9 Gravity

9.1 The Universal Law of Gravity
9.2 The Universal Gravitation Constant, G
9.3 Gravity and Distance: The Inverse-Square Law
9.4 Weight and Weightlessness
9.5 Ocean Tides
Tides in Earth and Atmosphere
Tidal Bulges on the Moon
9.6 Gravitational Fields
Gravitational Field Inside a Planet
Einstein's Theory of Gravitation
9.7 Black Holes
9.8 Universal Gravitation

Dutch friend Ed Van den Berg uses balls to pose questions about the inverse-square law in the opening photo to this chapter. I am still moved by photos of astronauts performing space walks! Photo 3 shows Eric Mazur engaging students in class. When engagement occurs between professor and student, learning can occur. Without this engagement, likely less learning occurs. Hats off to Eric! Tomas Brage, physics department head at Lund University in Sweden shows a version of the Cavendish apparatus to measure G.

The personality profile is of Eric Mazur.

This chapter begins with a historical approach and ends on an astronomical theme. It offers a good place to reiterate the idea of a scientific theory, and comment on the all-too common and mistaken idea that because something has the status of scientific theory, it is somehow short of being valid. This view is evident in those who say, "But it's *only* a theory." Bring the essence of the first and last footnotes in the chapter into your discussion (about scientific homework and being unable to see radically new ways of viewing the world). The last chapter on *Cargo Cult Science* of Feynman's book, *Surely You're Joking Mr. Feynman* (Norton, 1985), expands nicely on this. (When I first read this delightful book I allowed myself only one chapter per day—to extend the pleasure. It's THAT good!)

Kepler's 3rd law follows logically from Newton's law of gravitation. Equate the force of gravity between planet *m* and the Sun *M* to the centripetal force on *m*. Then,

$$\frac{GmM}{r^2} = \frac{mv^2}{r} = \frac{m(2\pi r/T)^2}{r}$$

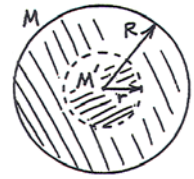
where the speed of the planet is 2π per period *T*. Cancel and collect terms,

$$\frac{GM}{4\pi^2} = \frac{r^3}{T^2}$$

This is Kepler's 3d law, for $GM/4\pi^2$ is a constant.

The idea of the force field is introduced in this chapter and is a good background for the electric field treated later in Chapter 22. The gravitational field here is applied to regions outside as well as inside the Earth.

In the text I say without explanation that the gravitational field increases linearly with radial distance inside a planet of uniform density. Figure 9.24 shows that the field increases linearly from zero at its center to maximum at the surface. This is also without explanation. The text states that "perhaps your instructor will provide the explanation." Here it is: We know that the gravitational force *F* between a particle *m* and a spherical mass M, when m is outside M is simply $F = GmM/d^2$. But when m is inside a uniform density solid sphere of mass M, the force on m is due only to the mass M' contained within the sphere of radius $r < R$, represented by the dotted line in the figure. Contributions from the shell $> r$ cancel out (Figure 9.25, and again for the analogous case of the electric field in Figure 22.20, later in the book). So, $F = GmM'/r^2$. From the ratio of M'/M, you can show that $M' = Mr^3/R^3$. [That is, $M'/M = V'/V = (^4/_3\, \pi r^3)/(^4/_3\, \pi R^3) = r^3/R^3$.] Substitute M' in Newton's equation for gravitation and we get $F = GmMr/R^3$. All terms on the right are constant except r. So $F = kr$; force is linearly proportional to radial distance when $r < R$.



Interestingly enough, the condition for simple harmonic motion is that the restoring force be proportional to displacement, $F = kr$. Hence the simple harmonic motion of one who falls in the tunnel through the Earth. (Hence also the simple harmonic motion of one who slides without friction to-and-fro along any straight line tunnel through any part of the Earth. The displacement is then the component $r \sin \theta$.) The period of simple harmonic motion, $T = 2\pi\sqrt{(R^3/GM)}$ is the same as that of a satellite in close circular orbit about the Earth. Note that it is independent of the length of the tunnel. I treat falling through a vertical tunnel in the screencast *Tunnel Through Earth*.

You can compare the pull of the Moon that is exerted on you with the pull exerted by more local masses, via the gravitational equation. Consider the ratio of the mass of the Moon to its distance squared:

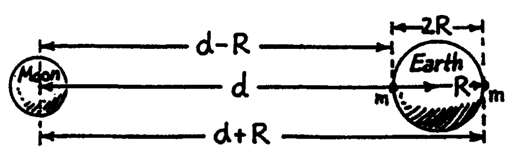
$$7.4 \times 10^{22}\ \text{kg}/(4 \times 10^5\ \text{km})^2 = 5 \times 10^{12}\ \text{kg/km}^2.$$

This is a sizeable ratio, one that buildings in your vicinity cannot match. (City buildings of greatest mass are typically on the order of 10^6 or 10^7 kilograms.) However, if you stand 1 kilometer away from the foot of a mountain of mass 5×10^{12} kilograms (about the mass of Mount Kilimanjaro), then the pull of the mountain and the pull of the Moon are about the same on you. Simply put, with no friction you would tend to gravitate from your spot toward the mountain—but you experience no tendency at all to gravitate from your spot toward the Moon! That's because the spot you stand on undergoes the same gravitational acceleration toward the Moon as you do. Both you and the whole Earth are accelerating toward the Moon. Whatever the lunar force on you, it has no tendency to pull you off a weighing scale—which is the essence of Think and Discuss 97 and 98. This is not an easy notion to grasp—at first.

Not covered in this edition is the inverse-cube nature of tidal forces. This follows from subtracting the tidal force on the far side of a body from the tidal force on the near side. Consider a kilogram of water on the side of the Earth nearest the Moon that is gravitationally attracted to the Moon with a greater force than a kilogram of water on the side of the Earth farthest from the Moon. The difference in force per kilogram of mass, $\Delta F/m$, which we'll call the tidal force T_F is

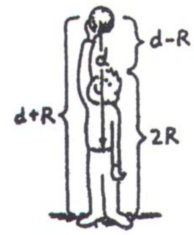
$$T_F = F_{d+R} - F_{d-R}$$
$$= GM\left[\frac{1}{(d+R)^2} - \frac{1}{(d-R)^2}\right] = \frac{4GMdR}{\left(d^2 - R^2\right)^2}$$

where M is the Moon's mass, ($d+R$) is the distance to the far side of Earth, (d-R) is the distance to the near side.



