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Analytic stochastic regularization: various applications in gauge and supersymmetric theories

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Abstract We consider analytic stochastic regulariration for various gauge theories. We find a breakdown of gauge invariance in spinor and scalar QCD by an explicit one-loop computation of the two-, three- and four-point vertex function of the gluon field. As a result, we prove that these theories require non gauge invariant counterterms. We observe, on the other hand, that in the supersymmetric multiphts there is a cancellation of unwanted terms, rendering the counterterms gauge invariant. The case of supersymmetric matter fields and supersymmetric gauge contributions are considered at one loop order.

1. Introduction

Non abelian gauge theories are in a rather distinguished position in the set of field theories. This is so because local symmetries are very difficult to be maintained in the process of quantization. For example, dimensional regularization is, in practice, the only regularization scheme preserving gauge symmetry. On the other hand, this procedure breaks supersymmetry and must be discarded in those cases where this latter symmetry is an important issue[1]. Besides that, non perturbatively there are the Gribov ambiguities which prevent a clear gauge specification[2]. It must be mentioned also that although computer simulations using Monte Carlo methods have unveiled a lot about the structure of gauge theories, the inclusion of fermions still poses a difficult problem not only theoretically but also concerning computer time as well[3].

In view of the above problems, the stochastic quantization method[4] is a positive proposal which can circumvent, all at the same time, the mentioned difficulties.

To start with, the gauge fixing procedure is not necessary, at least in the usual sense, since it is already incorporated in the initial conditions. Therefore, Gribov ambiguities are simply not an issue to be discussed.

Computer simulations of gauge theories on a lattice require much less computer time, since the introduction of the Langevin time permits the updating of the whole Iattice data at one step, permitting studies of spinor fields using a reasonable amount of computer time.

The issue in the present paper, however, concerns the obtention of a new regularization scheme based on the Langevin equation with a non Markovian process. This is a procedure not related to space time, and some authors claim that indeed, such a regularization scheme is able to preserve all symmetries of the lagrangian, including gauge symmetry and supersymmetry[5,6]. As a matter of fact, there are many results in this direction, partially confirming this hypothesis. It has been shown that QED vertex functions with zero externa1 momenta vanish, as in dimensional regularization; therefore, the highest divergence in the corresponding diagram cancels, and there is no mass counterterm. A one-loop calculation in 2-dimensional scalar QCD confirms this fact, and it can be shown, in fact, that all counterterms in that model are gauge invariant. However, in four-dimensional gauge theories there is an induction of non gauge invariant counterterms, containing derivatives of the gauge field. We present a detailed computation of the polarization tensor and of the three and four vertex function of scalar QCD. Besides the usual transverse (gauge invariant) terms we found a divergent part of the form $A^\mu \partial^2 A_\mu$. This may appear quite innocuous in the abelian case, but constitutes a breaking of gauge symmetry in the non abelian case. We show how one can possibly handle the problem in perturbation theory but non perturbatively this is an open problem.

In spinor QCD case, the situation is not better, and again, non gauge invariant counterterms have to be added. Already at one loop level again, we find a counterterm $A^\mu \partial^2 A_\mu$, spoiling the explicit gauge independence.

However, when QCD is coupled to supersymmetric matter fields, namely 2 bosonic and one fermionic field with the same charge, the matter contribution to that dangerous counterterm vanishes. Moreover, if we add fermions in the adjoint representation, a cancellation of gauge dependent counterms arising from the gauge field self energy occurs as well, showing that cancellation of gauge dependent counterterms happens for supersymmetric Yang-Mills coupled to supersymmetric matter fields. Therefore, the scheme is gauge independent, at leat to one loop order, for supersymmetric gauge theories.

A note on possible difficulties about current conservation in the framework of stochastic quantization has already appeared[6]. However, there was no clear indication of how it would appear for quantized gauge theories. In the present work we discuss the appearance of infinite counterterms spoiling that symmetry in the Langevin equation, disproving previous claims that gauge symmetry is automatically preserved in this scheme.

In section 2 we review the general rules of stochastic quantization for scalar, vector and spinor fields. Then we compute the polarization tensor for two-dimensional scalar QCD. We compare the results with dimensional regularization. Next, in section 3 four-dimensional scalar and spinor QCD are discussed in the external field approximation. In section 4 we discuss supersymmetric models. Discussions are in section 5.

2. Stochastic quantization and analytic stochastic regularization

2.1. Feynman rules

We start from the Langevin equation for an arbitrary field $\varphi(x,t)$ and its corresponding noise $\eta(x,t)$.

$$\frac{\partial\varphi_\eta(x,t)}{\partial t} = -\frac{\delta S}{\delta\varphi_\eta(x,t)} + \eta(x,t) \tag{2.1}$$

The variable t is called the fifth time, S is the classical action, and the noise has two point function given by

$$< \eta_i(x,t)\eta_j(x',t') >= 2\delta_{ij}\delta(x-x')\delta(t-t') \tag{2.2}$$

and **any** higher funtions are given by **Wick's** decomposition (higher connected functions vanish).

