'microscope' in the phase-space region between the primary resonances—the secondary resonances are, loosely speaking, magnified and are the new primary resonances of the renormalized system.

The renormalization process is then continued, such that the transformation between the old and new parameter sets is actually a four-dimensional mapping T. It has two fixed points, when i goes to infinity: (a) $M_i \to 0$, $P_i \to 0$ (stable), corresponding to convergence to a KAM torus with rotation number $g+1$ (where $g = (1+\sqrt{5})/2 \approx 1.618$ is the golden number); (b) $M_i \to \infty$, $P_i \to \infty$ (unstable), which corresponds to a cantorus.

In the parameter space (k, M, P), the analysis of the stable manifold related to the stable fixed point leads to a stochasticity criterion. If $k = 1$ and $\frac{1}{25} < M/P < 25$, or $\frac{1}{4} < k < 4$ and $M/P = 1$, it reads

$$MP^{g-1}[1 + C(k)P^2] = R(k) \tag{16}$$

Plots of the functions $C(k)$ and $R(k)$ can be found in Escande (1985). If $k = M/P = 1$ (the so-called 'central case') this relation reduces to the practical formula (Pettini & Torricelli-Ciamponi, 1988)

$$MP^{g-1} \approx 0.003 \tag{17}$$

which can be used, with good results, for the interval $0.7 < M/P < 1.3$ or even wider, as we shall see in the following.

5 Numerical applications

Let us apply the theory so developed to a small Tokamak, like the TBR-1, operating at the Universidade de São Paulo, Brazil. Its main parameters are listed in Table 1. A typical value of m is 6 for a limiter ring. After equation (10), the radial position of the two outermost resonances, namely the (6, 1) and (6, 2) ones, are given in Table 2, in terms of the plasma current. Note that I_P is the only parameter characterizing the equilibrium model in this work, since we have ignored the details of the plasma current density profile.

The half-widths $\Delta y_{m/n}$ of these two resonances are plotted in Fig. 3 as a function of the limiter current I, with a fixed value of $I_P = 10$ kA for the plasma current.

Table 1. Main parameters of the TBR-I Tokamak, according to Nascimento *et al.* (1994)

Parameter	Value
Major radius (R_0)	0.3 m
Minor radius (b)	0.11 m
Plasma radius (a)	0.08 m
Toroidal field (B_0)	0.5 T (at magnetic axis)
Plasma current (I_P)	10.0 kA (typical value)
Safety factor ($q(a)$)	5.0 (at plasma edge)
Central electron temperature (T_{eo})	200 eV
Central electron density (n_{eo})	$7.0 \times 10^{18}\ \mathrm{m}^{-3}$
Pulse duration (τ_p)	7–9 ms
Filling pressure (p)	10^{-4} torr

Table 2. Radial location of the resonances for $n_0 = 1$ and some values of the plasma current I_P

I_P (kA)	y_0^* (m)	y_1^* (m)
10.0	0.037	0.056
13.0	0.016	0.035
16.0	0.003	0.021

Both islands increase their width as $I^{1/2}$, and the resonance (6, 2) is lower than (6, 1) by a factor of $e^{1/2} \approx 0.606$. Evidently, taking more resonant terms in the Hamiltonian (4) would imply more internal island chains in the Tokamak, but their width would decrease by the same factor when n increases, so their effect is not significant for small values of EML current. This ensures that the inner plasma core would not be noticeably affected by the limiter action.

As we have seen previously, the Chirikov criterion (with the two-thirds rule) yields a critical value of $M_c \approx 0.043$ for global stochasticity resulting from interaction between the two peripheral resonances, which we take to be the (6, 1) and (6, 2) ones. The EDB criterion, according to equation (16), gives a threshold of $M_c \approx 0.040$. The agreement between these two criteria is more clearly observed in Fig. 4, where the critical limiter current I_c is plotted against the plasma current I_P for the two cases. The EDB criterion always furnishes a slightly lower value for the threshold. For the typical value of 10 kA for plasma current, the threshold is achieved with a EML current of less than 100 A.

Moreover, the critical stochasticity parameter in the modified Chirikov prescription (13) is actually higher than the assumed value of $S_c = \frac{2}{3}$. In fact, a value roughly about $S_c = 0.70$ would be necessary for the onset of stochasticity. We have verified this fact by numerical integration of Hamilton's equations for the paradigm Hamiltonian (7). The theoretical explanation for this deviation is that the two-thirds rule is strictly valid only in the 'central case' $M/P = 1$ (Pettini & Torricelli-Ciamponi, 1988), when both resonances are of equal width. As $M/P = e \approx 2.718$, this empirical rule is no longer expected to be rigorously valid. This was the main reason that led us to compare these two prescriptions.

