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Breaking of gauge invariance in spinor QED induced by a stochastic regulator

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Abstract We analyse the effects of the Stochastic Analytic Regularieation method on the gauge invariance, computing the vacuum polarieation tensor for spinor QED in one-loop order. Consequences in the non-abelian and supersymmetric cases are discussed.

1. Introduction

Some years ago, Parisi and Wu[1] developed a new original method to deal with Euclidean field theories in the continuum: the so-called Stochastic Quantization method (SQM). One of the most important features of SQM is a new regularization scheme which resembles the Analytic Regularization method[2]. We will call it Analytic Stochastic Regularization (ASR)[3,4]. There was some evidence that ASR could preserve all physical symmetries, like gauge invariance, in a given field theory. However, the validity of this statement has been questioned. Recently, the ASR method was used to verify the breaking of gauge symmetry for abelian and non-abelian scalar gauge theories in four dimensions[5]. We extended the analysis to theories containing spin 1/2 fermions.

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The paper is organized as follows: in section **2** we outline the basics of SQM and the necessary tools to include fermions in the formalism. In section **3** the ASR scheme is quickly reviewed, and in the subsequent section we apply ASR to Spinor QED. We calculate the one-loop correction to the photon propagator showing the breaking of gauge invariance induced by ASR. The physical consequences and further developments are discussed in the last section.

2. A survey of stochastic quantization

The cornerstone of the SQM[6] is the well-known formal analogy between Euclidean field theory and classical statistical mechanics. In the Euclidean n-point Green function, one can associate the functional $exp(-S[\phi]/\hbar)$ to the equilibrium distribution for a statistical system. In SQM we consider this system as performing a stochastic process.

In order to study the evolution of the system, the classical field is endowed with an additional parameter, here called ***ficticious time*** τ. Moreover, the stochastic dynamics may be described by a Langevin equation. For the simple case of a boson field with Euclidean action $S[\phi]$ this equations reads

$$\frac{\partial\phi(x,\tau)}{\partial\tau} = -\frac{\delta S[\phi]}{\delta\phi(x,\tau)} + \eta(x,\tau) \tag{2.1.}$$

where $\eta(x,\tau)$ is the (white) noise field, whose correlations are given by

$$\begin{aligned} &< \eta(x,\tau) >^{\text{conn.}} = 0 \\ &< \eta_i(x,\tau)\eta_j(x',\tau') > = 2\delta_{ij}\delta(x,x')\delta(\tau-\tau') \end{aligned} \tag{2.2}$$

Higher noise correlations are obtained by a Wick decomposition.

The n-point correlation furictions, in the stationary limit (equal and large ficticious times), reduce to the Euclidean Green functions. Perturbative calculations using SQM consist in:

(i) a choice of initial conditions for the Langevin equation so that it can be rewritten in an integral form;

(ii) solving the resulting integral equation by iterative procedures, in powers of the coupling constant;

(iii) a graphical convention: we assign full lines to propagators and crosses to noises. Vertices are linked to coupling constants as in usual field theories;

(iv) joining these tree expansions we obtain n-point correlation functions. Crosses are fused in accordance with their white noise properties, so we need to consider all possible contractions.

(v) in these (stochastic) diagrams, lines containing fused crosses describe composite or crossed propagators.

We will exemplify matters directly with fermion fields, although some remarks must be made for the sake of completeness. Stochastic Quantization of fermions is a non-trivial matter, because there is no classical analog for anticommuting variables. In fact, by means of a direct approach one is lead to non-positive operators and ill-defined probability distributions (in the sense of the equivalent Fokker-Planck equation). The most accepted prescription to circumvent these problems is the introduction of a kernel K_{ij} into the Langevin equation[7].

$$\frac{\partial\psi_i(x,\tau)}{\partial\tau} = -\int d^D y K_{ij}(x,y)\frac{\delta S[\phi]}{\delta\psi_j(y,\tau)} + v_i(x,\tau) \tag{2.3}$$

The noise correlations are also changed, giving

$$< v_i(x,\tau)v_j(x',\tau') >= 2K_{ij}(x,x')\delta(\tau,\tau') \tag{2.4}$$

where v is a Grassmann noise.

