Let us look at the physical meaning of (9) and (11):

A. In (9): For a bulk superconductor, all energy levels $E^{(0)}$ ($n = 0, 1, 2, \ldots$) will drop to $E_n^{(0)} - |\Delta E|$ upon taking into account the exterior electric field $\mathscr{E}$.

B. In (11): Taking into account $\mathscr{E}$, the lowest superconductive state vector $\Phi_0^{(1)}(x)$ is a mixed wave function of $\Phi_0^{(0)}(x)$ and $\Phi_1^{(0)}(x)$. The eigenvalue $E_1^{(0)}$ of the latter is the energy level neighboring $E_0^{(0)}$.

3. CONCLUSION AND DISCUSSION: THE CRITICAL VALUE OF $\mathscr{E}$

The reversal between normal and superconductive states of a bulk superconductor could be fine-tuned by changing $\mathscr{E}$. That is, from (9) we get

$$E_0^{(1)} = |\alpha| = |\Delta E| \tag{9a}$$

We call $\mathscr{E} = \mathscr{E}_c$, which preserves $E^{(1)}$ unchanged, the critical electric field strength. In the presence of $|\alpha|$, we can expect $\mathscr{E}_c$ to play an important part in the nucleating area near the surface of a bulk superconductor. In other words, we may be concerned with the relationship between $\mathscr{E}_c$ and H_{c3}. This will be discussed in another paper.

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Stochastic Quantization of the Nonlinear Sigma Model and the Background Field Method

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The background field method is a useful scheme for calculation of the effective action in conventional quantum field theory. In stochastic quantization this approach is introduced by using auxiliary fields, as suggested by Okano. In this work, we implement the background field method, using the normal coordinate expansion, for the nonlinear sigma model on a general Riemannian manifold in the context of stochastic quantization. We also calculate, making use of this novel formulation, the action necessary for investigation of the divergences, at least at the one-loop level.

1. INTRODUCTION

The stochastic quantization method (Parisi and Wu, 1981) provides us with many applications in scalar, gauge, tensor, and string field theories (Damgaard and Hüffel, 1987). The central point of the idea employed in this method was to introduce a fifth (or fictitious) time τ and postulate a stochastic Langevin dynamics for the system by means of a noisy field $\eta(x, \tau)$ with Gaussian correlations. In the equilibrium limit ($\tau \to \infty$), stochastic correlation functions become the N-point Euclidean Green functions. Solving the Langevin equations by an iterative procedure, it is possible to develop perturbation theory for finite τ. One of the advantages of this approach is a prescription for regularization of divergent amplitudes in stochastic diagrams, which is realized by considering a non-Markovian stochastic process (Abdalla and Viana, 1989). A formulation based in the

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path integral representation of the generating functional of the stochastic correlation function has been developed in order to discuss several aspects related to perturbative calculations (Chaturvedi *et al.*, 1986), and is convenient for discussion of renormalization. Recently the background field method was introduced in the stochastic quantization scheme (Okano, 1987).

The background field method (Abbott, 1981) has been a useful technique to investigate divergences of the nonlinear σ-model on a general Riemannian manifold (Alvarez-Gaumé *et al.*, 1981) in order to study possible connections with strings. In this work we implement the background field method for this model (defined on a Riemannian manifold) in the context of stochastic quantization method.

This paper is organized as follows: in Section 2 we briefly review the functional-integral approach and the extended stochastic correlation functions for scalar fields. In Section 3 we display the background field generating functional. In Section 4 we make contact with the formalism of the stochastic quantization of the nonlinear σ-model. In Section 5 we develop the background field method for the nonlinear σ-model on a general Riemannian manifold. We obtain an effective action for further one-loop calculations. Finally, a summary and conclusions are given in Section 6.