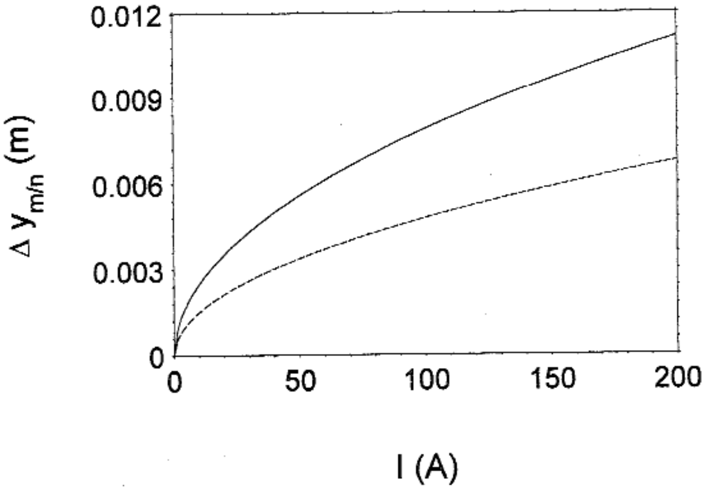

Fig. 3. Half-widths of the (6, 1) (solid line) and (6, 2) (dashed line) resonances versus limiter current.

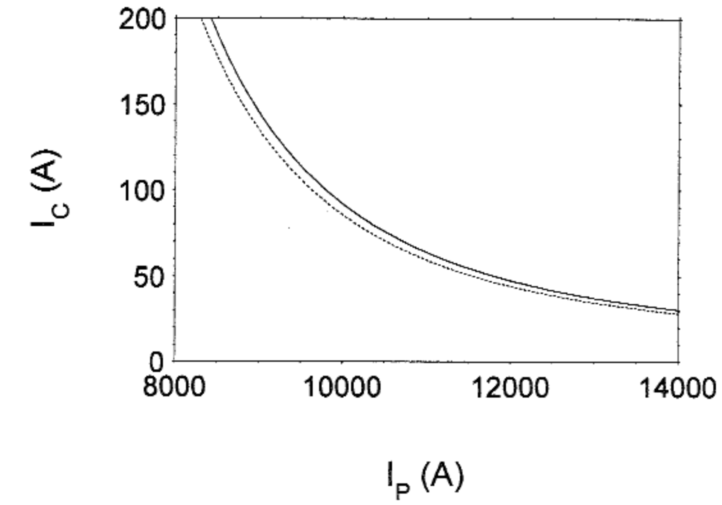

Fig. 4. Critical limiter current for stochasticity generation versus plasma current, according to two global stochasticity criteria: Chirikov (solid line) and EDB (dashed line).

6 Conclusions

A Hamiltonian description for field-line flow, parameterized by a spatial (ignorable) coordinate has been used in this work to describe magnetic island generation in the peripheral region of a large aspect ratio Tokamak. This formulation allows the treatment of global stochasticity in a concise and rather elegant way, since standard results of KAM theory can be applied, jointly with prescriptions for estimating the onset of large-scale stochasticity. However, the question of particle and heat diffusion in this region is far more complex, since field-line stochasticity is not a sufficient condition for that (Atlee-Jackson, 1989). Nevertheless, experimental results have indicated the usefulness of the cold boundary layer created by a limiter in the control of plasma–wall interactions (Grosman *et al.*, 1995; McCool *et al.*, 1989; Takamura *et al.*, 1987).

We have used two criteria for describing the onset of global peripheral stochasticity leading to this boundary layer—the modified Chirikov (with an empirical correction) and the EDB criteria. Both indicate a critical limiter current in the 50–100 A range, for typical parameters of a small Tokamak with plasma current in the 10 kA region. As a result, the ratio I/I_P is not greater than 1%, which is an adequate value for practical operation of such a limiter device. Moreover, the comparison between these two stochasticity criteria indicates that the use of the 'two-thirds rule' is not very accurate in this case, since the resonances have different widths. Numerical evidences suggest a slightly higher value for the critical stochasticity parameter.

