35 Special Theory of Relativity

The Part Eight photo opener is my granddaughter, Grace Hewitt.

The photo openers feature Ken Ford, Albert Einstein, and Edwin F. Taylor, three relativity experts—the one in the center being the most famous.

The personality profile for this chapter is Albert Einstein! (Regrettably, his picture on a bicycle in previous editions wasn't "approved" for this edition ☹. The profile continues in the next chapter.

The ideas discussed in this chapter are perhaps the most exciting in the book. But for most students they are difficult to comprehend. Regardless of how clearly and logically this material is presented, students will find that they do not sufficiently "understand" it. This is understandable for so brief an exposure to a part of reality untouched by conscious experience. The purpose of this chapter is to develop enough insight into relativity to stimulate further student interest and inquiry.

Note the important significance of "The Twin Trip" section in the text, in that it completely bypasses the equations for time dilation and the relativistic Doppler effect. The reciprocity of relativistic Doppler frequencies for approach and recession stems only from Einstein's 1st and 2nd postulates and is established without the use of a single mathematical formula. This is abbreviated in the long footnote in the chapter and is detailed in the 4-step classroom presentation in the following suggested lecture. [This reciprocal relationship is not valid for sound, where the "moving" frame is not equivalent to the "rest" (relative to air) frame. If the ratio of frequency received to frequency sent for hearing in the rest frame is 2, the ratio for hearing in the moving frame is $^3/_2$ (clearly not 2!). For sound, the speed as well as the frequency depends on the motion of the receiver.] From this and the simple flash-counting sequence, time dilation follows without the use of any mathematical formulas. The results of the twin-trip flash sequence agree with Einstein's time dilation equation. So this treatment is completely independent to the time dilation equation and the relativistic Doppler equation! (Who says that good physics can't be presented without high-powered math?)

If your class is in a more mathematical mood, you may wish to show an alternative approach to The Twin Trip and consider straightforward time dilation plus corrections for the changing positions of the emitting or receiving body between flashes. Instead of bypassing the time dilation equation, use it to show that at 0.6 *c*, 6-minute flash intervals in the emitting frame compute to be $7^1/_2$-minute flash intervals in the receiving frame. The flashes would *appear* at $7^1/_2$-minute intervals if the spaceship were moving crosswise, neither approaching or receding, such that each flash travels essentially the same distance to the receiver. In our case the spaceship doesn't travel crosswise, but recedes from and then approaches the receiver—so corrections must be made in the time interval due to the extra distance the light travels when the spaceship is receding and the lesser distance the light travels when the spaceship is approaching. This turns out to be $4^1/2$ minutes; [Δt = (extra distance)/c = (0.6 $c \times 7^1/2$ min)/c = $4^1/2$ min]

So when receding, the flashes are seen at $7^1/2 + 4^1/2$ = 12-min intervals; when approaching, the flashes are seen at $7^1/2 - 4^1/2$ = 3-min intervals. The results of this method are the same as those of the 4-step conceptual presentation in the following suggested lecture.

My 12-minute animated film, *Relativistic Time Dilation*, amplifies the section on The Twin Trip. It is part of the videotape of relativity in the *Conceptual Physics Alive!* series. Contact your Addison-Wesley representative or Arbor Scientific (arborsci.com) for availability.

As in previous editions, relativistic momentum, rather than relativistic mass, is treated. The very early editions of Conceptual Physics, and some other physics textbooks, speak of *relativistic mass*, given by the equation $m = m_0/\sqrt{1 - (v^2/c^2)}$. This idea has lost favor to the somewhat more complex idea of relativistic momentum, rather than mass. One problem with the idea of increased mass is that mass is a scalar: It has no direction. When particles are accelerated to high speeds, their increase in mass is directional. Increase occurs in the direction of motion in a manner similar to the way that length contraction occurs only in the direction of motion. Moving mass is, after all, momentum. So it is more appropriate to speak of increases in momentum rather than mass. Either treatment of relativistic mass or relativistic momentum, however, leads to the same description of rapidly moving objects in accord with observations.

The symbol γ (gamma) simplifies expressions for time dilation and relativistic momentum.

Because of the interest in physics that relativity generates, this chapter may be treated earlier in the course—even to begin your course.