The Langevin equation for free spin 1/2 fermions, whose Euclidean action is

$$S[\psi,\bar\psi] = -i\int d^4x d\tau\bar\psi(x,\tau)(\partial\!\!\!/ + iM)\psi(x,\tau) \tag{2.5}$$

is obtained with use of a Dirac kernel

$$K^F_{ij}(x,y) = i(\partial\!\!\!/_x + M)_{ij}\delta^4(x-y) \tag{2.6}$$

and reads as a boson-like expression

$$\frac{\partial\psi(x,\tau)}{\partial\tau} = (\partial^2 + M^2)\psi(x,\tau) + v(x,\tau) \tag{2.7}$$

together with its conjugate counterpart.

With a suitable choice of initial conditions for the above equation, and the help of the Green function (in momentum space)

$$G_F(k;\tau-\tau')_{ij} = \delta_{ij}\exp(-(k^2+M^2)(\tau-\tau'))\theta(\tau-\tau') \tag{2.8}$$

the integral form of eq. (2.7) is written as

$$\psi_i(k,\tau) = \int d\tau' G_F(k;\tau-\tau')_{ij} v_j(k,\tau') \tag{2.9}$$

The convolution of eq.(2.9) - also called *uncrossed* propagator - and the Dirac kernel yields a ***fermionic*** Green function.

$$\Gamma_{ij}(k;\tau-\tau') = (-\not{k}+M)_{ij}\exp(-(k^2+M^2)(\tau-\tau'))\theta(\tau-\tau') \tag{2.10}$$

3. Stochastic analytic regularization

The stochastic processes described up to now are Markovian, due to their white noise properties. Breit, Gupta and Zaks[8] introduced a non-Markovian element, smearing the delta function which involves the ficticious time, using a parameter-dependent regulator function. Alfaro took the Mellin transform of the Gupta regulator and found a, different function, which we adopt

$$f_\epsilon(\tau) = \epsilon|\tau|^{\epsilon-1} \tag{3.1}$$

and whose Fourier-transformed expression is

$$\begin{aligned}\tilde{f}_\epsilon(\omega) &= \int_{-\infty}^{+\infty}\frac{d\tau}{2\pi}f_\epsilon(\tau)\exp(= i\omega\tau)\\ &= 2\hat{f}_\epsilon|\omega|^{-\epsilon}\end{aligned} \tag{3.2}$$

where

$$\hat{f}_\epsilon = \epsilon\Gamma(\epsilon)\sin\left[\frac{\pi}{2}(1-\epsilon)\right] \tag{3.3}$$

The (non-white) noise correlations are

$$< \eta_i(x,\tau)\eta_j(x',\tau') = K_{ij}(x,x')f_\epsilon(\tau - \tau') \tag{3.4}$$

When the regulator parameter (in Alfaro's case) approaches zero one recovers the unregularized theory, i.e.

$$\lim_{\epsilon\to 0} f_\epsilon(\tau - \tau') = 2\delta(\tau - \tau') \tag{3.5}$$

Equation (3.1) enables us to compute the 2-point fermionic correlation function, whose lowest order contribution gives the crossed propagator (in momentum space)

$$\Delta_{ij}(k;\tau,\tau') =< \psi_i(k,\tau)\bar{\psi}_j(-k,\tau') > \tag{3.6}$$

Using eqs. (2.8), (2.9) and (3.3) we obtain

$$\begin{aligned}\Delta_{ij}(k;\tau,\tau') &= 2\int_0^\tau dt" \int_0^{\tau'} G_{i\ell}(k,\tau-\tau") \\ &\otimes K_{\ell n}(k)G_{nj}(-k;\tau'-\tau")f_\epsilon(\tau"-\tau"') \\ &= \hat{f}_\epsilon \frac{(\not{k}-M)_{ij}}{(k^2+M^2)^{1+\epsilon}}\int_{-\infty}^{+\infty}\frac{dx}{\pi}\frac{\exp(-ix(k^2+M^2)(\tau-\tau'))|x|^{-\epsilon}}{1+x^2}\end{aligned} \tag{3.7}$$

An outstanding feature of the ASR method is that one is led to meromorphic amplitudes for stochastic diagrams, i.e., the ultraviolet divergences show up as poles in ϵ, like in Analytic Regularization[2].