From the Langevin **equations** we compute the field $\varphi_\eta(x,t)$. Averages **in** η are, by definition, usual averages for functions of φ

$$< F[\varphi(x,t)] >_\eta = \int \mathcal{D}\eta\ F[\varphi_\eta(x,t)] \exp\Big(-\int_0^\infty dt \int dy \eta_i(y,t)\eta_j(y,t)\Big) \tag{2.3}$$

and with this Markovian process we are **able** to define the field theory. This is done defining Green functions as the **stationary** (t $\to \infty$) limit of the statistical average

$$< T\varphi_{i_1}(x_1)...\varphi_{i_N}(x_N) >= \lim_{t\to\infty} < \varphi_{i_1}(x_1,t)...\varphi_{i_N}(x_N.t) >_\eta \tag{2.4}$$

Originally developed for bosonic models, it **was** only recently that **stochastic** quantization of fermions received a physically consistent treatment. The starting point is a generalization of **the** original Langevin equation by means of the introduction of a Kernel K_{ij} (notice that (2.1) is incompatible with the dimension of fermion fields, since $\dim[t] = -2$)

$$\frac{\partial\varphi_i(x,t)}{\partial t} = -\int d^D y K_{ij}(x,y)\frac{\delta S}{\delta\varphi_j(y,t)} + \eta_i(x,t) \tag{2.5}$$

where η_i is the classical noise with correlations

$$< \eta_i(x,t)\eta_j(x',t' >= 2K_{ij}(x,x')\delta(t-t') \tag{2.6}$$

In the case of free fermion with classical Euclidean action[8]*

$$S[\bar{\psi},\psi] = -i\int d^4x\bar{\psi}(x)(\partial\!\!\!/ + im)\psi(x) \tag{2.7}$$

the kernel is given by

$$K_{\alpha\beta}(x,y) = (i\,\partial\!\!\!/_x + m)_{\alpha\beta}\delta(x-y) \tag{2.8}$$

and the free fermionic Langevin equation is

$$\frac{\partial\psi(x,t)}{\partial t} = (\partial^2 + m^2)\psi(x,t) + O(x,t) \tag{2.9}$$

With these considerations out of the way, we write down the general Feynman rules and propagators.

In perturbation theory we separate the quadratic part of the action, from the interaction

$$S = \int dx\{\frac{1}{2}\varphi_i D_{ij}\varphi_j + V(\varphi)\} \tag{2.10}$$

and the Langevin equation is rewritten as

$$\frac{\partial\varphi_i(x,t)}{\partial t} + \int dy K_{ij}(x,t)D_{kj}\varphi_j(y) = -K_{ij}\frac{\partial V(x,t)}{\partial\varphi_j} + \eta_i(x,t) \tag{2.11}$$

where

$$D_{ij} = \delta_{ij}(-\partial^2 + m^a) \qquad \text{for a scalar field,}$$

$$D_{\mu\nu} = -\delta_{\mu\nu}\mathrm{d}^2 + \partial_\mu\partial_\nu \qquad \text{for a gauge field,}$$

$$D_{\alpha\beta} = -\mathrm{i}\,\partial\!\!\!/_{\alpha\beta} + m\delta_{\alpha\beta} \qquad \text{for a fermion field.}$$

* Our Euclidean γ-matrices satisfy $\{\gamma_\mu,\gamma_\nu\} = -2\delta_{\mu\nu}$

The propagator is given by

$$G_{ik} = \left[\frac{\partial}{\partial t} + K.D\right]^{-1}_{ik} \tag{2.12}$$

It exists even in the case of a gauge field, due to the presence of the fifth time derivative. Indeed, we have

$$\tilde{G}_{ik}(k;t) = \delta_{ik}\theta(t)\exp(-t(k^2+m^2)) \tag{2.13}$$

for a scalar field

$$\tilde{G}_{\mu\nu}(k;t) = \left\{\left(\delta_{\mu\nu} - \frac{k_\mu k_\nu}{k^2}\right) \exp(-tk^2) + \frac{k_\mu k_\nu}{k^2}\right\}\theta(t) \tag{2.14}$$

for a gauge field, while for a fermion we have

$$G^F_{\alpha\beta}(k,t) = \delta_{\alpha\beta}\exp(-t(k^2+m^2))\theta(t) \tag{2.15}$$

which is very similar to eq.(2.13). There is a longitudinal piece in the gauge field propagator which is worth noting. It does not contribute to the Green functions of gauge invariant objects, as has been noted elsewhere. Inside Greens functions it does not contribute either, as far as the gauge field is coupled to a conserved current, in the presence of a cut-off. Thus it is dangerous if one uses a non gauge invariant regularization scheme.

The Langevin equations (2.11) can be solved iteratively as usual, and using the correlation functions of the η fields, an arbitrary Green function of the φ fields can be computed. We obtain a set of rules[7,9] :

i) draw the topologically inequivalent diagrams.

ii) two contracted η's form a crossed propagator, to be computed below.

iii) every loop contains a crossed line.

iv) two external vertices can not be connected by a continuous path of lines without crosses.

v) any crossed line can be connected to an external leg by a sequence of uncrossed lines.

vi) the number of crossed lines is given by

$$N_c = (\text{loops}) + N(\text{external lines}) - 1 \tag{2.16}$$

following,

vii) to the lines we associate the propagators

uncrossed line:

$$G(x,t)$$

crossed line:

$$D(x-x',t-t') = \int_0^t d\tau \int dy G(x-y;t-\tau)G(x'-y;t'-\tau) \tag{2.17}$$

After setting the Feynman rules, we are able to build any unregularized diagram, which is, in general, divergent. There are always diagrams as divergent (in the power counting sense) as those of the usual formulation of field theory. Since any loop contains a crossed line (rule (iii) above) we introduce a regularization in the noise, which is a non Markovian element in the statistical process[6]

$$< \eta_i(x,t)\eta_j(x',t' >= \delta_{ij}\delta(x-x')f_\epsilon(t-t') \tag{2.18}$$

with

$$\lim_{\epsilon\to 0} f_\epsilon(t) = 2\delta(t) \tag{2.19}$$

We will make a choice very similar to analytic regularization

$$f_\epsilon(t) = \epsilon|t|^{\epsilon-1} \tag{2.20}$$

With this regularization, the Green functions are meromorphic in c with poles on the real axis. Different c's could be used in different lines, as in analytic regularization[10,11].