Further work on this subject has to consider more appropriate geometries to describe this problem, to include toroidicity effects for example. Another improvement to be made is to include the effects caused by a plasma current density profile, since some portion of the outer plasma column is affected by the stochastic boundary layer.

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References

Atlee-Jackson, E. (1989) *Perspectives on Nonlinear Dynamics* (Cambridge University Press, Cambridge).

Caldas, I. L., Pereira, J. M., Ullmann, K. and Viana, R. L. (1996) Magnetic field line mappings for a Tokamak with ergodic limiters. *Chaos, Solitons and Fractals* 7, 991–1010.

Chirikov, B. V. (1979) A universal instability of many-dimensional oscillator systems. *Physics Reports* **52**, 265–379.

Engelhardt, W. and Feneberg, W. (1978) Influence of an ergodic magnetic limiter on the impurity content in a Tokamak. *Journal of Nuclear Materials* **76/77**, 518–520.

Escande, D. F. (1985) Stochasticity in classical Hamiltonian systems: universal aspects. *Physics Reports* **121**, 165–261.

Escande, D. F., Mohamed-Benkadda, M. S. and Doveil, F. (1984) Threshold of global stochasticity and universality in Hamiltonian systems. *Physics Letters* **101A**, 309–313.

Feneberg, W. (1977) The use of external helical windings for the production of a screening layer in ASDEX and a Tokamak with a material limiter. *Proceedings of the 8th European Conference on Controlled Fusion and Plasma Physics*, Vol. 1, p. 3.

Filonenko, N. N., Sagdeev, R. Z. and Zaslawsky, G. M. (1967) Destruction of magnetic surfaces in Tokamaks by magnetic field irregularities: Part II. *Nuclear Fusion* **7**, 253–256.

Freis, R. P., Hartman, C. W., Hamzeh, F. M. and Lichtenberg, A. J. (1973) Magnetic island formation and destruction in a Levitron. *Nuclear Fusion* **13**, 533–548.

Goldstein, H. (1980) *Classical Mechanics* (Addison-Wesley, New York), Chapter 9, pp. 382–385.

Greene, J. M. (1979) A method for determining a stochastic transition. *Journal of Mathematical Physics* **20**, 1183–1201.

Greene, J. M. and Johnson, J. L. (1965) Hydromagnetic equilibrium and stability. *Advances in Theoretical Physics*, Vol. 1 (ed. K. A. Brueckner; Academic Press, New York), pp. 195–244.

Grosman, A., Monter-Garbet, P., Vallet, J. C., *et al.* (1995) Radiative layer control experiments with an ergodized edge zone in tore-supra. *Journal of Nuclear Materials* **220/222**, 188–192.

Hamzeh, F. M. (1974) Magnetic surface destruction in toroidal systems. *Nuclear Fusion* **14**, 523–536.

Karger, F. and Lackner, K. (1977) Resonant helical divertor. *Physics Letters* **61A**, 385–387.

Kerst, D. W. (1962) The influence of errors on plasma-confining magnetic fields. *Journal of Nuclear Energy* **4C**, 253–262.

Lichtenberg, A. J. (1984) Stochasticity as the mechanism for the disruptive phase of the $m = 1$ Tokamak oscillations. *Nuclear Fusion* **24**, 1277–1289.

Lichtenberg, A. J. and Lieberman, M. A. (1983) *Regular and Stochastic Motion* (Springer, New York).

Martin, T. J. and Taylor, J. B. (1984) Ergodic behaviour in a magnetic limiter. *Plasma Physics and Controlled Fusion* **26B**, 321–340.

McCool, S. C., Wootton, A. J., Aydemir, A. Y., *et al.* (1989) Electron thermal confinement studies with applied resonant fields on TEXT. *Nuclear Fusion* **29**, 547–562.

Nascimento, I. C., Caldas, I. L. and Galvão, R. M. O. (1994) Tokamak research at University of São Paulo. *Journal of Fusion Energy* **12**, 529.

Pettini, M. and Torricelli-Ciamponi, G. (1988) Stochastic magnetic field generation in MHD resistive instabilities: validity limits of linear stability analysis. *Journal of Plasma Physics* **40**, 419–440.

Takamura, S., Ohnishi, N., Yamada, H. and Okuda, T. (1987) Electric and magnetic structure of an edge plasma in a tokamak with a helical magnetic limiter. *Physics of Fluids* **30**, 144–147.

Viana, R. L. (1995) Hamiltonian representation for magnetic field lines in an exactly soluble model. *Brazilian Journal of Physics* **25**, 215–218.