2. EXTENDED STOCHASTIC CORRELATION FUNCTIONS

In the essence of the background field method lies the calculation of the effective action, and is necessary to compute trees of proper (1PI) diagrams. However, in its original sense, stochastic quantization does not allow the concept of a proper diagram (Damgaard and Hüffel, 1987). We can circumvent this problem, introducing auxiliary π-fields at the external legs of usual diagrams (Namiki and Yamanaka, 1986).

The main object to be studied here is the extended stochastic correlation function, which includes (for scalar theories) both ϕ and π-fields:

$$\langle \phi(x_1, \tau_1) \cdots \phi(x_N, \tau_N)\pi(y_1, \tau_1') \cdots \pi(y_N, \tau_N')\rangle \tag{1}$$

We can get these N-point correlation functions by using a suitable defined generating functional $Z[J]$ by functional differentiation:

$$\langle \phi(x_1, \tau_1) \cdots \pi(y_1, \tau_1') \cdots \rangle = \left(-\frac{\delta}{\delta J_\alpha}\right)^N Z[J_\alpha]\Big|_{J_\alpha=0} \tag{2}$$

where we have assigned two external sources to ϕ and π-fields, $J_{(1)}$ and $J_{(2)}$, respectively, such that $J_\alpha = (J_{(1)}J_{(2)})$.

The calculation of this generating functional can be done from the operator point of view (Chaturvedi *et al.*, 1986). The stochastic scalar field ϕ is considered as a Heisenberg operator: $\phi(x,\tau) \to \hat{\phi}(x,\tau)$, and there is a canonically conjugated momentum $\hat{\pi}(y,\tau)$, obeying equal-fictitious time commutation rules:

$$[\hat{\phi}(x,\tau), \hat{\pi}(y,\tau')]_{\tau=\tau'} = \delta(x-y) \tag{3}$$

$$[\hat{\phi}, \hat{\phi}] = [\hat{\pi}, \hat{\pi}] = 0 \tag{4}$$

The stochastic field satisfies a Heisenberg-type equation

$$\frac{\partial \hat{\phi}(x,\tau)}{\partial \tau} = [\hat{\mathscr{H}}_{FP}, \hat{\phi}] \tag{5}$$

where $\hat{\mathscr{H}}_{FP}$ is the Fokker–Planck Hamiltonian. Recall that the Fokker–Planck equation (which is equivalent to the usual Langevin equation of the stochastic quantization method) is written as (we omit the carets for ease of notation)

$$\frac{\partial P}{\partial \tau}(\phi, \tau) = \mathscr{H}_{FP} P(\phi, \tau) \tag{6}$$

where $P(\phi,\tau)$ is the probability distribution describing the stochastic process, and $\mathscr{H}_{FP}$ is an operator given by

$$\mathscr{H}_{FP} = \int dx \frac{\delta}{\delta\phi}\left(\frac{\delta S}{\delta \phi} + \frac{\delta}{\delta\phi}\right) \tag{7}$$

In the operator approach we assign functional derivatives to canonical momenta, as

$$\frac{\delta}{\partial\phi(x,\tau)} \to -\pi(x,\tau) \tag{8}$$

and get

$$\mathscr{H}_{FP} = \int dx \left[-\pi(x,\tau)\frac{\delta S[\phi]}{\delta\phi} + \pi^2(x,\tau)\right] \tag{9}$$

Performing a Legendre transformation, we readily obtain the Lagrangian density as

$$\mathscr{L} = \pi\frac{\partial\phi}{\partial\tau} - \mathscr{H}_{FP} = \pi\frac{\partial\phi}{\partial\tau} - \pi^2 + \pi K[\phi] \tag{10}$$

where $K[\phi]$ is a Langevin kernel, whose simpler expression (for a bosonic or gauge field) is $(\partial S/\partial\phi)[\phi]$. Notice that in fermionic theories other kernels are necessary. Integrating over the ϕ and π-fields, in the Euclidean D-

dimensional space, we write the path-integral representation of the stochastic generating functional:

$$Z[J_\alpha] = \int \mathscr{D}\phi\, \mathscr{D}\pi \exp\left[-\int d^D x\, d\tau\, (\mathscr{L} + J_\alpha \Phi)\right] \tag{11}$$

where Φ is the two-component vector $\binom{\phi}{\pi}$.