The regularized crossed propagators is given by

$$D^\epsilon(x-x';t,t') = \int_0^t d\tau \int_0^{t'} d\tau' \int dy G(x-y;t-\tau)G(x'-y';t'-\tau')f_\epsilon(\tau-\tau') \quad (2.21)$$

The above function may be explicitly computed to provide, for a scalar field

$$\tilde{D}^\epsilon(p) = \int_{-\infty}^{\infty} \frac{d\omega}{\pi} \frac{\exp(-i\omega(t_1-t_2))}{(p^2+m^2)^2+\omega^2} f_\epsilon |\omega|^{-\epsilon}$$

$$= \hat{f}_\epsilon (p^2+m^2)^{-1-\epsilon} \quad (2.22)$$

while for fermionic fields we have

$$\Delta_{\alpha\beta}(k;t,t') = \hat{f}_\epsilon \frac{(\not{k}-m)_{\alpha\beta}}{(k^2+m^2)^{1+\epsilon}} \int_{-\infty}^{\infty} \frac{d\omega}{\pi} \frac{\exp(-i\omega(k^2+m^2)(t_1-t_2))}{1+\omega^2} |\omega|^{-\epsilon} \quad (2.23)$$

where

$$\hat{f}_\epsilon = \epsilon\Gamma(\epsilon)\sin\frac{\pi}{2}(1-\epsilon) \quad (2.24)$$

Relevant terms to a one loop computation are

$$\tilde{D}^\epsilon(p) = \frac{\hat{f}_\epsilon}{(p^2+m^2)^{1+\epsilon}} \Big\{ \exp(-(p^2+m^2)(t_2-t_1))$$

$$- \epsilon \int_{-\infty}^{\infty} \frac{d\omega}{\pi} \frac{\ln w}{1+\omega^2} \exp(-(i\omega(p^2+m^2)(t_1-t_2)) + \theta(\epsilon^2) \Big\} \quad (2.25)$$

The first term is exatly the same as that obtained with analytic regularization, the only difference lying in the fact that only crossed propagators are regularized.

2.2. An example

To illustrate the use of the regularization method introduced before, we will calculate the lowest order contributions to the polarization tensor of two-dimensional scalar QED. The photon polarization tensor, $\pi_{\mu\nu}$, is a convenient object to focus our attention as it must be transverse if gauge symmetry holds[12].

This is also a good test on the advantages of the new regularization method because, as the reader probably knows, the usual analytic regularization method of field theory induces a **mass** counterterm, breaking gauge invariance.

The model is described by the Lagrangian density

$$\mathcal{L} = \frac{1}{4} F_{\mu\nu} F^{\mu\nu} - \varphi^* D^\mu D_\mu \varphi + m^2 \varphi^* \varphi \tag{2.26}$$

where

$$F_{\mu\nu} = \partial_\mu A'' - \partial_\nu A_\mu$$

and

$$D, = \partial_\mu + ieA_\mu$$

is the covariant derivative. The Langevin equations governing the evolution of the fields A,, φ and φ^* are

$$\dot{A}_\mu = -\frac{\partial S}{\partial A_\mu} + \eta_\mu = \partial_\rho F^\rho{}_\mu - i\varphi^* D_m u\varphi + \eta_\mu$$

$$\dot{\varphi} = -\frac{\partial S}{\partial \varphi^*} + \eta = D^2 \varphi - m^2 \varphi + \eta \tag{2.27}$$

$$\dot{\varphi}^* = -\frac{\partial S}{\partial \varphi} + \eta^* = D^2 \varphi^* - m^2 \varphi^* + \eta^*$$

with the **random** field η_μ, η and η^* satisfying

$$< \eta_\mu (\mathrm{x}, t) \eta_\nu (x', t') > = 2\delta_{\mu\nu} f_\epsilon (t - t') \delta(x - \mathrm{x}')$$

$$< \eta_\mu (x, t) \eta^* (x', t') > = f_\epsilon (t - t') \delta(x, x')$$

$$< \eta(x, t) \eta(x', t') > = 0 \tag{2.28}$$

Solving these equations iteratively we found in lowest order of perturbation the graphs shows in fig. I. Note that there is one graph contributing to **fig.I.a,** four graphs contributing fig. I.b - they correspond to different graphs with the same topology having two crossed lines, one external and the other internal - and two graphs for fig. I.c. To get a better idea of the details of the calculation, we divide it in two parts. In the first part we calculate the amplitudes for the graphs, neglecting the contributions of the second term in eq.(2.25) to the crossed propagators. Also, for simplicity, in computing correlation functions, we suppose that the fifth times of the fields are all equal and very large. We then integrate over the fifth times of the internal vertices and keep only the dominant terms (i.e., only those surviving in the infinite fifth time limit). In the appendix A, for the reader's convenience, we have included more details of the calculation. For the graph I.a we get

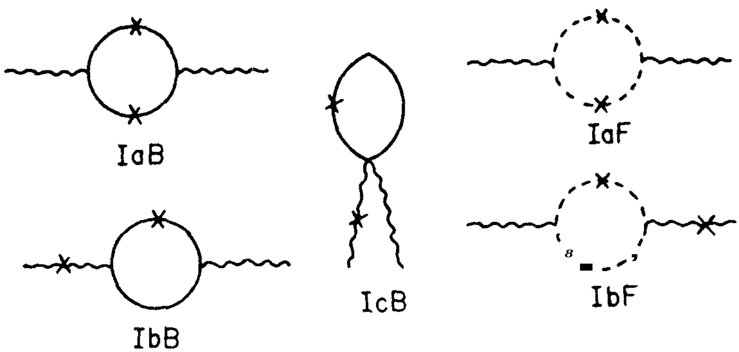

Fig.I - Diagrams contributing to vacuum polarization functions. B stands for interna1 bosonic Iines and F for fermionic.