In order to calculate stochastic amplitudes in gauge theories we need expressions for uncrossed and crossed gauge field propagators as well. In the Feynman gauge, they are[10]

$$G_{ij}(k;\tau-\tau') = \delta_{ij}\exp(-k^2(\tau-\tau'))\theta(\tau-\tau') \tag{3.8}$$

and

$$D_{ij}(k;\tau,\tau') = \hat{f}_\epsilon \frac{\delta_{ij}}{(k^2)^{1+\epsilon}} \otimes \int_{-\infty}^{+\infty} \frac{dx}{\pi} \frac{\exp(-ixk^2(\tau-\tau'))|x|^{-\epsilon}}{1+x^2}$$

respectively. In fig. 1 our graphical conventions for fermion and gauge field propagators are depicted.

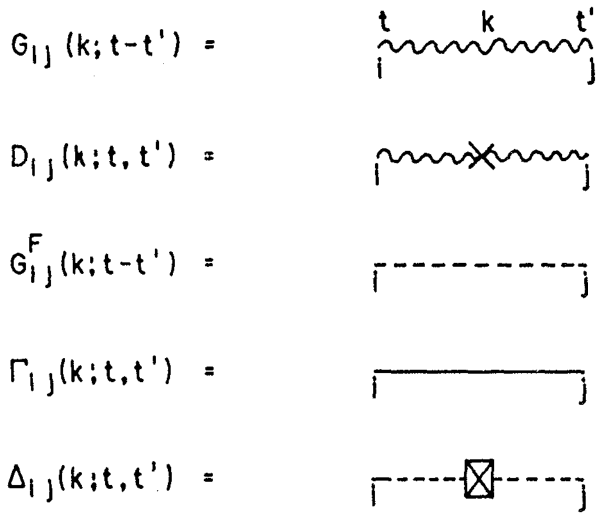

Fig.1 - Fermion and Gauge Field Propagators.

4. Vacuum polarization tensor in spinor electrodynamics

With the Feynman rules shown in the preceding section, we are able to perform loop computations in spinor QED whose Euclidean action is

$$S[A_\mu,\psi,\bar{\psi}] = \int d^4 s d\tau \left[\frac{1}{4}F_{\mu\nu}F_{\mu\nu} - i\bar{\psi}(\not{\partial} - ie\not{A} + iM)\psi\right] \qquad (4.1)$$

The related Langevin equations are (T stands for matrix transposition)

$$\frac{\partial\psi_i}{\partial\tau} = -[(\not{D} - iM)(\not{D} + iM)]_{ij}\psi_j + v_i \tag{4.2}$$

$$\frac{\partial\bar{\psi}_i}{\partial\tau} = -\bar{\psi}_i[(\not{D}' - iM)^T(\not{D}' + iM)^T]_{ji} + \bar{v}_i \tag{4.3}$$

$$\frac{\partial A_\mu}{\partial\tau} = -\partial_\nu F_{\mu\nu} + e\bar{\psi}\gamma_\mu\psi + \eta_\mu \tag{4.4}$$

where D, is the usual slashed covariant derivative and

$$D'_\mu = -\not{\partial}_\mu - ieA_\mu \tag{4.5}$$

The commuting (η) and anticommuting (v) noises obey regularized correlations (3.4).

Due to an inherent shortcoming of the SQM method, namely the lack of a Noether's theorem approach, we are forced to deal with indirect methods to study physical symmetries, specially gauge invariance. A common way to do it is computing the mass correction to photon propagator. In this sense, the non-transversality of the latter implies a breaking in gauge invariance induced by this regularization prescription (ASR).

Thus, we calculate the vacuum polarization tensor at one loop order. We show the relevant diagrams in fig. 2. As the ultraviole divergences occur as simple poles in ϵ, the approximation

$$|x|^{-\epsilon} \simeq 1 + O(\epsilon) \tag{4.6}$$

suffices for crossed propagators. Vertices in this stochastically quantized theory are of the same type as that appearing in conventionally quantized one (in Euclidean space).

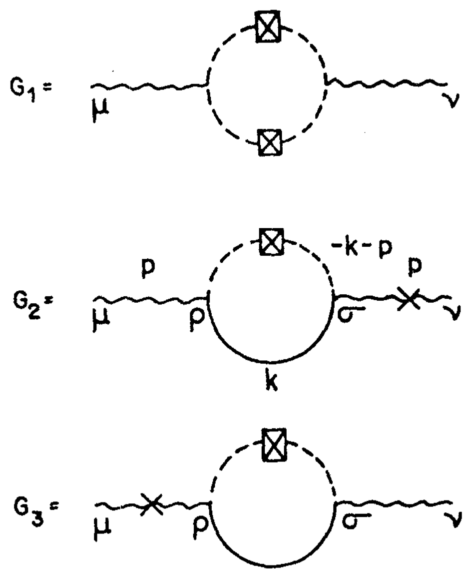

Fig.2 - Diagrama with one and two internal crossed lines for the calculation of vacuum polarization.