Practicing Physics Book:
• Time Dilation

Problem Solving Book:
There are problems on relativity. Have your math-oriented students have a go at them!

Laboratory Manual:
Not surprisingly, there are no activities or experiments on special relativity

Next-Time Questions:
• Astronaut Travel Time
• Shrinking Spear

Hewitt-Drew-It Screencasts: •*Special Relativity* •*Time Dilation* •*Relativistic Velocities* •*Length Contraction* •*E = mc*^{2} •*Correspondence Principle*

SUGGESTED LECTURE PRESENTATION

After discussing Einstein and a broad overview of what special relativity is and is not, point out somewhere along the line that the theory of relativity is grounded in *experiment*, and in its development it explained some very perplexing experimental facts (constancy of the speed of light, muon decay, solar energy, the nature of mass, etc.). It is not, as some people think, only the speculations of one man's way of thinking.

Motion Is Relative
Ask your class to pretend they are in a parking lot playing ball with someone driving toward them and away from them in an open vehicle. A pitcher in the vehicle tosses a ball at them, always with the same pitching speed—no variation. Ask for the relative speed of catching a ball when the car approaches, and when it recedes. They know there will be a difference. Ask how they would react if the speeds of the ball in

catching were the same, whether the thrower was moving toward them, at rest, or moving away from them. This would be most perplexing. Now discuss the null result of the Michelson-Morley experiment.

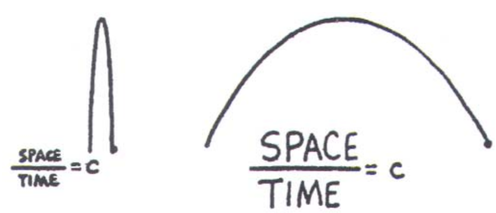
Michelson-Morley Experiment
Treat the Michelson-Morley experiment very briefly, and I suggest you do not delve into the mechanics of the interferometer. Instead direct your students' mental energies to the broad ideas of special relativity. Explain what it means to say the velocity of light is invariant.

First Postulate
The laws of physics are the same in all uniformly moving reference frames. A bee inside a fast-moving jet plane executes the same flying maneuvers regardless of the speed of the plane. If you drop a coin to the floor of the moving plane, it will fall as if the plane were at rest. Physical experiments behave the same in all uniformly moving frames. This leads, most importantly, to the development of special relativity, to the speed of light that is seen to be the same for all observers. There is no violation of common sense in this first postulate. It rules out any effect of uniform motion on any experiment, however. For any observed effect violates this postulate and the foundation of relativity.

Second Postulate
Stand still and toss a pencil up and down, catching it as you would when flipping a coin. Ask the class to suppose that in so doing that all measurements show the pencil to have a constant average speed. Call this constant speed *c* for short. That is, both they and you see only one average speed for the tossed pencil. Then proceed to walk at a fairly brisk pace across the room and again toss the pencil. State that from your frame of reference you again measure the same speed. Ask if the speed looked any different to them. They should respond that the pencil was moving faster this time. Ask them to suppose that their measurement yielded the same previous value. They may be a bit perplexed, which again is similar to the perplexed state of physicists at the turn of the century. On the board, write with uniformly sized letters

$$\boldsymbol{c} = \frac{\text{SPACE}}{\text{TIME}} .$$

This is the speed as seen by you in your frame of reference. State that from the frame of reference of the class, the space covered by the tossed pencil appeared to be greater, so write the word SPACE in correspondingly larger letters. Underline it. State that if they see the same speed, that is, the same ratio of space to time, then such can be accounted for if the time is also measured to be greater. Then write the enlarged word TIME beneath the underline, equating it to *c*.



Analogy: Just as all observers will measure the same ratio of circumference to diameter for all sizes of circles, all observers will measure the same ratio of space to time for electromagnetic waves in free space.

Simultaneity
Show by way of Figures 35.9 and 35.10, and the footnoted diagram on page 665 that an interesting result of the constancy of the speed of light in all reference frames is the nonsimultaneity of events in one frame that are simultaneous in another. Contrast this to classical nonsimultaneity, like different observers hearing gun blasts at different time intervals.