$$\text{I.a} = 2\hat{f}_\epsilon^2 \int_{\tau_2>\tau_1} d\tau_1\, d\tau_2 \exp(-p^2(t-\tau_1))\exp(-p^2(t-\tau_1)$$
$$\times \int \frac{d^2k}{(2\pi)^2} \frac{(2k+p)_\mu (2k+p)_\nu}{[k^2+m^2]^{1+\epsilon}[(k+p)^2+m^2]^{1-\epsilon}} \exp\left(-[(k+p)^2+p^2+2m^2]|\tau_2-\tau_1|\right) \tag{2.29}$$

As explained in the appendix, the factor 2 on the right hand side of this equation comes from the two possible orderings of the internal times ($\tau_1 > \tau_2$ or $\tau_1 < \tau_2$). Integrating over τ_1 and τ_2 we get

$$\text{I.a} = \frac{\hat{f}_\epsilon^2}{p^2}\int\frac{d^2k}{(2\pi)^2}$$
$$\times\left\{\frac{1}{(k^2+m^2)^{1+\epsilon}}\;\frac{(2k+p)_\mu(2k+p)_\nu}{[(k+p)^2+m^2]^{1+\epsilon}[(k+p)^2+k^2+p^2+2m^2]}\right\} \quad (2.30)$$

This expression is very difficult (even for $\epsilon = 0$) to be evaluated. Although in two dimensions we could still obtain a closed form for it, we find it more instructive to employ a different procedure which **has** the advantage of being generalizable to four dimensions. The **basic** observation is that $\pi_{\mu\nu}$ is analytic in m for big enough m (equivalently, for small p). Then $\pi_{\mu\nu}$ can be expanded in powers of m^{-1} (or, equivalently, in powers of the externa1 momenta) and the transversality property of $\pi_{\mu\nu}$ will be correct only if it is satisfied at **each** order of the expansion. In the forthcoming calculation we will analyse the **terms** of the mentioned expansion, up to the first one to be finite when the regularization is removed. With this approximation we have

$$\text{I.''} = \frac{\hat{f}_\epsilon^2}{p^2}\int\frac{d^2k}{(2\pi)^2}\frac{2k_\mu k_\nu}{(k^2+m^2)^{3+2\epsilon}} = \frac{\delta_{\mu\nu}}{8\pi p^2 m^2} + O(\epsilon) \quad (2.31)$$

For the **graph** of fig. **(I.b)** the calculation is **also** straightforward but a little bit more extensive. From the appendix A we get

$$\text{I.b} = \frac{\delta_{\mu\nu}}{2\pi(p^2)^2}\left(\frac{1}{\epsilon}-1\right) - \frac{1}{12\pi m^2(p^2)^2}\left(\frac{5}{2}\delta_{\mu\nu}p^2 - p_\mu p_\nu\right) \quad (2.32)$$

Finally, the graph of fig. (I.c) gives

$$\text{I.c} = -\frac{2\hat{f}_\epsilon^2\delta_{\mu\nu}}{(p^2)^{2+\epsilon}}\int\frac{d^2k}{(2\pi)^2}\frac{1}{(k^2+m^2)^{1+\epsilon}} = -\frac{\delta_{\mu\nu}}{2\pi\epsilon(p^2)^2} \quad (2.33)$$

where the extra factor of two comes from the two graphs of **fig.(I.c)**.

Adding the contributions **eqs.(2.31-33)** we note that the divergent pieces exactly cancel, leaving the result

$$\frac{1}{12\pi p^2 m^2}\left(-\delta_{\mu\nu}+\frac{p_\mu p_\nu}{p^2}\right) - \frac{\delta_{\mu\nu}}{2\pi(p^2)^2} \quad (2.34)$$

which is, evidently, non transverse. This expression should be compared with the one employing the usual analytical regularization of field theory. In that case the regularised integrand is obtained by replacing the free propagator ($p^2 + m^2$) by $(p^2 + m^2)^\lambda$. With this substitution we get a polarization tensor differing from eq. (2.34) just by a term which vanishes after a judicious choice of the participating lambdas. Thus, up to this point there is no great advantage in using stochastic analytic regularization instead of the more usual one. However, we still have to compute the corrections coming from the neglected terrns in eq.(2.25). These terms are very important because, as we shall see right now, they will make the final result gauge invariant. Firstly notice that there is no correction coming from the graph of fig. I.a since it is finite without the regularization. The contribution of the remaining graphs is not difficult to be evaluated (see appendix A) and we get the following additional terms

$$\frac{\delta_{\mu\nu}}{\pi(p^2)^2} \qquad \text{from Fig.I.b}$$

$$\frac{-\delta_{\mu\nu}}{2\pi(p^2)^2} \qquad \text{from Fig. I.c}$$

Adding all these contributions we get a miraculous cancellation of the non transverse parts, leaving the net result

$$\frac{1}{12\pi m^2}\left(\frac{p_\mu p_\nu}{p^2} - \delta_{\mu\nu}\right) \tag{2.35}$$

The cancellation of the non transverse terms is a consequence of a general theorem proven[13], asserting the non-existence of mass corrections to the photon field. However, tbe mentioned theorem does not preclude the induction of non gauge invariant terms, ccntaining derivatives of the A_μ field.

3. Gauge invariance in 4-dimensional scalar and spinor QCD

The computation of the polarization tensor in 2-dimensional QED, with a transverse result, indicates that stochastic regularization may be a very efficient method for the study of gauge theories, specially in cases where dimensional regularization is not apropriate. However, there are problems in four dimensional non-abelian QCD[12]. We study the theory defined by the lagrangean density

$$\mathcal{L} = \mathrm{Tr}\frac{1}{4}F_{\mu\nu}F^{\mu\nu} - \varphi_i^* D_\mu D_\mu \varphi_i + m^2\varphi_i^*\varphi_i - \bar{\psi}_\ell(i \not{D} - m)\psi_\ell \tag{3.1}$$

where

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu + ie[A_\mu, A_\nu] \tag{3.1a}$$

$$D_\mu = \partial_\mu + ieA_\mu \tag{3.1b}$$

$$A_\mu = A_\mu^a \tau^a \tag{3.1c}$$

with τ^a the generators of the gauge algebra. In the above we introduced flavor indices for bosonic and fermionic fields; in the following we treat the two contributions separately in order to analyse them.