When d is very much greater than R, the $(d^2-R^2)^2$ is very nearly equal to d^4. Then the inverse-cube nature of tidal force is evident, for

$$T_F \sim \frac{4GMR}{d^3}\ .$$



Some interesting results occur when calculating the tidal force of the Moon on planet Earth. T_F is 2.2 × 10^{-6} N/kg. In contrast T_F of an overhead Moon on a person on Earth is 3 × 10^{-13} N/kg, a hundred million times weaker because of the tiny differences in pulls across the body. The tidal force of the Earth on the same person is 6 × 10^{-6} N/kg, more than the Moon's influence. And as the text reports, the tidal force due to a 1-kg mass held 1 m above your head is about 200 times as much effective as the Moon! Have those who believe the tidal effects of planets influence people make the calculations themselves.

A brief treatment of black holes is included in this chapter. The idea that light is influenced by a gravitational field isn't treated until Chapter 36, so may merit further explanation. You'll probably want to acknowledge that light bends in a gravitational field as does a thrown baseball. We say light travels in straight lines much for the same reason that some people say that a high-speed bullet doesn't curve downward in the first part of its trajectory. Over short distances the bullet doesn't *appear* to drop only because of its high speed and the short time involved. Likewise for light's speed, which we don't notice because of the vast distance it travels in the brief time it's in the strong part of Earth's gravitational field. Look ahead to the treatment of this idea in Figure 36.6.

Black holes at the center of galaxies are bigger than those found in binary star systems. The biggest recently reported galactic black holes have equivalent masses of some 10 to 40 billion Suns.

Dark matter is briefly mentioned in this chapter and is discussed in Chapter 11. Present consensus among astrophysicists is that dark energy is working against the force of gravity to accelerate the expansion of the universe. These findings downplay the oscillating universe scenario speculated about in the earlier editions of this text (although there remains speculation that the present outward acceleration may change to rapid deceleration and lead to a Big Crunch). The concepts of dark matter and dark energy are at the forefront of physics at this point, and are quite mysterious. Dark matter is out of sight, but not out of mind.

This chapter is prerequisite to the following chapter on satellite motion. It also provides useful background information for Chapter 22 (the inverse-square law, and the analogy between a gravitational and electric field) and Chapter 36 (general relativity). This chapter may be skipped without complicating the treatment of material in Chapters other than 22 and 36. It's an especially interesting chapter because the material is high interest, historical, quite understandable, and closely related to areas of space science that are currently in the public eye.

Practicing Physics Book:
• Inverse-Square Law • Our Ocean Tides

Problem Solving Book:
Some 30 problems

Laboratory Manual:
No labs for this chapter

Next-Time Questions:
• Earth-Moon Cable
• Moon Tides
• Solar Black Hole
• Earth Rise
• Normal Force and Weight
• Giant Plane
• Body Tide
• Gravity Force on Shuttle
• Weight

Hewitt-Drew-It! Screencasts:
• *Weight/Weightlessness*
• *Gravity*
• *Gravity Inside Earth*
• *Tunnel Through Earth*
• *Ocean Tides*

SUGGESTED LECTURE PRESENTATION

Begin by briefly discussing the simple codes and patterns that underlie the complex things around us, whether musical compositions or DNA molecules, and then briefly describe the harmonious motion of the solar system, the Milky Way and other galaxies in the universe—stating that the shapes of the planets, stars, and galaxies, and their motions are all governed by an extremely simple code, or if you will, a pattern. Then write the gravitational equation on the board. Give examples of bodies pulling on each other to convey a clear idea of what the symbols in the equation mean and how they relate. (Acknowledge that many other texts and references use the symbol *r* instead of the *d* in this text. The *r* is used to indicate the radial distance from a body's CG, and to emphasize the center-to-center rather than surface-to-surface nature for distance, and to prepare for *r* as a displacement vector. We don't set our plow that deep, however, and use *d* for distance.)

Inverse-Square Law

Discuss the inverse-square law and go over Figures 9.5 and 9.6 or their equivalents with candlelight or radioactivity.

Plot to scale an inverse-square curve on the board, showing the steepness of the curve—$^1/_4$, $^1/_9$, and $^1/_{16}$, for twice, three times, and four times the separation distance. This is indicated in Figures 9.5 and 9.6. (You may return to the curve of Figure 9.6 when you explain tides.)

> CHECK QUESTIONS: A photosensitive surface is exposed to a point source of light that is a certain distance away. If the surface were instead exposed to the same light four times as far away, how would the intensity upon it compare? A radioactive detector registers a certain amount of radioactivity when it is a certain distance away from a small piece of uranium. If the detector is four times as far from the uranium, how will the radioactivity reading compare?
>
> CHECK QUESTIONS: How is the gravitational force between a pair of planets altered when one of the planets is twice as massive? When both are twice as massive? When they are twice as far apart? When they are three times as far apart? Ten times as far apart? [The screencast on *Gravity* explains this.]
>
> CHECK QUESTION: What do you say to a furniture mover who claims that gravity increases with increased distance from the Earth, as evident to him when he's carrying heavy loads up flights of stairs?

Weight and Weightlessness

Note that we define weight as a support force. Even in a gravity-free region inside a rotating toroid, you'd experience weight. So weight needn't always be related to gravity. Discuss weightlessness and relate it to the queasy feeling your students experience when in a car that goes too fast over the top of a hill. State that this feeling is what an astronaut is confronted with all the time in orbit! Ask how many of your class would still welcome the opportunity to take a field trip to Cape Canaveral and take a ride aboard an orbiting vehicle. What an exciting prospect!