Space-time
An interesting way to look at how space and time are related to the speed of light is to think of all things moving through space-time at a constant speed. When movement is maximum through space, movement in

time is minimum. When movement in space is minimum, movement in time is maximum. For example, something at rest relative to us moves not at all in space and moves in time at the maximum rate of 24 hours per day. When something approaches the speed of light relative to us, it moves at its near maximum speed in space, and moves near zero in time—it doesn't age.

The box about clockwatching on a trolley car ride was motivated by remarks made by Jacob Bronowski in his *Accent of Man* series, where he cites the notion of clock information traveling to distant locations, and how that one traveling at the speed of light would see a clock frozen in time.

The Twin Trip

You have a choice of a short treatment of this or a longer more detailed treatment. The short treatment begins without fanfare and as a matter of fact presents the half rate of flashes seen when a spaceship approaches (Figure 35.15) and the doubled rate seen when the spaceship recedes (Figure 35.16). The fact that the half rate and doubled rate are reciprocals is not developed. For the vast majority of my students this is fine. More sophisticated students may be uneasy with this and wish to see this reciprocal relationship developed. This is the longer treatment. This longer treatment is shown by the 4 steps below. For the shorter treatment, jump ahead to paragraph 2, with the * on page 364.

We will in effect bypass the derivation of the relativistic Doppler effect, namely,

$$f = f_0\sqrt{\frac{1 + v/c}{1 - v/c}}$$

with the following four-step conceptual presentation:

Step 1: Consider a person standing on Earth directing brief flashes of light at 3-min intervals to a distant planet at rest relative to the Earth. Some time will elapse before the first of these flashes reaches the planet, but since there is no relative motion between the sender and receiver, successive flashes will be observed at the distant planet at 3-min intervals. While you are making these remarks, make a sketch on the board of Figure 1.

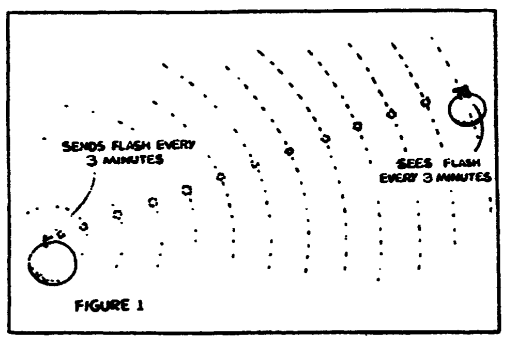


FIGURE 1

Step 2: How frequently would these flashes encounter an observer in a fast-moving spaceship traveling between the Earth and the planet? Although the speed of the flashes would be measured by the spaceship to be *c*, the frequency of flashes would be greater or less than the emitting frequency depending on whether the spaceship were receding or approaching the light source. After supporting this idea with some examples of the Doppler effect (car horns, running into versus away from a slanting rain, etc.) make the supposition that the spaceship recedes from the light source at a speed great enough for the frequency of light flashes to decrease by half—so they're seen from the spaceship only half as often, at 6-min intervals. By now your chalkboard sketch looks like Figure 2.

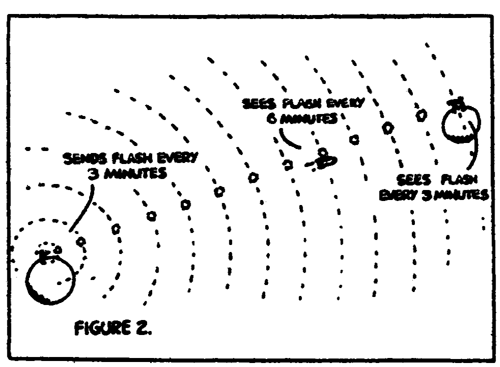


FIGURE 2.

Step 3: Suppose further that each time a flash reaches the spaceship, a triggering device activates a beacon on the spaceship that sends its own flash of light toward the distant planet. According to a clock in the spaceship then, this flash is emitted every 6 min. Since the flashes from Earth and the flashes emitted by the spaceship travel at the same speed *c*, both sets of flashes travel together, and an observer on the distant planet sees not only the Earth flashes at 3-min intervals, but the spaceship flashes at 3-min intervals as well (Figure 3). At this point you have established that 6-min intervals on the approaching spaceship are seen as 3-min intervals on the stationary planet.