Using the standard rules for diagrammatic calculations in SQM[11], we find for the contribution (G – 1) (we attach a factor two due to the different orderings of internal ficticious times)

$$(G-1)=\frac{\hat{f}_\epsilon^2 e^2}{2p^2}\int\frac{d^4k}{(2\pi)^4}\otimes$$

$$\otimes\frac{\mathrm{Tr}[\gamma_\mu(-\not{p}-\not{k}+M)\gamma_\nu(-\not{k}+M)]}{[(p+k)^2+M^2]^{1+\epsilon}(k^2+M^2)^{1+\epsilon}(p^2+k^2+p.k+M^2)} \qquad (4.7)$$

The diagrams (G – 2) and (G – 3) are topologically similar and one can add them, with an overall combinatorial factor, giving (more details of the algebra are found in the appendix)

$$(G-2)+(G-3)=4e^2\int_{-\infty}^{\tau}d\tau_2'\int_{-\infty}^{\tau_2'}d\tau_1'\int\frac{d^4k}{(2\pi)^4}$$
$$\otimes G_{\mu\rho}(p;\tau-\tau_1')D_{\sigma\nu}(p;\tau\tau_2')$$
$$\otimes \mathrm{Tr}[\gamma_\mu\Delta(k+p.\tau_1',\tau_2')\gamma_\nu\Gamma(k;\tau_1',\tau_2')] \tag{4.8}$$

$$(G-2)+(G-3)=\frac{4\hat{f}_\epsilon e^2}{(p^2)^2}\int\frac{d^4k}{(2\pi)^4}$$
$$\otimes\frac{-\delta_{\mu\nu}(M^2+k^2+k.p)+2k_\mu k_\nu+p_\nu k_\mu+p_\mu k_\nu}{[(k+p)^2+M^2]^{1+\epsilon}(p^2+k^2+k.p+M^2)} \tag{4.9}$$

These integrals are very difficult to evaluate in a closed form. Although we may obtain some exact results in two dimensions, it seems more interesting to use an expansion, removing the divergent (and perhaps finite) terms from the expressions.

This is possible because of the analiticity of the polarization tensor for large mass. Hence re rescale the loop momentum k $\rightarrow$ $M\,k$ and expand the thoublesome integrand in powers of k/M until the order which shows dívergent pieces, i.e., until one isolates the poles. Further terms in this expansion are finite.

We apply this procedure to the integrals above, getting (see appendix)

$$(G-1)=-\frac{\delta_{\mu\nu}}{32p^2\pi^2\epsilon} \tag{4.10}$$

$$(G-2)+(G-3)=\frac{7\delta_{\mu\nu}}{48p^2\pi^2\epsilon}-\frac{p_\mu p_\nu}{12\pi^2(p^2)^2\epsilon} \tag{4.11}$$

such that the polarization tensor reads (for its divergent piece)

$$\pi_{\mu\nu}^{\mathrm{div}}(p)=\frac{11p^2\delta_{\mu\nu}}{96\pi^2\epsilon}-\frac{p_\mu p_\nu}{12\pi^2\epsilon} \tag{4.12}$$

which is clearly non-transverse due to the double-crossed diagram (G – *1)*. It is half the necessary value - notice that 2(G – 1) + (G – 2) + (G – 3) is transverse - in the sense of dimensional regularization (with analytically continued dimension

$D = 4 - 2\epsilon$). The other two diagrams are equal to the dimensionally regularized ones.

5. Conclusions

At first sight, any regularization scheme which involves a ficticious time variable would respect physical symmetries. Our results, however, show that there is an over-simplification in these beliefs. Althoug a certain choice of regulator could keep gauge invariance, as shown by Gavela and Huffel[12] (who had studied spinor QED by means of a Gupta-type regulator), other possibilities will no longer give the same result. We proved this latter statement using the Alfaro regulator, up to one-loop order, which means a perturbative breaking of gauge invariance induced by a specific regulator function.