3. THE BACKGROUND FIELD GENERATING FUNCTIONAL

In ordinary quantum field theories, the background field method is a scheme for calculation of the effective action (DeWitt, 1967; Honerkamp, 1972; Abbott, 1981). Starting from this object, we construct the S-matrix stringing trees of 1PI diagrams to generate connected Green functions, cutting external legs and putting all momenta on-shell. This method was extended to stochastic quantization by Okano (1987). We will restrict our discussion to scalar theories, using as an example a nonmassive $\lambda\phi^4$ theory.

When defined in Ricci-flat (Euclidean) manifolds, scalar theories allow the simple field splitting

$$\phi \to \varphi_{cl} + \psi \tag{12}$$

where φ_{cl} is the background part and ψ is the quantum part. As stressed by Okano, it is not necessary to split π-fields. The background stochastic generating functional $\tilde{Z}[J_\alpha, \varphi]$ is obtained by noting that (i) the integration is performed only over the quantum parts, (ii) we do not couple the external source to the background field. Hence

$$\tilde{Z}[J_\alpha, \varphi] = \int \mathscr{D}\psi\, \mathscr{D}\pi \exp\left\{-\int d^D x\, d\tau\, [\mathscr{L}(\varphi + \psi) + J_{(1)}\psi + J_{(2)}\pi]\right\} \tag{13}$$

where

$$\mathscr{L}[\varphi + \psi] = \pi(\partial_\tau \varphi + \partial_\tau \psi) - \pi^2 + \pi K[\varphi + \pi] \tag{14}$$

We write the stochastic effective action by means of a Legendre transform:

$$\tilde{\Gamma}[\tilde{\psi}, \tilde{\pi}, \varphi] = \tilde{W}[J_\alpha, \varphi] - \int d^D x\, d\tau (J_{(1)}\tilde{\psi} + J_{(2)}\tilde{\pi}) \tag{15}$$

where

$$\tilde{W}[J_\alpha, \varphi] = -\ln \tilde{Z}[J_\alpha, \varphi] \tag{16}$$

generates connected extended stochastic correlation functions, and the "classical" fields $\tilde{\pi}$ and $\tilde{\psi}$ are given by

$$\tilde{\psi} = \frac{\delta \tilde{W}[J_\alpha, \varphi]}{\delta J_{(1)}} \tag{17}$$

$$\tilde{\pi} = \frac{\delta \tilde{W}[J_\alpha, \varphi]}{\delta J_{(2)}} \tag{18}$$

As in ordinary field theories, $\tilde{\Gamma}[0, \tilde{\pi}, \varphi]$ is the sum of 1PI extended stochastic diagrams. They have two types of composite lines, namely (a) external $\pi - \varphi$ and $\varphi - \varphi$ legs, and (b) internal $\pi - \psi$ and $\psi - \psi$ legs. The propagators are obtained from the free part of the stochastic generating functional by double functional differentiation [see equation (2)] at zero external sources and taking the inverse of the kinetic terms. Vertices are obtained, as usual, from the interaction part.