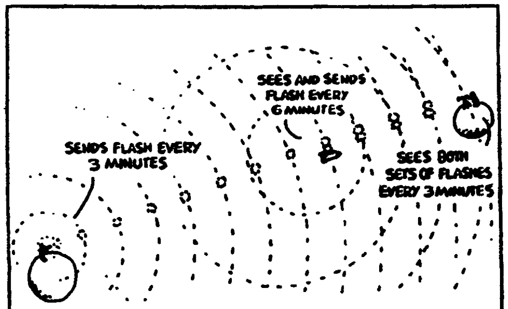


FIGURE 3.

Step 4: To establish that the 6-min flashes emitted by the spaceship are seen at 12-min intervals from the Earth, go back to your earlier supposition that 3-min intervals on Earth are seen as 6-min intervals from the frame of reference of the receding spaceship. Ask your class: If instead of emitting a flash every 3 min, the person on Earth emits a flash every 6 min, then how often would these flashes be seen from the receding spaceship? And then ask if the situation would be any different if the spaceship and Earth were interchanged—if the spaceship were at rest and emitted flashes every 6 min to a receding Earth? After a suitable response erase from your board drawing all the flashes emitted from the Earth. Replace the Earth-twin's light source with a telescope while asking how often the 6-min flashes emitted by the moving spaceship are seen from Earth. Class response should show that you have established the reciprocity of frequencies for the relativistic Doppler effect without a single equation. This is summarized in Fig. 4.

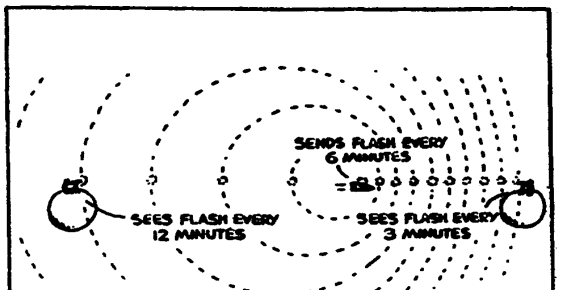


FIGURE 4.

Note that you have employed Einstein's postulates in the last two steps, that is, the second postulate in Step 3 (constancy of the speed of light) and the first postulate in Step 4 (equivalence of the Earth and spaceship frames of reference).

Whether you have established this reciprocity from the Doppler equation or from the preceding four steps, you are now ready to present the twin trip and demonstrate time dilation while also presenting a resolution to the so-called twin paradox.

*With a sketch of Figure 4 above in an upper corner of your board, proceed to make a rendition of textbook Figure 35.17 on the main part of your board. You'll draw only one picture of the Earth and let your eraser be the spaceship which you move away from Earth (to the right as the top part of the figure, *a*, suggests). State the spaceship sends 10 flashes at 6-min intervals. At the 10th flash 1 hour has passed for the spaceship. Ask how these 6-min intervals appear to Earth observers? [At 12 min apart.] What time is it on Earth when the 10th flash is received? [Two hours later!] The spaceship quickly turns around and continues homeward, again sending 10 flashes at 6-min intervals. When the 10th flash is emitted, another hour passes for the spaceship and its total trip time is 2 hours. But ask how often the incoming flashes occurred to Earth types. [At 3-min intervals.] Ten incoming 3-min flashes take only 30 min, Earth time. So from the Earth frame of reference the spaceship took a grand total of $2^{1}/_{2}$ hours.

During your discussion, summarize this on the board as the lettering of Figure 35.18.

Depending on lecture time, switch frames of reference and repeat the similar sequence suggested by Figures 35.19 and 35.20 for the case where the flashes are emitted from Earth and viewed by the moving spaceship. You get the same results.

Space Travel
Speculate on the idea of "century hopping," the future version of today's "jet hopping." In this scenario future space travelers may take relatively short trips of a few years or less and return in decades, or even centuries. This is, of course, pending the solution to two present problems: rocket engines and sufficient fuel supplies for prolonged voyages, and a means of shielding the radiation that would be produced by impact of interstellar matter.