A marvelous space station called Skylab was in orbit in the 1970s. When it underwent unavoidable orbital decay the space shuttle was not yet in operation to give it the boost it needed to keep it in orbit. Quite unfortunate. But fortunately, there is movie footage of antics of astronauts aboard Skylab. The NASA film is "Zero *g*," which I showed every semester in my classes. It not only is fascinating in its shots of astronaut acrobatics in the orbiting lab, but illustrates Newton's laws as they apply to intriguing situations. The film shows the good sense of humor of the astronauts. A must! Also check out the screencast on *Weight/ Weightlessness*.

Discuss the differences in a baseball game on the Moon, and your favorite gravity-related topics.

Tides: Begin your treatment of tides by asking the class to consider the consequences of someone pulling your coat. If they pulled only on the sleeve, for example, it would tear. But if every part of your coat were pulled equally, it and you would accelerate—but it wouldn't tear. It tears when one part is pulled harder than another—or it tears because of a *difference* in forces acting on the coat. In a similar way, the spherical Earth is "torn" into an elliptical shape by differences in gravitational forces by the Moon and Sun.

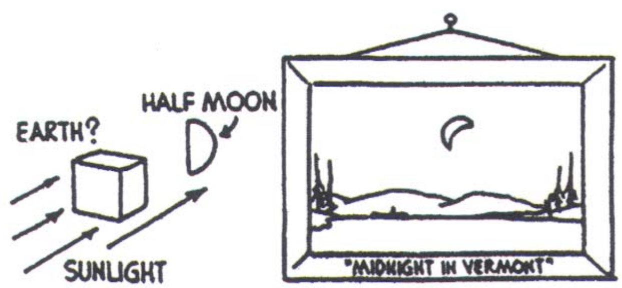
> CHECK QUESTION: Why do the tides not occur at the same time each day? [As the Earth takes 24 hours to rotate, the Moon advances in its orbit one hour ahead of the Earth. If the Moon didn't move in its orbit, the high-tide bulge would be at the same time each day as the Earth spins beneath the water.]

Misconceptions About the Moon: This is an appropriate place for you to dispel two popular misconceptions about the Moon. One is that since one side of the Moon's face is "frozen" to the Earth it doesn't spin like a top about its polar axis; and two, that the crescent shape commonly seen is *not* the Earth's shadow. To convince your class that the Moon spins about its polar axis, simulate the situation by holding your eraser at arms length in front of your face. Tell your class that the eraser represents the Moon and your head represents the Earth. Rotate slowly keeping one face of the eraser in your view. Call attention to the class that from your frame of reference, the eraser doesn't spin as it rotates about you—as evidenced by your observation of only one face, with the backside hidden. But your students occupy the frame of reference of the stars. (Each of them *is* a star.) From their point of view they can see all sides of the eraser as it rotates because it spins about its own axis as often as it rotates about your head. Show them how the eraser, if not slowly spinning and rotationally frozen with one face always facing the same stars, would show all of its sides to you as it circles around you. See one face, then wait 14 days later and the backside is in your view. The Moon's spin rate is the same as its rotational rate .

Misconception 2: Draw a half moon on the board. The shadow is along the diameter and is perfectly straight. If that were the shadow of the Earth, then the Earth would have to be flat, or be a big block shape! Discuss playing "flashlight tag" with a suspended basketball in a dark room that is illuminated by a flashlight in various locations. Ask your class if they could estimate the location of the flashlight by only looking at the illumination of the ball. Likewise with the Moon illuminated by the Sun!

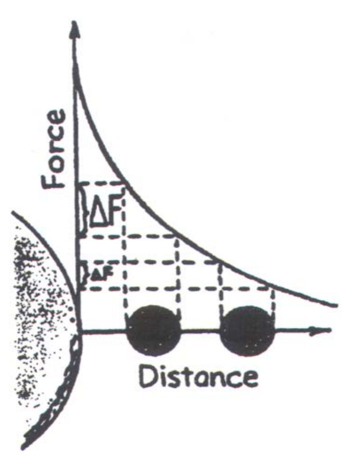
Sketch the picture on the right on the board and ask what is wrong with it.

[Answer: The Moon is in a daytime position as evidenced by the upper part of the Moon being illuminated. This means the Sun must be above. Dispel notions that the crescent shape of the Moon is a partial eclipse by considering a half moon and the shape of the Earth to cast such a shadow.]



Back to Tides

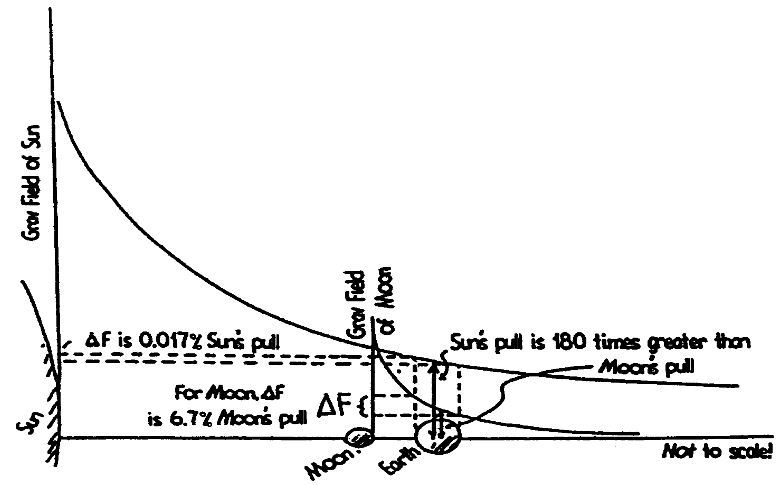
Explain tides via the accelerating ball of Jell-O as in Figure 9.14. Equal pulls result in an undistorted ball as it accelerates, but unequal pulls cause a stretching. This stretching is evident in the Earth's oceans, where the side nearest the Moon is appreciably closer to the Moon than the side farthest away. Carefully draw Figure 9.16 on the board, which explains why closeness is so important for tides. The figure shows that the magnitude of ΔF rather than F itself is responsible for tidal effects. Hence the greater attraction of the distant Sun produces only a small *difference* in pulls on the Earth, and compared to the Moon makes a small contribution to the tides on Earth.