The Langevin equations are

$$\dot{A}_\mu = D_\rho F_{\rho\mu} - ie\varphi^* D_\mu \varphi + \eta_\mu \tag{3.2a}$$

$$\dot{\varphi} = D_\mu D_\mu \varphi - m^2\varphi + \eta \tag{3.2b}$$

$$\dot{\varphi}^* = D_\mu D_\mu \varphi^* - m^2\varphi^* + \eta^*$$

$$\dot{\psi}_\alpha = -\{(\not{D} - im)(\not{D} + im)\psi\}_\alpha + \theta_\alpha \tag{3.2c}$$

$$\dot{\bar{\psi}}_\alpha = -\{\bar{\psi}(\not{D}' - im)^T(\not{D}' + im)^T\}_\alpha + \bar{\theta}_\alpha \tag{3.2d}$$

with $D'_\mu = -\partial_\mu - ieA_\mu$.

Where, as proposed in ref.(8) we used the modified covariant Kernels

$$K_{\alpha\beta}(x,y) = i(\not{D} - im)_{\alpha\beta}\delta(x-y) \tag{3.3}$$

However, in the one loop computation we can drop the gauge field contribution in some expression and we use

$$< \theta_\alpha(x,t)\bar{\theta}_\beta(x',t') >= (i \not{\partial} + m)_{\alpha\beta}\delta^4(x-x')f_\epsilon(t-t') \tag{3.4a}$$

$$< \eta_\mu(x,t)\eta_\nu(x',t') >= \delta_{\mu\nu}\delta(x-x')f_\epsilon(t-t') \tag{3.4b}$$

$$< \eta(x,t)\eta^*(x',t') >= \delta(x-x')f_\epsilon(t-t') \tag{3.4c}$$

If we use dimensional regularization, the diagrams may be grouped in such a way as to respect gauge invariance. In this case, we find a counterterm

$$(Z_1^F + Z_1^B)F_{\mu\nu}^r F_{\mu\nu}^r \tag{3.5}$$

where

$$F_{\mu\nu}^r = \partial_\mu A_\nu - \partial_\nu A_\mu + ieZ_2[A_\mu, A_\nu]$$

But as it turns out, the result of the computation is given by

$$(Z_1^F + Z_1^B)(F_{\mu\nu}^r)^2 + (Z_3^F + Z_3^B)A_\mu \partial^2 A_\mu \tag{3.6}$$

We will consider explicitly the contribution of each diagram, the calculation being rather simplified by noting that since the relevant terms are divergent when the cut-off is eliminated we disregard finite terms, thus leaving the second term in eq.(2.25).

We calculate the following sets of diagrams.

1. Graphs with two external gluon lines, diagrams shown in fig. I. The diagram with two internal crossed lines is divergent and will be responsible for the gauge symmetry problems.

The bosonic contribution (I.a. B) is given by

$$\text{I.a.B} = \frac{\hat{f}_\epsilon}{p^2}\int \frac{d^4k}{(2\pi)^4}\left\{\frac{1}{(k^2+m^2)^{1+\epsilon}}\frac{(2k+p)_\mu(2k+p)_\nu}{[(k+p)^2+m^2]^{1+\epsilon}[(k+p)^2+k^2+p^2+2m^2]}\right.$$

$$= \frac{\hat{f}_\epsilon^2}{p^2}\int \frac{d^4k}{(2\pi)^4}\frac{2k_\mu k_\nu}{(k^2+m^2)^{3+2\epsilon}} + \text{finite terms}$$

$$\left. = \frac{\delta_{\mu\nu}}{64\pi^2 p^2 \epsilon} + \text{finite terms}\right\} \tag{3.7}$$

Notice the presence of the exponent 2ϵ in the next to last expression, implying a factor 1/2 in the final result. In particular, the value of dimensional regularization is 1/2 with $D = 4 - 2\epsilon$.

The fermionic contribution with two internal crossed lines, corresponding to fig. (I.a.F) is given by

$$\text{I.a.F} = \frac{2\hat{f}_\epsilon^2}{p^2} \int \frac{d^4k}{(2\pi)^4}$$

$$\times \frac{-\delta_{\mu\nu}(k^2 + m^2 + k.p) + 2k_\mu k_\nu + k_\mu p_\nu + k_\nu p_\mu}{[(k+p)^2 + m^2]^{1+\epsilon}(k^2 + m^2)^{1+\epsilon}(p^2 + k^2 + k.p + m^2)} \tag{3.8}$$

where the trace has already been performed. The resulting divergent piece is computed expanding in the external momenta, as in eq.(3.7)

$$\text{I.a.F} = -\frac{\hat{f}_\epsilon^2}{p^2} \int \frac{d^4k}{(2\pi)^4} \frac{4k_\mu k_\nu}{(k^2 + m^2)^{3+2\epsilon}}$$

$$= -\frac{\delta_{\mu\nu}}{32p^2\pi^2\epsilon} + \text{finite terms} \tag{3.9}$$

This value is one-half the result obtained in dimensional regularization with $D = 4 - 2\epsilon$, and minus twice the corresponding bosonic value. We shall comment further on these results later on.