The Centrifuge
Follow this up with this interesting but fictitious example to show that one needn't go far in space for significant time dilation: Suppose that one could be whirled in a giant centrifuge up to relativistic speeds without physical injury. Of course one would be crushed to death in such a case, but pretend that somehow one is physically unaffected by the crushing centripetal forces (the fictitiousness of this example). Then cite how one taking a "ride" in such a centrifuge might be strapped in his seat and told to press a button on the seat when he or she wishes the ride terminated. And suppose that after being whirled about at rim speeds near the speed of light the occupant decides that 10 minutes is enough. So he or she presses the button, signaling those outside to bring the machine to a halt. After the machine is halted, those outside open the door, peer in, and ask, "Good gosh, what have you been doing in there for the past 3 weeks!" In the laboratory frame of reference, 3 weeks would have elapsed during a ten-minute interval in the rotating centrifuge. Motion in space, rather than space itself is the key factor.

Length Contraction
Hold up a meterstick, horizontally, and state that if your students made accurate measurements of its length, their measurements would agree with your own. Everyone would measure it as 1 meter long. People at the back of the room would have to compensate for it appearing smaller due to distance, but nevertheless, they would agree it is 1 meter long. But now walk across the room holding the meterstick like a spear. State that your measurements and those of your students would differ. If you were to travel at 87% the speed of light, relative to the class, they would measure the meterstick to be half as long, 0.5 m. At 99.5% the speed of light, they would see it as only 10 cm long. At greater speeds, it would be even shorter. At the speed of light it would contract to zero length. Write the length-contraction formula on the board:

$$L = L_o\sqrt{1 - (v^2/c^2)}$$

State that contraction takes place only in the direction of motion. The meterstick moving in spear fashion appears shorter but it doesn't appear thinner.

> CHECK QUESTION: Consider a pair of stars, one on each "edge" of the universe. That's an enormous distance of separation from our frame of reference. Now consider a photon traveling from one star across the entire universe to the other. From the frame of reference of the photon, what is the distance of separation between stars? (How big is the universe?) [Zero! From a frame of reference traveling at *c*, the length contraction reaches zero.]

The implication of the above question is that at high speeds, future space travelers may not face the restrictions of traveling distance that seem formidable without relativity! There is much food for thought on this!

The Mass-Energy Relationship
Write $E = mc^2$ on the board, the most celebrated equation of the previous century. It relates energy and mass. Every material object is composed of energy—"energy of being." This "energy of being" is appropriately called *rest energy*, which is designated by the symbol *E*. (Some instructors label this E_o, to distinguish it from a generalized symbol *E* for total energy. Since we're concerned only with rest energy here, there isn't a need for the subscript. We are here defining rest energy as *E*, which shouldn't pose a problem. If we were to extend our treatment, then such a subscript would be useful.) So the mass of something is actually the internal energy within it. This energy can be converted to other forms—light for example.

On the 4.5 million tons of matter converted to radiant energy by the Sun each second: That tonnage is carried by radiant energy through space, so when we speak of matter being "converted" to energy, we are merely converting from one form to another—from a form with one set of units, to another. Because of the mass and energy equivalence, in any reaction that takes into account the whole system, the total amount of mass + energy does not change.

Discuss the interesting idea that mass, every bit as much as energy, is delivered by the power utilities through the copper wires that run from the power plants to the consumers.

From a long view, the significance of the twentieth century will be most likely seen as a major turning point with the discovery of the $E = mc^2$ relationship. We can speculate about what the equation of this 21st century will turn out to be.

Relativistic Momentum

State that if you push an object that is free to move, it accelerates in accord with Newton's 2nd law, $a = F/m$. As it turns out, Newton originally wrote the 2nd law not in terms of acceleration, but in terms of momentum, $F = \Delta p/\Delta t$, which is equivalent to the familiar $F = ma$. Here we use the symbol p for momentum, $p = mv$. In accord with the momentum version of Newton's 2nd law, push an object that is free to move and we increase its momentum. The acceleration or the change-of-momentum version of the 2nd law produces the same result. But for very high speeds, it turns out that the momentum version is more accurate. $F = \Delta p/\Delta t$ holds for all speeds, from everyday speed to speeds near the speed of light—as long as the relativistic expression of momentum is used.

Write the expression for relativistic momentum on the board: $p = mv/\sqrt{1 - (v^2/c^2)}$. Or $p = \gamma mv$.