We can consider, as an example, a quartic self-interactive nonmassive bosonic model defined by the Euclidean action

$$S[\phi] = \int d^D x \left[\frac{1}{2}\partial_\mu \phi\, \partial_\mu \phi + \frac{\lambda}{4!}\phi^4\right] \tag{19}$$

so that

$$K[\phi] = -\Box\phi + \frac{\lambda}{3!}\phi^3 \tag{20}$$

Performing the splitting prescribed in (3.1), we have, using (3.2),

$$\tilde{Z}[J_\alpha, \phi] = \int \mathscr{D}\psi\, \mathscr{D}\pi \exp\left\{-\int d^D x\, d\tau\, [\pi(\partial_\tau\varphi + \partial_\tau\psi) - \pi^2 + \pi\left[-\Box(\varphi+\psi) + \frac{\lambda}{3!}(\varphi+\psi)^3\right] + J_{(1)}\psi + J_{(2)}\pi\right\} \tag{21}$$

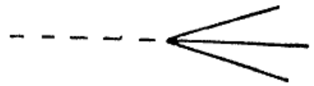
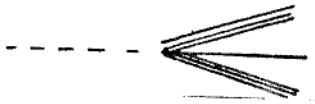
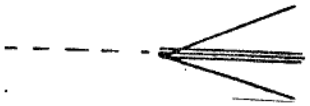
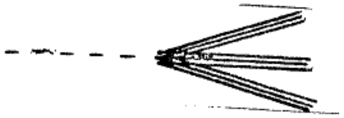
The diagrammatic rules for this model are depicted in Table I, in momentum space. It is worthwhile to note that we also Fourier transform over the fictitious time, so that $\tau \to \omega$ and $\tau' \to z$ in momentum-space propagators.

4. STOCHASTIC QUANTIZATION OF THE NONLINEAR SIGMA MODEL

Constrained systems, when stochastically quantized, have nontrivial features. A formal procedure to handle this situation is to consider the holonomic constraint on the hypersurface:

$$M: \quad F[\phi] = 0 \tag{22}$$

Table I. Diagrammatic Rules for $\lambda\phi^4$ Model Through Stochastic Generating Functional Approach

Propagators	
k	$\langle\psi\pi\rangle = -\dfrac{1}{k^2+i\omega}$
k	$\langle\psi\psi\rangle = -\dfrac{1}{(k^2)+\omega^2}$
k	$\langle\pi\pi\rangle = \dfrac{1}{4}(k^2-i\omega)^2$
p	$\langle\varphi\varphi\rangle = \dfrac{1}{(p^2)^2+z^2}$
Vertices	
	λ
	3λ
	3λ
	λ

This constraint can be imposed directly on the Langevin equation by means of Lagrange multipliers (Namiki and Yamanaka, 1986):

$$\frac{\partial\phi_i(x,\tau)}{\partial\tau} = -\left(K[\phi_i] + \lambda\frac{\partial F}{\partial\phi_i}[\phi]\right) + \eta_i(x,\tau) \tag{23}$$

where $\eta_i(x,\tau)$ is the white-noise field, with Gaussian correlations:

$$\langle\eta\rangle = 0, \qquad \langle\eta_i(x,\tau)\eta_j(x',\tau')\rangle = 2\delta_{ij}\delta^4(x-x')\delta(\tau-\tau')$$

Notice that in $O(N)$ σ-models the constraint condition

$$M\colon\ \sum_{i=1}^{N}\phi_i^2 = \frac{N}{2f} \tag{24}$$

ensures that ϕ-fields lie on a N-sphere. It is possible to use this approach so as to study the $1/N$ expansion of the model (Brunelli and Gomes, 1992).

A different pathway is to put the constraint into the metric of the differentiable manifold defined by ϕ-fields. For σ-models the following classical action has been proposed:

$$S[\phi] = \frac{1}{2\lambda^2}\int d^2x\, g_{ij}[\phi]\,\partial_\mu\phi^i\,\partial_\mu\phi^j \tag{25}$$

where $i, j = 1, \ldots, N$ and $\mu = 0, 1$. Without loss of generality we omit the coupling constant.