Further diagrams are computed in the same way. For bosons, we have (I.b.B) and (I.c.B), which are given by the following expressions (equality holds for infinite parts)

$$\text{I.b.B} = \frac{2}{(p^2)^2} \int \frac{d^4k}{(2\pi)^4} \frac{(2k+p)_\mu (2k+p)_\nu}{(k^2 + m^2)^{1+\epsilon}[p^2 + k^2 + (p+k)^2 + 2m^2]}$$

$$= \frac{1}{(4\pi)^2}\left[-\frac{2m^2\delta_{\mu\nu}}{(p^2)^2} - \frac{5\delta_{\mu\nu}}{6p^2} + \frac{p_\mu p_\nu}{3(p^2)^2}\right]\frac{1}{\epsilon} \tag{3.10}$$

where again expansion in the external momenta has been performed to separate the divergent parts.

For diagrams (I.c.B) we have

$$\text{I.c.B} = -\frac{2\delta_{\mu\nu}}{(p^2)^2}\int \frac{d^4k}{(2\pi)^4}\frac{1}{(k^2+m^2)^{1+\epsilon}}$$

$$= \frac{2m^2\delta_{\mu\nu}}{(4\pi)^2(p^2)^2\epsilon} \tag{3.11}$$

We note that the mass counterterm for the gauge field cancels between eq.(3.10) (first term on the r.h.s.) and eq.(3.11).

The fermionic contributions are given by

$$\text{I.b.F}_1 + \text{I.bF}_2 = 4\int_{-\infty}^{\tau} dt_2 \int_{-\infty}^{t_2} dt_1 G_{\mu\rho}(p;\tau.t_1)$$

$$\times D_{\sigma\nu}(p;\tau,t_2)\text{tr}\gamma_\rho\Delta(k+p;t_1,t_2)\gamma_\sigma(-\not k+m)G(k;t_1,t_2) \tag{3.12}$$

and the result for the divergent part is

$$\text{I.b.F}_1 + \text{I.b.F}_2 = \frac{7\delta_{\mu\nu}}{48p^2\pi^2\epsilon} - \frac{p_\mu p_\nu}{12\pi^2(p^2)^2\epsilon} \tag{3.13}$$

The next step is the computation of diagrams with three and four gluon lines. Let us first give the detailed computation of the scalar case, and afterwards the results for spinor QCD_4. In the set presented in fig. (II.a) there are 18 linearly divergent diagrams, which cancel (their divergent contribution) in groups of two. This is expected, otherwise there would be charge-conjugation-violating counterterms $\partial_\mu A$, A, A,. These diagrams do not have counterparts in spinor QCD. The other set of topological similar diagrams with 3 external gluon lines has also 18 diagrams. A simple computation is presented by the representative diagrams (II.b.B)

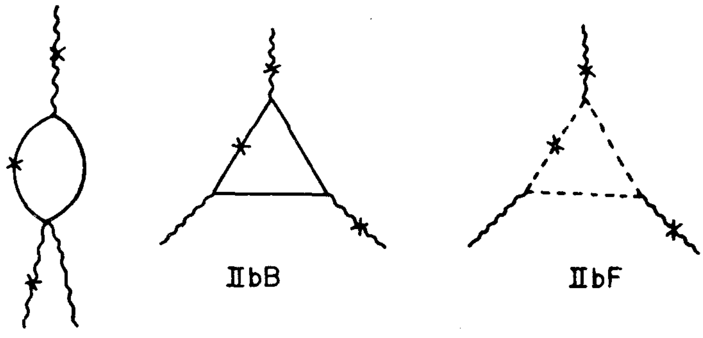

Fig.II - Representatives of 3 external gluon line diagrams. In order to obtain the full function, one muat add appropriate permutations.

$$\text{II.b.B} = -e^3 p_1^2 \frac{\operatorname{tr}(A_\mu A_\nu A_\rho)}{p_1^2 + p_2^2 + p_3^2}$$

$$\times \int \frac{d^4k}{(2\pi)^4} \frac{(2k+p_1)_\mu (2k+2p_1+p_2)_\nu (2k-p_3)_\rho}{(k^2+m^2)^{1+\epsilon}[p_3^2 + k^2 + (k-p_3)^2 + 2m^2][p_2^2 + (k+p_1)^2 + k^2 + 2m^2]} \tag{3.14}$$

where the external field propagators have been eliminated. After the usual momentum expansion we have

$$\text{II.b.B} = -\frac{e^3 p_1^2 \operatorname{tr} A_\mu A_\nu A_\rho}{p_1^2 + p_2^2 + p_3^2} \left[p_{1\mu}\delta_{\nu\rho} - p_{3\rho}\delta_{\mu\nu} + (p_1 - p_3)_\nu \delta_{\mu\rho}\right] \frac{1}{(4\pi)^2 \epsilon} \tag{3.15}$$

which, after addition of other diagrams with the corresponding changes of momenta and indices, gives

$$\sum(\text{II.b.B}) = -\frac{2e^3}{(4\pi)^2\epsilon} \operatorname{tr}(\partial_\mu A_\nu - \partial_\nu A_\mu) A^\mu A^\nu \tag{3.16}$$

The corresponding fermionic contributions are given in (II.b.F)

$$\text{II.b.F} = \frac{e^3}{2(4\pi)^2\epsilon} \operatorname{tr}(\partial_\mu A_\nu - \partial_\nu A_\mu) A^\mu A^\nu \tag{3.17}$$

We come to diagrams with four external gluon lines. They can be grouped in sets of topologically similar diagrams. Consider the set whose typical diagram is (III.a.b) which has two internal lines. They are 48 in number. This diagram is given by

$$\text{III.a.B} = \frac{2e^4 p_1}{p_1^2 + p_2^2 + p_3^2 + p_4^2} \operatorname{tr}(A_\mu A_\mu)^2$$

$$\times \int \frac{d^4 k}{(2\pi)^4} \frac{1}{(k^2+m^2)^{1+\epsilon}(p_2^2 + p_3^2 + k^2 + (k+p_2+p_3)^2 + 2m^2)}$$

$$- \frac{2e^4 p_1^2}{(2\pi)^2 \epsilon (p_1^2 + p_2^2 + p_3^2 + p_4^2)} + \text{finite terms} \tag{3.18}$$