Viana, R. L. and Caldas, I. L. (1991) Comments on the magnetic field generated by an infinite current grid. *European Journal of Physics* **12**, 293–296.

Viana, R. L. and Caldas, I. L. (1992) Peripheral stochasticity in Tokamaks—the Martin–Taylor model revisited. *Zeitschrift für Naturforschung* **47A**, 941–944.

Wesson, J. (1987) *Tokamaks* (Oxford University Press, Oxford).

Whiteman, K. J. (1977) Invariants and stability in classical mechanics. *Reports on Progress in Physics* **40**, 1033–1069.

Yu, X. Y. and DeGrassie, J. S. (1986) Mapping techniques for the GA ergodic magnetic limiter experiment on TEXT. Fusion Research Center, University of Texas Report FRC-292, November.

Appendix A: Derivation of the paradigm Hamiltonian

The truncated Hamiltonian $H^{(t)}(x, y, z)$ which describes the two-resonance system is expressed by equation (6). Let us make a first canonical transformation $(x, y) \rightarrow (p, q)$ by using a generating function of the second kind

$$F_1(x, p, z) = \left(\frac{mx}{b} - \frac{n_0 z}{R_0}\right)\frac{bp}{m} \tag{A1}$$

From the transformation equations (Goldstein, 1980) $q = \partial F_1/\partial p, y = \partial F_1/\partial x$ and $H^{(1)} = H^{(t)} + \partial F_1/\partial z$, we obtain the new Hamiltonian

$$H^{(1)}(p, q, z) = \frac{1}{2\pi R_0}\left(\hat{\alpha} p + \frac{\beta}{2} p^2\right) + \Gamma\, e^{-mp/b}\left[\cos\left(\frac{mq}{b}\right) + \cos\left(\frac{mq}{b} - \frac{z}{R_0}\right)\right] \tag{A2}$$

where $\hat{\alpha} \equiv \alpha - (2\pi b n_0/m)$.

Making a second canonical transformation $(p, q) \rightarrow (\bar{p}, \bar{q})$ through the generating function

$$F_2(q, \bar{p}) = \left(\bar{p} - \frac{\hat{\alpha}}{\beta}\right) q \tag{A3}$$

we have a Hamiltonian without the linear term in $\bar{p}$

$$H^{(2)}(\bar{p}, \bar{q}, z) = \frac{\beta}{2\pi F_0}\frac{\bar{p}^2}{2} + \Omega\, e^{-mp/b}\left[\cos\left(\frac{m\bar{q}}{b}\right) + \cos\left(\frac{m\bar{q}}{b} - \frac{z}{R_0}\right)\right] \tag{A4}$$

where $\Omega \equiv \Gamma \exp\,(m\hat{\alpha}/b\beta)$, and the constant term has been ignored.

Let us introduce dimensionless variables as

$$Y \equiv \frac{m\bar{p}}{b}, \qquad X \equiv \frac{m\bar{q}}{b}, \qquad t \equiv \frac{z}{R_0} \tag{A5}$$

as well as a dimensionless Hamiltonian

$$H_p = \frac{H^{(2)}}{(\beta b^2/2\pi R_0 m^2)} \tag{A6}$$

so that, dividing the Hamiltonian (A4) by the factor $(\beta b^2)/(2\pi R_0 m^2)$ we have

$$H_p(X, Y, t) = \frac{1}{2} Y^2 + T(Y) \cos X + T(Y) \cos\,(X - t) \tag{A7}$$

where the non-dimensional function $T(Y)$ is given by

$$T(Y) \equiv \frac{2\pi m^2 R_0 \Omega}{\beta b^2} e^{-Y} \tag{A8}$$

The two resonances exhibited by (A8) are located at $Y = 0$ and $Y = 1$. If these resonances are not too wide, we approximate the amplitudes in $T(Y)$ to their values at resonance positions. A theoretical justification for this procedure is given in (Escande, 1985). Hence

$$M \equiv T(Y=0) = \frac{2m^2 R_0 \mu_0 I}{\beta b^2 B_0} e^{m\hat{\alpha}/b\beta}, \qquad \text{for} \quad n_1 = n_0 \tag{A9}$$

$$P \equiv T(Y=1) = \frac{M}{e}, \qquad \text{for} \quad n_2 = n_0 + 1 \tag{A10}$$

which gives the paradigm Hamiltonian expressed by equation (7).