Explain why the highest high tides occur when the Earth, Moon, and Sun are aligned—at the time of a new and a full Moon.

Discuss tides in the molten Earth and in the atmosphere.

Amplify Figure 9.16 with a comparison of ΔFs for both the Sun and the Moon as sketched at the upper right.



Clearly ΔF is smaller for the larger but farther Sun.

The text treats tides in terms of forces rather than fields. In terms of the latter, tidal forces are related to differences in gravitational *field* strengths across a body, and occur only for bodies in a nonuniform gravitational field. The gravitational fields of the Earth, Moon, and Sun, for example, are inverse-square fields—stronger near them than farther away. The Moon obviously experiences tidal forces because the near part to us is in a stronger part of the Earth's gravitational field than the far part. But even an astronaut in an orbiting space shuttle strictly speaking experiences tidal forces because parts of her body are closer to the Earth than other parts. This tidal force, the difference between the forces on near and far parts of her body, follow an inverse-cube law (in this manual as derived earlier). The micro differences produce **microtides**. Farther away in deep space, the differences are less. Put another way, the Earth's gravitational field is more uniform farther away. The "deepness" of a deep-space location can in fact be defined in terms of the amount of microtides experienced by a body there. Or equivalently, by the uniformity of any gravitational field there. There are no microtides in a body located in a strictly uniform gravitational field.

If there are microtides of an astronaut in orbit, would such microtides are even greater on the Earth's surface? The answer is yes. This brings up Exercise 42 that concerns **biological tides**. Interestingly enough, microtides in human bodies are popularly attributed to not the Earth, but the Moon. This is because popular knowledge cites that the Moon raises the ocean an average of 1 meter each 12 hours. Point out that the reason the tides are "stretched" by 1 meter is because part of that water is an Earth diameter closer to the Moon than the other part. In terms of fields, the near part of the Earth is in an appreciably stronger part of the Moon's gravitational field than the far part. To the extent that part of our bodies are closer to the Moon than other parts, there would be lunar microtides—but enormously smaller than the microtides produced by not only the Earth, but massive objects in one's vicinity.

Is there a way to distinguish between a gravity-free region and orbital free-fall inside the International Space Station? The answer is yes. Consider a pair of objects placed side by side. If the ISS were floating in a gravity-free region, the two objects would remain as placed over time. Since the ISS orbits the Earth, however, each object is in its own orbit about the Earth's center, in its own orbital plane. All orbital planes pass through the center of the Earth and intersect, which means that depending on the proximity of the objects, they may collide by the time the ISS makes a quarter orbit—a little more than 23 minutes! If the pair of objects are placed one in front of the other, with respect to their direction of motion, there will be no such effect since they follow the same orbital path in the same orbital plane. If the objects are one above the other, one farther from Earth, they will migrate in seemingly strange ways relative to each other because they are in distinct orbits with different PEs. Gravity makes itself present to astronauts by secondary effects that are not directly related to weight. See the "Bob Biker" Practice Pages 49 and 50 for Chapter 8.

> CHECK QUESTION: Consider the tiny tidal forces that DO act on our bodies, as a result of parts of our bodies experiencing slightly different gravitational forces. What planetary body is most responsible for microtides in our bodies? [The Earth, by far. When we are standing, there is a greater difference in Earth gravity on our feet compared to our heads than the corresponding differences in gravity due to farther away planetary bodies.]

Simulated Gravity in Space Habitats

The tallness of people in outer space compared to the radius of their rotating space habitats is very important. A *gravitational gradient* is appreciable in a relatively small structure. If the rim speed is such that the feet are at Earth-normal one *g*, and the head is at the hub, then the gravitational gradient is a full 1-*g*. If the head is halfway to the hub, then the gradient is $^1/_2$-*g*, and so forth. Simulated gravity is directly proportional to the radius. To achieve a comfortable $^1/_{100}$-*g* gradient, the radius of the structure must be 100 times that of one's height. Hence the designs of large structures that rotate to produce Earth-normal gravity.

Tidal forces reach an extreme in the case of a **black hole**. The unfortunate fate of an astronaut falling into a black hole is not encountering the singularity, but the tidal forces encountered far before getting that close. Approaching feet first, for example, his closer feet would be pulled with a greater force than his midsection, which in turn would be pulled with a greater force than his head. The tidal forces would stretch him and he would be killed before these forces literally pulled him apart.

Gravitational Fields

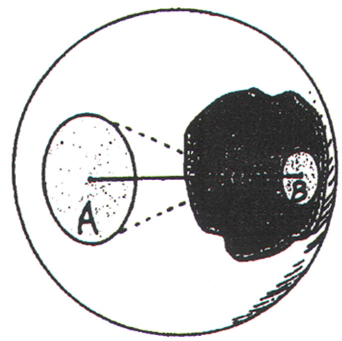
Introduce the idea of force/mass for a body, and the gravitational force field. Relate the gravitational field to the more visible magnetic field as seen via iron filings (Look ahead to Figures 24.2 and 22.4). Since the field strength of the gravitational field is simply the ratio of force per mass, it behaves as force—it follows an inverse-square relationship with distance. Pair this with student viewing of my screencast on *Earth's Gravity*, where the field strength inside a planet is treated. Follow this with *Tunnel Through Earth*.

It's easy to convince your students that the gravitational force on a body located at the exact center of the tunnel would be zero—a chalkboard sketch showing a few symmetrical force vectors will do this. Hence the gravitational field at the Earth's center is zero. Then consider the magnitude of force the body would experience between the center of the Earth and the surface. A few more carefully drawn vectors will show that the forces don't cancel to zero. The gravitational field is between zero and the value at the surface. You'd like to easily show that it's half for an Earth of uniform density, to establish the linear part of the graph of Figure 9.24. Careful judgment should be exercised at this point. For most classes I would think the geometrical explanation would constitute "information overload" and it would be best to simply say "It can be shown by geometry that halfway to the center the field is half that at the surface…" and get on with your lecture. For highly motivated students it may be best to develop the geometrical explanation (given earlier in this manual). Then the pedagogical question is raised; how many students profit from your display of the derivation and how many will not?