Adding all contributions we have the result

$$\text{lII.a.B} = \frac{3e^4}{4\pi^2 \epsilon} \operatorname{tr}(A_\mu A_\mu)^2 \tag{3.19}$$

There are also **144** triangular diagrams, which may be grouped in 12 groups of twelve digrams, in such a way that in each subset the label of the external line is fixed. Let us take the diagram shown in fig. (II1.b.B)

$$\text{III.a.B} = \frac{2e^4 p_1^2}{p_1^2 + p_2^2 + p_3^2 + p_4^2} \operatorname{tr} A_\mu A_\rho A_\nu A_\nu \times \int \frac{d^4 k}{(2\pi)^4} \frac{1}{(k^2+m^2)^{1+\epsilon}}$$

$$\times \frac{(2k+p_1)_\mu (2k + 2p_1 + p_2)_\rho}{(k^2 + (k+p_1+p_2)^2 + p_3 + p_4 + 2m^2][(k+p_1)^2 + k^2 + p_2^2 + p_3^2 + p_4^2 + 2m^2]}$$

$$- \frac{e^4 p_1^2 \operatorname{tr}(A_\mu A_\mu)^2}{p_1^2 + p_2^2 + p_3^2 + p_4^2} \frac{1}{2(4\pi)^2 \epsilon} + \text{finite terms} \tag{3.20}$$

Further diagrarns are similarly computed, and the result of this group is

$$-\frac{6e^4}{4\pi^2 \epsilon} \operatorname{tr}(A_\mu A_\mu)^2 \tag{3.21}$$

Finally, we get to the box diagram (III.c.B). There are 96 diagrams, separated in group of 16, fixing the labels of the external legs. The exemplified case of (III.c.B) is given by

$$\text{III.c.B} = \frac{e^4 p_1^2}{p_1^2 + p_2^2 + p_3^2 + p_4^2} \text{tr } A_\mu A_\nu A_\rho A_\sigma \int \frac{d^4 k}{(2\pi)^4} \frac{1}{(k^2 + m^2)^{1+\epsilon}}$$

$$\times \frac{(2k + p_1)_\mu (2k + 2p_1 + p_2)_\nu (2k + 2p_1 + 2p_2 + p_3)_\rho (2k - p_4)_\sigma}{[k^2+(k-p_4)^2+p_4^2+2m^2][k^2+(k+p_1+p_2)^2+p_3^2+p_4^2+2m^2][k^2+(k+p_1)^2+p_2^2+p_3^2+p_4^2+2m^2]}$$

$$= \frac{e^4 p_1^2}{12(4a\pi)^2 \epsilon} \frac{\text{tr}\{3(A_\mu A_\mu)^2 + \frac{1}{2}[A_\mu, A_\nu]^2\}}{p_1^2 + p_2^2 + p_3^2 + p_4^2} \tag{3.22}$$

Further diagrams are computed similarly. Taking into account the correct combinatorial factors we obtain for this set

$$\text{III.c.B} + \text{other} = \frac{3e^4}{4\pi^2 \epsilon} \text{tr}(A_\mu A_\mu)^2 + \frac{e^4}{9\pi^2 \epsilon} \text{tr}[A_\mu, A_\nu]^2 \tag{3.23}$$

Notice that the terms $(A_\mu A_\mu)^2$ cancel between eqs.(3.19), (3.21) and (3.23).

In the fermionic case we have only the set corresponding to the box diagrams, fig. (III.c.F), which can be computed similarly

$$\text{III.F} = \frac{e^4}{2\pi^2 \epsilon} \text{tr}[A_\mu, A_\nu]^2 \tag{3.24}$$

We may now gather all results, and write down the counterterms, once we have the constants

$$Z_1^B = \frac{1}{(4\pi)^2 12\epsilon} \qquad Z_1^F = \frac{-1}{(4\pi)^2 3\epsilon}$$

$$Z_2^B = \frac{-1}{(4\pi)^2 8\epsilon} \qquad Z_2^F = \frac{1}{(4\pi)^2 4\epsilon}$$

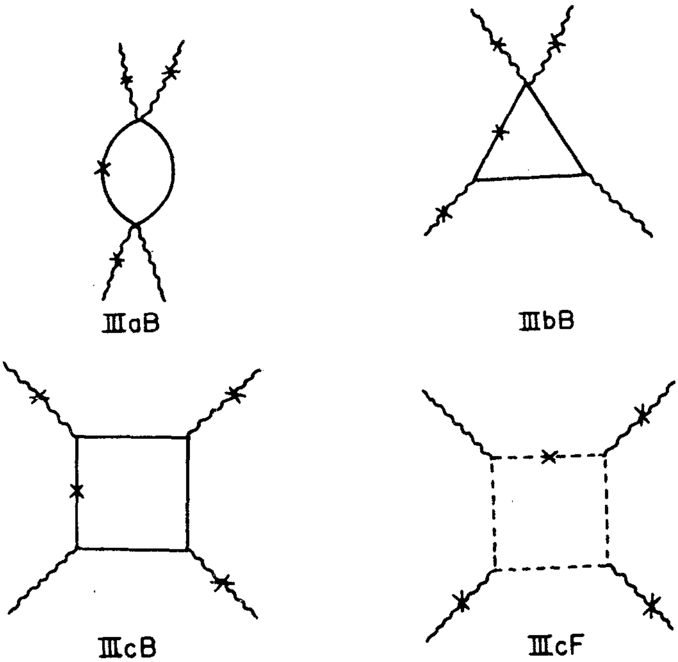

Fig.III - Representatives of 4 external gluon diagrams.