In recent papers[13,14], dealing with abelian as well as non-abelian scalar gauge theories in the context of ASR, this breaking has been observed many times. Surprisingly, even when a Gupta regulator is applied in a pure non-abelian theory[15] there is some trouble with gauge invariance. The question of whether this or that regulator furnishes non-gauge invariant corrections to a gauge field remains open, in our opinion.

On the other hand, as a byproduct of our calculations, we found evidence that the ASR scheme can work nicely in some supersymmetrical models[16]. A comparison between results already found for scalar non-abelian theories with spinor ones show that the contribution $(G-1)$, given by eq.(4.10) is minus twice the corresponding bosonic value. This fact indicates that ASR would be a reasonable method to regularize supersymmetric theories, because the problematic bosonic and fermionic contributions cancel in some multiplets, namely:

(i) The coupling of a gauge field with a supersymmetric matter multiplied (two bosonic charged fields and one Dirac field);

(ii) The case of $N = 1$ supersymmetric Yang Mills theory with one Majorana fermion in the adjoint representation. The scalar matter contribution is given by the non-abelian self-interaction.

Appendix

In this appendix, we show the intermediate steps necessary to write down the stochastic amplitudes associated with the diagrams (G – 2) + (G – 3). In order to integrate over the ficticious internal times, we adopt the ordering

$$\tau_1 < \tau_2 < \tau_1' = \tau_2' = \text{finite ficticious time}$$

because we start our Langevin process with $\tau \to -\infty$.

Using the Feynman *rules* (fig.1), we rewrite eq.(4.8) as (we take $e = 1$)

$$(G-2)+(G-3) = 4\hat{f}_\epsilon^2 \int_{-\infty}^{\tau} d\tau_2' \int_{-\infty}^{\tau_2'} d\tau_1' \int \frac{d^4k}{(2\pi)^4} \int_{-\infty}^{+\infty} \frac{dx_1}{\pi} \otimes$$

$$\otimes \frac{\exp(-ix_1 p^2(\tau-\tau_2'))}{1+x_1^2} \int_{-\infty}^{+\infty} \frac{dx_2}{\pi} \frac{\exp(-ix_2[(k+p)^2+M^2](\tau_2'-\tau_1'))}{1+x_2^2} \otimes$$

$$\otimes \frac{\exp(-p^2(\tau-\tau_1') - (k^2+M^2)(\tau_2'-\tau_1'))\mathrm{Tr}[\gamma_\mu(-\not{k}-\not{p}+M)\gamma_\nu(-\not{k}+M)]}{(p^2)^{1+\epsilon}[(k+p)^2+M^2]^{1+\epsilon}} \qquad (A.1)$$

The trace in the above expression is computed using the Euclidean Clifford algebra

$$\{\gamma_\mu, \gamma_\nu\} = -2\delta_{\mu\nu} \qquad (A.2)$$

so that we obtain eq. (4.9).

We may substitute $k \to -k - p$ and obtain an expression in powers of the external momenta, as follows

$$(G-2)+(G-3) = \frac{4\hat{f}_\epsilon^2 M^2}{(p^2)^2} \int \frac{d^4k}{(2\pi)^4} \frac{1}{(k^2+1)^{2+\epsilon}} \otimes$$

$$\otimes \left[-\delta_{\mu\nu}(1+k^2+k_\lambda p_\lambda) + 2k_\mu k_\nu + \left(\frac{p}{M}\right)_\nu k_\mu + \left(\frac{p}{M}\right)_\nu k_\nu \right] \otimes$$

$$\otimes\left\{1-\frac{k_\rho\left(\frac{p}{M}\right)_\rho+\left(\frac{p}{M}\right)^2}{k^2-1}-\frac{k_\rho k_\sigma\left(\frac{p}{M}\right)_\rho\left(\frac{p}{M}\right)_\sigma}{(k^2+1)^2}\right\} \qquad (A.3)$$

giving twelve momentum integrals which to be evaluated, whose sum is **eq.** (4.11).

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Resumo

Analisamos os efeitos do método de Regularização Analítica estocástica sobre a invariança de calibre, calculando o tensor de polarização do vácuo em QED espinorial, em ordem de 1 loop. As consequências nos casos não abeliano e supersimétrico são discutidos.