Class time might better be spent on speculating further about the hole drilled through the Earth. Show with the motion of your hand how if somebody fell in such a tunnel they would undergo simple harmonic motion—and that this motion keeps perfect pace with a satellite in close circular orbit about the Earth. The time for orbit, nearly 90 minutes, is the time to make a to and fro trip in the tunnel. Consider going further and explain how ideally the period of oscillation of a body traveling in such a tunnel under the influence of only gravity would be the same for any straight tunnel—whether from New York to Australia, or from New York to Hawaii or China. You can support this with the analogy of a pendulum that swings through different amplitudes with the same period. In non-vertical tunnels, of course, the object must slide rather than drop without friction. But the period is the same, and timetables for travel in this way would be quite simple; any one-way trip would take nearly 45 minutes. See the screencast *Tunnel Through Earth*.

Gravitational Field Inside a Hollow Planet

Consider the case of a body at the center of a completely *hollow* planet. Again, the field at the center is zero. Then show that the field everywhere inside is zero—by careful explanation of the following sketch. [Consider sample point P, twice as far from side A than side B. A solid cone defines area A and area B. Careful thought shows A has 4 times the area of B, and therefore has 4 times as much mass as B. That would mean 4 times as much gravitational pull, but being twice as far has only $^{1}/4$ as much pull. $^{1}/4$ of 4 gives the same gravitational pull as the pull toward B. So the forces cancel out (as they of course do in the center). The forces cancel everywhere inside the shell provided it is of uniform composition. If you stress this material (which will likely be on the heavy-duty side for many students) the following Check Question will measure the worth of your lecture effort.]



> CHECK QUESTION: Sketch a graph similar to that in Figure 9.24 to represent the gravitational field inside and outside a hollow sphere. (The graphical answers should look like the following: A thin shelled planet is on the left, a thick shelled one is on the right.)

Speculate about the living conditions of a civilization inside a hollow planet. Expanding on Exercise 54 that considers a hollow planet, consider what happens to the *g* field inside when a massive spaceship lands on the outside surface of the hollow planet. The situation is interesting!

Black Holes

Begin by considering an indestructible person standing on a star, as in Figure 9.27. Write the gravitational equation next to your sketch of the person on the star, and show how only the radius changes in the equation as the star shrinks, and how the force therefore increases. Stress that the force on the person who is able to remain at distance *R* as the star shrinks experiences no change in force—the field there is constant as the star shrinks, even to a black hole. It is near the shrinking surface where the huge fields exist.

> CHECK QUESTION: Consider a satellite companion to a star that collapses to become a black hole. How will the orbit of the companion satellite be affected by the star's transformation to a black hole? [Answer is not at all. No terms in the gravitation equation change. What does happen, though, is that matter streams from the visible star to the black hole companion, emitting x-rays as it accelerates toward the black hole, providing evidence of its existence.]

Cosmological Constant

It was long believed that gravity is only attractive. Newton worried that it would cause the universe to collapse and assumed God kept that from happening. Einstein sought a natural explanation and added a constant term to his gravity equation to give a repulsion to stabilize the universe. This was term called the *cosmological constant*. A few years later, after discovery that the universe is expanding, Einstein dropped it, calling it his "biggest blunder." However, in 1998 it was discovered that the expansion of the universe is accelerating under the action of some yet unidentified energy field called **dark energy**, which carries about 68 percent of the energy and mass of the universe. The cosmological constant is thought to be the source of the dark energy. However, calculations give a result that is fifty orders of magnitude greater than what is observed. This "cosmological constant problem" is one of the biggest unanswered questions in physics.

Answers and Solutions for Chapter 9

Reading Check Questions

1. Newton discovered that gravity is universal.
2. The Newtonian synthesis is the union between terrestrial laws and cosmic laws.
3. The Moon falls away from the straight line it would follow if there were no gravitational force acting on it.
4. Every body in the universe attracts every other body with a force that, for two bodies, is directly proportional to the product of their masses and inversely proportional to the square of the distance between their centers:

   $$F = G\frac{m_1 m_2}{d^2}$$

5. The gravitational force between is 6×10^{-11} N.
6. The gravitational force is about 10 N, or more accurately, 9.8 N.
7. Actually the mass of Earth could then be calculated, but calling it "weighing of Earth" seemed more dramatic.
8. The force of gravity is one-fourth as much.
9. Thickness is one-fourth as much.
10. You're closer to Earth's center at Death Valley, below sea level, so you weigh more there than on any mountain peak.
11. Springs would be more compressed when accelerating upward; less compressed when accelerating downward.
12. No changes in compression when moving at constant velocity.
13. Your weight is measured as *mg* when you are firmly supported in a gravitational field of *g* and in equilibrium.
14. In an upward accelerating elevator your weight is greater than *mg*, in free fall your weight is zero.
15. The occupants are without a support force.
16. Tides depend on the difference in pulling strengths.
17. One side is closer.
18. Spring tides are higher.
19. Yes, interior tides occur in Earth and are caused by unequal forces on opposite sides of Earth's interior.
20. At the time of a full or new moon, Sun, Moon, and Earth are aligned.
21. No, for no lever arm would exist between Earth's gravitational pull and the Moon's axis.
22. A gravitational field is a force field about any mass, and can be measured by the amount of force on a unit of mass located in the field.
23. At Earth's center, its gravitational field is zero.
24. Half way to the center, the gravitational field is half that at the surface.
25. Anywhere inside a hollow planet the gravitational field of the planet is zero.
26. Einstein viewed the curve in a planet's path as a result of the curvature of space itself.
27. Your weight would increase.
28. Field strength increases as the star surface shrinks.
29. A black hole is invisible because even light cannot escape it.
30. Perturbations of Uranus' orbit not accounted for by any known planet led to the discovery of Neptune.