From the details of our computation it is not difficult to see the root of the problem, and verify why gauge invariance is broken. We look at the diagram with two crossed internal lines, (I.a.B) and (I.a.F), respectively for bosons and fermions. Then we verify that there is a doubling of the regulator (in this diagram it is always 2ϵ instead of ϵ appearing).This implies an extra factor of $1/2$ only for this diagram. A cross-check is the computation of this diagram with dimensional regularization, with $D = 4 - 2\epsilon$. It turns out that all results are the same, with the exception of diagrams (I.a.F,B) where a factor $1/2$ appears, generating a non zero $Z_2^{F,B}$.

In non stochastic approachs we have to add gauge fixing terms (and Fadeev-Popov fields) to the Lagrange density. In Lorentz gauge we add $\frac{1}{1\alpha}(\partial_\mu A_\mu)^2$. In the abelian case this term could absorb Z_2, but that is not the case with non abelian symmetries. In general, observables must be ai-independent, and that is the case only if A, is coupled to a conserved current. In stochastic quantization the problem is very **severe**[14], since the longitudinal part of A, field has a piece proportional to the fifth time, and it is mandatory that the gauge field be coupled to a conserved current.

4. Supersymmetric gauge theories

Although last section presented the method in a very bas shape, we will now comment on the status of supersymmetric gauge theories and claim that in this case all gauge breaking terms cancel between bosonic and fermionic contributions.

First notice that the phenomenon happens in the supersymmetric matter contribution to Z_2. The bosonic and fermionic counterterm lagrangian are

$$\delta L_B = \frac{n_B}{(4\pi)^2 12\epsilon}(F^a_{\mu\nu})^2 + \frac{n_B}{128\pi^2\epsilon}A_\mu \partial^2 A_\mu \tag{4.1}$$

$$\delta L_F = \frac{-n_F}{48\pi^2\epsilon}(F^a_{\mu\nu})^2 - \frac{n_F}{64\pi^2\epsilon}A_\mu \partial^2 A_\mu \tag{4.2}$$

where n_B and n_F are the numbers of bosonic and fermionic fields. In supersymmetric matter fields $n_B = 2n_F = 2n$ and the supersymmetric counter-lagrangian is given by

$$\delta L_{SUSY} = \frac{-n}{96\pi^2\epsilon}(F^a_{\mu\nu})^2 \tag{4.3}$$

which is gauge invariant.

In the case of N = 1 supersymmetric Yang-Mills multiplet the result is similar. We have to consider now the gauge field contribution to the loop, as well as a Majorana fermion contribution, in the adjoint representation. The gauge field contribution is the same as in scalar matter, with an extra factor of 2 coming from a combinatorial, while the fermionic contribution is the same as previously. Thus we get, as counterterm,

$$\delta L = \frac{1}{96\pi^2\epsilon}(F^a_{\mu\nu})^2 \tag{4.4}$$

which is, again, explicitly gauge invariant.

There is neither any break of supersymmetry, since the regularization process is independent of space-time, although we did not deal with supersymmetric Ward identities. But it is worthwhile mentioning, that usual supersyrnmetric cancellations of divergences occur.

5. Conclusions

We stablished, in this paper, case examples where stochastic quantization can be used to define a stochastic regularization prescription. Although the process is perfectly well defined as a regularization procedure, there may be problems in connection with gauge invariances in non perturbative scheme.

For abelian gauge theories the method is clearly advantageous, and possibly provides a good alternative to the use of dimensional regularization in chiral theories. Indeed, in lower dimensions (d $<$ 4) the polarization tensor is transverse and gauge invariance is not broken (even in the non abelian case). In four dimensions because of quadratic divergences, there is induction of a non gauge invariant counterterm $A_\mu \partial^2 A$,. Technically, this happens because the number of interna1 crossed lines (which are affected by the regularization) varies from graph to graph, causing unbalance of weights among them due to difference in the regular action. Thus, non invariant pieces do not cancel one against the other. This is related to the fact that the gauge field is not coupled to a conserved current, which is a necessary condition if the scheme is to be gauge invariant. The problem is not very dangerous in the abelian case since the non invariant counterterm can be rended to a renormalization of the gauge fixing term in field theory, or of the fifth time in stochastic theory.

In the non abelian case the situation is really serious, since it is no longer possible to relate it to $(\partial_\mu A^\mu)^2$ plus gauge invariant counterterms. The problem is that renormalization of the gluon polarization tensor is now dependent on the renormalization of the three and four vertex function of the gauge field. At the perturbative level we could remedy this desease by attributing different epsilons to different crossed propagators. This is not the original spirit of the method, but one can obtain gauge-invariant amplitudes if the epsilons are chosen adequately. Non perturbatively however, there is not, up to the present, any prescription which guarantees gauge invariant results.

Nonetheless, in supersymmetric theories, at the one loop level, we have a positive result, since non gauge invariant counterterms cancel between bosonic and fermionic contributions. Although we verified this 'result only for the infinite part, and at one loop level, it is very rewarding, since there is no scheme of regularization preserving at the same time gauge and supersymmetry of shell.

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Resumo

Teorias de gauge não abelianas ocupam uma posição de destaque em teorias de campo. Isto acontece porque simetrias locais dificilmente se preservam depois do processo de quantização. Regularizacc dimensional é, na prática, o único esquema de regularização que preserva simetria de gauge. Por outro lado, este procedimento quebra supersimetria e não deve ser considerado quando esta última é a simetria relevante. Além disso, num tratamento perturbativo, as ambiguidades de Gribov impedem uma especificação de gauge clara. Deve-se mencionar ainda que apesar de simulações numéricas usando métodos de Monte Carlo terem revelado muito sobre a estrutura das teorias de gauge, o inclusão de fermions ainda coloca um problema difícil, não somente do ponto de vista teórico como o do tempo de computação.