The kernel is given by

$$K_i[\phi] = -g_{ij}[\phi][\Box\phi^j + \Gamma^j_{kl}\,\partial_\mu\phi^k\,\partial_\mu\phi^l] \tag{26}$$

(Γ^i_{kl} is the Christoffel symbol), and the generating functional is obtained from our preceding discussion, including also the π-fields:

$$Z[J_\alpha] = \int \mathscr{D}\phi\,\mathscr{D}\pi\,\exp\left\{-\int d^2x\,d\tau\left[-\pi^i\frac{\partial\phi_i}{\partial\tau} + \pi^i\pi^i + \pi^i g_{ij}[\phi](\Box\phi^j - \Gamma^j_{kl}\,\partial_\mu\phi^k\,\partial_\mu\phi^l)\right] + J_\alpha\Phi\right\} \tag{27}$$

This equation will be the starting point of the forthcoming analysis of the background field method.

5. BACKGROUND FIELD SPLITTING FOR NONLINEAR SIGMA MODEL

When defined in non-Ricci flat manifolds, scalar models do not allow the simple field splitting (12), because the ψ^i-field will not be transformed as a covariant vector. A common method for dealing with these difficulties is to expand the quantum part ψ^i in powers of the so-called Riemannian (or normal) coordinates ξ^i (Alvarez-Gaumé *et al.*, 1981). One attaches tangent vectors to the curved manifold M such that a geodesic is given by

$$s^2 = g_{ij}\xi^i\xi^j \tag{28}$$

If we define a variable $\lambda^i(t)$ as a geodetic parametrization (t is proportional to arc length), satisfying

$$\ddot{\lambda}^i + \Gamma^i_{jk}\dot{\lambda}^j\dot{\lambda}^k = 0 \tag{29}$$

in which dots indicate differentiation with respect to t, one gets, for $t = 0$,

$$\lambda(0) = \phi^i, \qquad \dot{\lambda}(0) = \xi^i \tag{30}$$

and for an arbitrary value of t, the expansion

$$\dot{\lambda}(t) = \phi^i + \xi^i t - \frac{1}{2}\Gamma^i_{j_1 j_2}\xi^{j_1}\xi^{j_2}t^2 - \frac{1}{3!}\Gamma^i_{j_1 j_2 j_3}\xi^{j_1}\xi^{j_2}\xi^{j_3}t^3 - \cdots \tag{31}$$

where $\Gamma^i_{j_1 j_2 \cdots j_N}$ is the Christoffel connection.

Choosing $t = 1$, we can write, for the quantum part of the splitting,

$$\psi^i = \xi^i - \frac{1}{2}\Gamma^i_{k_1 k_2}\xi^{k_1}\xi^{k_2} - \frac{1}{3!}\Gamma^i_{k_1 k_2 k_3}\xi^{k_1}\xi^{k_2}\xi^{k_3} - \cdots \tag{32}$$

In one-loop calculations, it is sufficient to retain terms up to ξ^3 in all expansions. For later use, we give some properties:

$$\text{(i)} \quad D_\mu \xi^i = \partial_\mu \xi^i + \Gamma^i_{jk}\xi^j \partial_\mu \varphi^k \tag{33}$$

$$\text{(ii)} \quad \partial_{(j_1}\partial_{j_2}\cdots\partial_{j_{n-2}}\Gamma^i_{j_{n-1} j_n)} = 0 \tag{34}$$

A covariant tensor field has the Taylor-like expansion within this procedure:

$$T_{k_1 k_2 \cdots k_n}(\varphi + \psi) = \sum_{n=0}^{\infty}\frac{1}{n!}\left(\frac{\partial}{\partial \xi_{i_1}}\cdots\frac{\partial}{\partial \xi_{i_N}}\right)T_{k_1 \cdots k_n}(\varphi)\xi^{i_1}\cdots\xi^{i_N} \tag{35}$$

in which we often use Riemann tensors, in order to take the coefficients covariant. It is often necessary to deal with quadratic terms like $D_\mu \xi^a D_\mu \xi^a$. In this case, we proceed as follows: using a n-bein, one rewrites normal coordinates in the tangent system:

$$\xi^a(x) = e^a_i \xi^i(x) \tag{36}$$

such that the covariant derivatives turn out to be given by

$$D_\mu \xi^a = \partial_\mu \xi^a + \omega^{ab}_i \partial_\mu \varphi^i \tag{37}$$

where ω^{ab}_i is the spin connection. It follows that $\omega^{ab}_i \partial_\mu \varphi^i$ stands for an $SO(N)$-gauge potential for φ-fields.