Think and Do

31. Open ended.
32. Hold it half way from your eye and it covers the same area of eyesight as the unfolded bill, nicely illustrating the inverse-square law.

Plug and Chug

33. $F = G\frac{m_1 m_2}{d^2} = 6.67 \times 10^{-11}\,\mathrm{N \cdot m^2/kg^2} \times \frac{(1\,\mathrm{kg})(6 \times 10^{24}\,\mathrm{kg})}{(6.4 \times 10^{6}\,\mathrm{m})^2} = 9.8\,\mathrm{N}$.

34. $F = G\frac{m_1 m_2}{d^2} = 6.67 \times 10^{-11}\,\mathrm{N \cdot m^2/kg^2} \times \frac{(1\,\mathrm{kg})(6 \times 10^{24}\,\mathrm{kg})}{[2(6.4 \times 10^{6}\,\mathrm{m})]^2} = 2.5\,\mathrm{N}$.

35. $F = G\frac{m_1 m_2}{d^2} = 6.67 \times 10^{-11}\ \mathrm{N\cdot m^2/kg^2} \times \frac{(6.0 \times 10^{24}\ \mathrm{kg})(7.4 \times 10^{22}\ \mathrm{kg})}{(3.8 \times 10^{8}\ \mathrm{m})^2} = 2.1 \times 10^{20}$ N.

36. $F = G\frac{m_1 m_2}{d^2} = 6.67 \times 10^{-11}\ \mathrm{N\cdot m^2/kg^2} \times \frac{(6.0 \times 10^{24}\ \mathrm{kg})(2.0 \times 10^{30}\ \mathrm{kg})}{(1.5 \times 10^{11}\ \mathrm{m})^2} = 3.6 \times 10^{22}$ N.

37. $F = G\frac{m_1 m_2}{d^2} = 6.67 \times 10^{-11}\ \mathrm{N\cdot m^2/kg^2} \times \frac{(3.0\ \mathrm{kg})(6.4 \times 10^{23}\ \mathrm{kg})}{(5.6 \times 10^{10}\ \mathrm{m})^2} = 4.1 \times 10^{-8}$ N.

38. $F = G\frac{m_1 m_2}{d^2} = 6.67 \times 10^{-11}\ \mathrm{N\cdot m^2/kg^2} \times \frac{(3.0\ \mathrm{kg})(100\ \mathrm{kg})}{(0.5\ \mathrm{m})^2} = 8.0 \times 10^{-8}$ N.

The obstetrician exerts about twice as much gravitational force.

Think and Solve

39. From $F = GmM/d^2$, three times d squared is $9\,d^2$, which means the force is one ninth of surface weight.

40. From $F = GmM/d^2$, $(2m)(2M) = 4\,mM$, which means the force of gravity between them is 4 times greater.

41. From $F = G2m2M/(2d^2) = 4/4\,(GmM/d^2)$, with the same force of gravitation.

42. From $F = GmM/d^2$, if d is made 10 times smaller, $1/d^2$ is made 100 times larger, which means the force is 100 times greater.

43. $g = \frac{GM}{d^2} = \frac{(6.67 \times 10^{-11})(6.0 \times 10^{24})}{[(6380 + 200)\mathrm{x}10^3]^2}$ = 9.24 N/kg or 9.24 m/s^2; 9.24/9.8 = 0.94 or 94%.

44. (a) Substitute the force of gravity in Newton's second law: $a = \frac{F}{m} = \frac{GmM/d^2}{m} = G\frac{M}{d^2}$.

(b) Note that m cancels out. Therefore the only mass affecting your acceleration is the mass M of the planet, not your mass.

Think and Rank

45. B=C, A, D
46. C, B, A
47 a. B, A=C, D b. D, A=C, B
48. C, B, A
49. B, A, C

Think and Explain

50. Nothing to be concerned about on this consumer label. It simply states the universal law of gravitation, which applies to *all* products. It looks like the manufacturer knows some physics and has a sense of humor.

51. This goes back to Chapter 4: A heavy body doesn't fall faster than a light body because the greater gravitational force on the heavier body (its weight), acts on a correspondingly greater mass (inertia). The ratio of gravitational force to mass is the same for every body—hence all bodies in free fall accelerate equally.

52. In accord with the law of inertia, the Moon would move in a straight-line path instead of circling both the Sun and Earth.

53. The force of gravity is the same on each because the masses are the same, as Newton's equation for gravitational force verifies.

54. The force of gravity is the same on each because the masses are the same, as Newton's equation for gravitational force verifies. When dropped the crumpled paper falls faster only because it encounters less air resistance than the sheet.

55. The force decreases as the square of increasing distance, or force increases with the square of decreasing distance.

56. The forces between the apple and Earth are the same in magnitude. Force is the same either way, but the corresponding accelerations of each are different.

57. In accord with Newton's 3rd law, the weight of the Earth in the gravitational field of Larry is 300 N; the same as the weight of Larry in Earth's gravitational field.

58. Less, because an object there is farther from Earth's center.

59. Letting the equation for gravitation guide your thinking, twice the diameter is twice the radius, which corresponds to 1/4 the astronaut's weight at the planet's surface.

60. Letting the equation for gravitation guide your thinking, twice the mass means twice the force, and twice the distance means one-quarter the force. Combined, the astronaut weighs half as much.

61. Your weight would decrease if the Earth expanded with no change in its mass and would increase if the Earth contracted with no change in its mass. Your mass and the Earth's mass don't change, but the distance between you and the Earth's center does change. Force is proportional to the inverse square of this distance.

62. A person is weightless when the only force acting is gravity, and there is no support force. Hence the person in free fall is weightless. But more than gravity acts on the person falling at terminal velocity. In addition to gravity, the falling person is "supported" by air resistance.

63. The high-flying jet plane is not in free fall. It moves at approximately constant velocity so a passenger experiences no net force. The upward support force of the seat matches the downward pull of gravity, providing the sensation of weight. The orbiting space vehicle, on the other hand, is in a state of free fall. No support force is offered by a seat, for it falls at the same rate as the passenger. With no support force, the force of gravity on the passenger is not sensed as weight.

64. Gravitational force is indeed acting on a person who falls off a cliff, and on a person in a space shuttle. Both are falling under the influence of gravity.

65. In a car that drives off a cliff you "float" because the car no longer offers a support force. Both you and the car are in the same state of free fall. But gravity is still acting on you, as evidenced by your acceleration toward the ground. So, by definition, you would be weightless (until air resistance becomes important).

66. The two forces are the normal force and mg, which are equal when the elevator doesn't accelerate, and unequal when the elevator accelerates.

67. The pencil has the same state of motion that you have. The force of gravity on the pencil causes it to accelerate downward alongside of you. Although the pencil hovers relative to you, it and you are falling relative to the Earth.

68. The jumper is weightless due to the absence of a support force.

69. You disagree, for the force of gravity on orbiting astronauts is almost as strong as at Earth's surface. They feel weightless because of the absence of a support force.

70. In a rotating habitat (as discussed in Chapter 8) rotation provides the required support force. The weight experienced would be a centrifugal force.

71. Your weight equals mg when you are in equilibrium on a horizontal surface and the only forces acting on you are mg downward and an equal-and-opposite normal force N upward.

72. The scale shows the normal force acting on you, which on an incline is less than the normal force that occurs on a firm horizontal surface. The force of gravity on you is *mg* whatever the support force. But for *mg* to align with the normal force, the scale must be supported on a horizontal surface. If you want to know how strongly gravity is pulling on you, you need to put your scale on a horizontal surface.

73. The force due to gravity, *mg*, does not vary with jouncing. Variations in the scale reading are variations in the support force *N*, not in *mg*.

74. Just as differences in tugs on your shirt will distort the shirt, differences in tugs on the oceans distort the ocean and produce tides.

75. The gravitational pull of the Sun on the Earth is greater than the gravitational pull of the Moon. The tides, however, are caused by the *differences* in gravitational forces by the Moon on opposite sides of the Earth. The difference in gravitational forces by the Moon on opposite sides of the Earth is greater than the corresponding difference in forces by the stronger pulling but much more distant Sun.

76. No torque occurs when the Moon's long axis is aligned with Earth because there is no lever arm. A lever arm exists when the Moon's CG and CM are not aligned with Earth.

77. No. Tides are caused by differences in gravitational pulls. If there are no differences in pulls, there are no tides.

78. Ocean tides are not exactly 12 hours apart because while the Earth spins, the Moon moves in its orbit and appears at its same position overhead about every 25 hours, instead of every 24 hours. So the two-high-tide cycle occurs at about 25-hour intervals, making high tides about 12.5 hours apart.

79. Lowest tides occur along with highest tides, spring tides. So the tide cycle consists of higher-than-average high tides followed by lower-than-average low tides (best for digging clams!).

80. Whenever the ocean tide is unusually high, it will be followed by an unusually low tide. This makes sense, for when one part of the world is having an extra high tide, another part must be donating water and experiencing an extra low tide. Or as the hint in the exercise suggests, if you are in a bathtub and slosh the water so it is extra deep in front of you, that's when it is extra shallow in back of you—"conservation of water!"

81. Because of its relatively small size, different parts of the Mediterranean Sea and other relatively small bodies of water are essentially equidistant from the Moon (or from the Sun). So one part is not pulled with any appreciably different force than any other part. This results in extremely tiny tides. Tides are caused by appreciable differences in pulls.

82. Tides are produced by *differences* in forces, which relate to differences in distance from the attracting body. One's head is appreciably closer than one's feet to the overhead melon. The greater proportional difference for the melon out-tides the more massive but more distant Moon. One's head is not appreciably closer to the Moon than one's feet.

83. In accord with the inverse-square law, twice as far from the Earth's center diminishes the value of *g* to 1/4 its value at the surface or 2.5 m/s^2.

84. For a uniform-density planet, *g* inside at half the Earth's radius would be 5 m/s^2. This can be understood via the spherical shell idea discussed in the chapter. Halfway to the center of the Earth, the mass of the Earth in the outer shell can be neglected—the gravitational contribution of all parts of the shell cancels to zero. Only the mass of the Earth "beneath" contributes to acceleration, the mass in the sphere of radius $r/2$. This sphere of half radius has only $^1/8$ the volume and only $^1/8$ the mass of the whole Earth (volume varies as r^3). This effectively smaller mass alone would find the acceleration due to gravity $^1/8$ that of *g* at the surface. But consider the closer distance to the Earth's center as well. This twice-as-close distance alone would make *g* four times as great (inverse-square law). Combining both factors, $^1/8$ of 4 = $^1/2$, so the acceleration due to gravity at $r/2$ is $g/2$.

85. Your weight would be less down in the mine shaft. One way to explain this is to consider the mass of the Earth above you which pulls upward on you. This effect reduces your weight, just as your weight is reduced if someone pulls upward on you while you're weighing yourself. Or more accurately, we see that you are effectively within a spherical shell in which the gravitational field contribution is zero; and

that you are being pulled only by the spherical portion below you. You are lighter the deeper you go, and if the mine shaft were to theoretically continue to the Earth's center, your weight moves closer to zero.

86. The increase in weight indicates that the Earth is more compressed—more compact—more dense—toward the center. The weight that normally would be lost when in the deepest mine shafts from the upward force of the surrounding "shell" is more than compensated by the added weight gained due to the closeness to the more dense center of the Earth. (Referring to our analysis of Exercise 49, if the mine shaft were deep enough, reaching halfway to the center of the Earth, you would, in fact, weigh less at the bottom of the shaft than on the surface, but more than half your surface weight.)

87. Open-ended.

Think and Discuss

88. Your friend's misconception is a popular one. But investigation of the gravitational equation shows that no matter how big the distance, force never gets to zero. If it were zero, any space shuttle would fly off in a straight-line path!