Our task is to find an expression for the generating functional, so that we outline some useful expansions (the fictitious time is artificially treated as an additional space-time coordinate):

$$\partial_\mu(\varphi^i + \psi^i) = \partial_\mu \xi^i + D_\mu \xi^i + \frac{1}{3}R^i_{k_1 k_2 j}\xi^{k_1}\xi^{k_2}\partial_\mu \varphi^j \tag{38}$$

$$\begin{aligned}\Box(\varphi^j + \psi^j) = {} & \Box\partial^j + \Box\xi^j + \partial_\mu(\Gamma^j_{kl}\xi^k \partial_\mu \varphi^l) \\ & + \frac{1}{3}\partial_j(R^j_{k_1 k_2 l}\xi^{k_1}\xi^{k_2}\partial_\mu \varphi^l)\partial_\mu \xi^j\end{aligned} \tag{39}$$

$$\partial_\tau(\varphi^i + \psi^i) = \partial_\tau \varphi^i + \partial_\tau \xi^i - \frac{1}{2}\partial_j(\Gamma^i_{k_1 k_2}\xi^{k_1}\xi^{k_2})\partial_\tau \varphi^j \tag{40}$$

$$g_{ij}(\varphi^k + \psi^k) = g_{ij}(\varphi^k) + \frac{1}{3}R_{ik_1 k_2 j}(\varphi^k)\xi^{k_1}\xi^{k_2} \tag{41}$$

$$\begin{aligned}\Gamma^i_{kl}(\varphi + \psi) = {} & \Gamma^i_{kl}(\varphi) + \frac{1}{3}[g^{jm}(\partial_k R_{mm_1 m_2 l} + \partial_l R_{km_1 m_2 m} \\ & - \partial_m R_{km_1 m_2 m}) + R_{jm_1 m_2 m}(\partial_k g_{ml} + \partial_l g_{km} - \partial_m g_{kl})]\xi^{m_1}\xi^{m_2}\end{aligned} \tag{42}$$

Applying these expressions to (26) and taking in account equation (13), we obtain, after some algebra, the generating functional:

$$\tilde{Z}[J_\alpha, \varphi] = \int \mathscr{D}\psi\, \mathscr{D}\pi \exp\left\{-\int d^2x\, d\tau\, [\mathscr{L}(\varphi + \psi) + J^i_{(1)}\psi^i + J^i_{(2)}\pi^i]\right\} \tag{43}$$

where

$$\begin{aligned}\mathscr{L}(\varphi + \psi) = {} & -\pi^i\left[\partial_\tau \varphi^i + \partial_\tau \xi^i - \frac{1}{2}\partial_j(\Gamma^i_{m_1 m_2}\xi^{m_1}\xi^2)\right] + \pi^i \pi^i \\ & - \pi^i\Big\{g_{ij}\Box\varphi^j + g_{ij}\Box\xi^j + g_{ij}\partial_\mu(\Gamma^j_{kl}\xi^k \partial_\mu \varphi^l) \\ & + \frac{1}{3}g_{ij}\partial_m(R^j_{k_1 k_2 l}\xi^{k_1}\xi^{k_2})\partial_\mu \varphi^m + \frac{1}{3}R_{ik_1 k_2 j}\xi^{k_1}\xi^{k_2}\Box\varphi^j \\ & + g_{ij}\Gamma^j_{kl}\Big(\partial_\mu \varphi^k \partial_\mu \varphi^l + \partial_\mu \varphi^k D_\mu \xi^k + \frac{1}{3}\partial_\mu \varphi^k R^l_{k_1 k_2 m}\xi^{k_1}\xi^{k_2}\partial_\mu \varphi^m \\ & + D_\mu \xi^k D_\mu \varphi^l + D_\mu \xi^k D_\mu \xi^l + \frac{1}{3}R^k_{k_1 k_2 m}\xi^{k_1}\xi^{k_2}\partial_\mu \varphi^m \partial_\mu \varphi^l\Big) \\ & + g_{ij}\Big\{\frac{1}{3}[g^{jm}(\partial_k R_{mk_1 k_2 l} + \partial_l R_{kk_1 k_2 m} - \partial_m R_{kk_1 k_2 l}) + R_{ik_1 k_2 m} \\ & \times(\partial_k g_{ml} + \partial_l g_{km} - \partial_m g_{kl})]\xi^{k_1}\xi^{k_2}\Big\}\partial_\mu \varphi^k \partial_\mu \varphi^l \\ & + \frac{1}{3}R_{ik_1 k_2 j}\xi^{k_1}\xi^{k_2}\Gamma^j_{kl}\partial_\mu \varphi^k \partial_\mu \varphi^l\Big\}\end{aligned} \tag{44}$$

This result then can be used to investigate the divergences of the model and discuss its renormalization, too, at least in the one-loop approximation.

6. CONCLUSIONS

In a recent paper, Okano presented a formulation of the background field method in stochastic quantization which performs the calculations starting from the path-integral representation of the stochastic generating functional, and showed the usefulness of this technique in the computation of stochastic effective action. We have outlined the basics of his method and considered as an example a nonmassive $\lambda\phi^4$ theory. Within this framework we showed the corresponding diagrammatic rules.

We were able to implement the background field method for the nonlinear sigma model on a general Riemannian manifold in the context of stochastic quantization. We first introduce the constraint into the manifold metric, and write the generating functional including the auxiliary π-fields and quantum fluctuations ψ around the classical solutions φ. The prescription used here is the introduction of the covariant vector field $\xi^i(x, t)$ at the point $\varphi^i(x, t)$ for the quantum field $\psi^i(x, t)$. In this case, we take a Riemannian, or normal coordinate expansion.

We have obtained the generating functional for the nonlinear sigma model in stochastic quantization. The result allows us to discuss, at least at the one-loop level, the renormalization of this model in stochastic quantization. This will be the subject of a forthcoming paper.

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Čerenkov Radiation at Finite Temperature

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A power formula for Čerenkov radiation at finite temperature is derived in the framework of the generalized finite-temperature Cutkosky rules. Spins 1/2 and 0 are considered.

1. INTRODUCTION

Reactions rate for quantum processes taking place in a heat bath in thermal equilibrium have been actively studied the past few years. Computations of discontinuities at finite temperature and their physical interpretations in the framework of the imaginary-time finite-temperature field theory (IT FTFT) were done by Weldon (1983).

The generalization of Cutkosky rules in the real-time FTFT (the circled diagrams algorithm) was found in Kobes and Semenoff (1986). Recently, Niégawa (1990) and Ashida *et al.* (1991) showed that any Kobes–Semenoff diagram can be cut. Therefore the total discontinuity at finite temperature is a collection of different reaction rates. We use the result of this theory for concrete physical situations.

In this article we derive a spectral formula for finite-temperature Čerenkov radiation, i.e., the energy loss of a charged particle moving faster than the speed of light in the medium. We discuss quantum particles (spin 1/2 and 0) and we consider that the medium is filled by equilibrium photon radiation (finite-temperature situation).

The energy loss per unit time of the particle is defined by (Tzytovich, 1962)

$$-\frac{dE}{dt} = \int d\omega\, \omega P(\omega) = \int d\omega\, \omega \{[N_B(\omega) + 1]\Gamma^+(\omega) - N_B(\omega)\Gamma^-(\omega)\} \quad (1)$$

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