89. The force of gravity on Moon rocks at the Moon's surface is considerably stronger than the force of gravity between Moon distant Earth. Rocks dropped on the Moon fall onto the Moon's surface. (The force of the Moon's gravity is about $^1/_6$ of the weight the rock would have on Earth; but the force of the Earth's gravity at that distance is only about $^1/_{3600}$ of the rock's Earth-weight.)

90. If gravity between the Moon and its rocks vanished, the rocks, like the Moon, would continue in their orbital path around the Earth. The assumption ignores the law of inertia.

91. Nearer the Moon, because of its smaller mass and lesser pull at equal distances.

92. The Earth and Moon equally pull on each other in a single interaction. In accord with Newton's 3rd law, the pull of the Earth on the Moon is equal and opposite to the pull of the Moon on the Earth. An elastic band pulls equally on the fingers that stretch it.

93. Earth and Moon *do* rotate around a common point, but it's not midway between them (which would require both Earth and Moon to have the same mass). The point around which Earth and Moon rotate (called the *barycenter*) is within the Earth about 4600 km from the Earth's center.

94. For the planet half as far from the Sun, light would be four times as intense. For the planet ten times as far, light would be $^1/_{100}$th as intense.

95. By the geometry of Figure 9.4, tripling the distance from the small source spreads the light over 9 times the area, or 9 m^2. Five times the distance spreads the light over 25 times the area or 25 m^2, and for 10 times as far, 100 m^2.

96. The gravitational force on a body, its weight, depends not only on mass but distance. On Jupiter, this is the distance between the body being weighed and Jupiter's center—the radius of Jupiter. If the radius of Jupiter were the same as that of the Earth, then a body would weigh 300 times as much because Jupiter is 300 times more massive than Earth. But the radius of Jupiter is about 10 times that of Earth, weakening gravity by a factor of 100, resulting in 3 times its Earth weight. (The radius of Jupiter is actually about 11 times that of Earth).

97. If Earth gained mass you'd gain weight. Since Earth is in free fall around the Sun, the Sun contributes nothing to your weight. Earth gravitation presses you to Earth; solar gravitation doesn't press you to Earth.

98. First of all, it would be incorrect to say that the gravitational force of the distant Sun on you is too small to be measured. It's small, but not immeasurably small. If, for example, the Earth's axis were supported such that the Earth could continue turning but not otherwise move, an 85-kg person would see a gain of 1/2 newton on a bathroom scale at midnight and a loss of 1/2 newton at noon. The key idea is *support*. There is no "Sun support" because the Earth and all objects on the Earth—you, your bathroom scale, and everything else—are continually falling around the Sun. Just as you wouldn't be pulled against the seat of your car if it drives off a cliff, and just as a pencil is not pressed against the floor of an elevator in free fall, we are not pressed against or pulled from the Earth by our gravitational

interaction with the Sun. That interaction keeps us and the Earth circling the Sun, but does not press us to the Earth's surface. Our interaction with the Earth does that.

99. The gravitational force varies with distance. At noon you are closer to the Sun. At midnight you are an extra Earth diameter farther away. Therefore the gravitational force of the Sun on you is greater at noon.

100. As stated in question 98, our "Earth weight" is due to the gravitational interaction between our mass and that of the Earth. The Earth and its inhabitants are freely falling around the Sun, the rate of which does not affect our local weights. (If a car drives off a cliff, the Earth's gravity, however strong, plays no role in pressing the occupant against the car while both are falling. Similarly, as the Earth and its inhabitants fall around the Sun, the Sun plays no role in pressing us to the Earth.)

101. The Moon does rotate like a top as it circles Earth. It rotates once per revolution, which is why we see only the same face. If it didn't rotate, we'd see the back side every half month.

102. Tides would be greater if the Earth's diameter were greater because the difference in pulls would be greater. Tides on Earth would be no different if the Moon's diameter were larger. The gravitational influence of the Moon is just as if all the Moon's mass were at its CG. Tidal bulges on the solid surface of the Moon, however, would be greater if the Moon's diameter were larger—but not on the Earth.

103. Earth would produce the largest microtides in your body. Microtides are greatest where the difference between your head and feet is greatest compared with the distance to the tide-pulling body, Earth.

104. Tides occur in Earth's crust and Earth's atmosphere for the same reason they occur in Earth's oceans. Both the crust and atmosphere are large enough so there are appreciable differences in distances to the Moon and Sun. The corresponding gravitational differences account for tides in the crust and atmosphere.

105. More fuel is required for a rocket that leaves the Earth to go to the Moon than the other way around. This is because a rocket must move against the greater gravitational field of the Earth most of the way. (If launched from the Moon to the Earth, then it would be traveling with the Earth's field most of the way.)

106. On a shrinking star, all the mass of the star pulls in a noncanceling direction (beneath your feet)—you get closer to the overall mass concentration and the force increases. If you tunnel into a star, however, there is a cancellation of gravitational pulls; the matter above you pulls counter to the matter below you, resulting in a decrease in the net gravitational force. (Also, the amount of matter "above" you decreases.)

107. $F \sim m_1 m_2/d^2$, where m_2 is the mass of the Sun (which doesn't change when forming a black hole), m_1 is the mass of the orbiting Earth, and d is the distance between the center of mass of Earth and the Sun. None of these terms change, so the force F that holds Earth in orbit does not change.

108. Letting the gravitational force equation be a guide to thinking, we see that gravitational force and hence one's weight does not change if the mass and radius of the Earth do not change. (Although one's weight would be zero inside a hollow uniform shell, on the outside one's weight would be no different than if the same-mass Earth were solid.)

109. Astronauts are weightless because they lack a support force, but they are well in the grips of Earth gravity, which accounts for them circling the Earth rather than going off in a straight line in outer space.

110. The misunderstanding here is not distinguishing between a theory and a hypothesis or conjecture. A theory, such as the theory of universal gravitation, is a synthesis of a large body of information that encompasses well-tested and verified hypotheses about nature. Any doubts about the theory have to do with its applications to yet untested situations, not with the theory itself. One of the features of scientific theories is that they undergo refinement with new knowledge. (Einstein's general theory of relativity has taught us that in fact there are limits to the validity of Newton's theory of universal gravitation.)