Point out that it differs from the classical expression for momentum by γ, or by the denominator $\sqrt{1 - (v^2/c^2)}$. A common interpretation is that of a relativistic mass $m = \gamma m_o$, or $m = m_o/\sqrt{1 - (v^2/c^2)}$, multiplied by a velocity v. Because the increase in mass with speed is directional (as is length contraction), and momentum rather than mass is a vector, the concept of momentum increase rather than mass increases is preferred in advanced physics courses. Either treatment of relativistic mass or relativistic momentum, however, leads to the same description of rapidly moving objects in accord with observations.

A good example of the increase of either mass or momentum for different relative speeds is the accelerated electrons and protons in high-energy particle accelerators. In these devices speeds greater than 0.99 c are attained within the first meter, and most of the energy given to the charged particles during the remaining journey goes into increasing mass or momentum, according to your point of view. The particles strike their targets with masses or momentum thousands of times greater than the physics of Newton accounts for. Interestingly enough, if you traveled along with the charged particles, you would note no such increase in the particles themselves (the v in the relativistic mass equation would be zero), but you'd measure a mass or momentum increase in the atoms of the "approaching" target (the crash is the same whether the elephant hits the mouse or the mouse hits the elephant).

Cite how such an increase must be compensated for in the design of circular accelerators such as cyclotrons, bevatrons, and the like, and how such compensation is not required for a linear accelerator (except for the bending magnets at its end).

Point out to your class the similarities of the equations: $t = \gamma t_o$; $p = \gamma mv$.

Show how for small speeds the relativistic momentum equation reduces to the familiar mv (just as for small speeds $t = t_o$ in time dilation). Then show what happens when v approaches c. The denominator of the equation approaches zero. This means that the momentum approaches infinity! An object pushed to the speed of light would have infinite momentum and would require an infinite impulse (force × time), which is clearly impossible. Nothing material can be pushed to the speed of light. The speed of light c is the upper speed limit in the universe.

Cars, planes, and even the fastest rockets don't approach speeds to merit relativistic considerations, but subatomic particles do. They are routinely pushed to speeds beyond 99% the speed of light where their momenta increase thousands of times more than the classical expression *mv* predicts. This is evidenced when a beam of electrons directed into a magnetic field is appreciably deflected from its normal path. The greater its speed, the greater its "moving inertia"—its momentum, and the greater it resists deflection (Figure 35.26). High-energy physicists must take relativistic momentum into account when working with high-speed subatomic particles in accelerators. In that arena, relativity is an everyday fact of life.

Optional—Relativistic Kinetic Energy

From where did Einstein's equation $E = mc^2$ originate? Einstein was the first to derive the relativistic expression for kinetic energy, KE $= mc^2/\sqrt{1 - v^2/c^2} - mc^2$ and the first to note the term mc^2, which is independent of speed. This term is the basis of the celebrated equation, $E = mc^2$.

Interestingly enough, for ordinary low speeds the relativistic equation for kinetic energy reduces to the familiar KE = $^1/_2\ mv^2$ (via the binomial theorem). In many situations where the momentum or energy of high-speed particles is known rather than speed, the expression that relates total energy E to the relativistic momentum p is given by $E^2 = p^2c^2 + (mc^2)^2$. This expression is derived by squaring the relation $E = mc^2$ to obtain $E^2 = m^2c^4 = m^2c^2\ (c^2 + v^2 - v^2)$, and combining the relativistic equation for momentum.

Like the argument for a speed limit via the infinite impulse required to produce infinite momentum, we find that doing more and more work to move an object or particle increases its kinetic energy disproportionately to its increase in speed. The accelerated matter requires more and more kinetic energy for each small increase in speed. An infinite amount of energy would be required to accelerate a material object to the speed of light. Since an infinite amount of energy is unavailable, we again conclude that material particles cannot reach the speed of light.

The Correspondence Principle

Show your students that when small speeds are involved, the relativity formulas reduce to the everyday observation that time, length, and the masses of things do not appear any different when moving. That's because the differences are too tiny to detect.

Answers and Solutions for Chapter 35

Reading Check Questions

1. Speed relative to the ground is 61 km/h.
2. Speed relative to the Sun is only slightly changed.
3. Fitzgerald hypothesized that the experimental apparatus shrunk.
4. Einstein rejected the notion that space and time are independent of each other.
5. The laws of nature are the same in uniformly moving reference frames.
6. The speed of light in free space is constant.
7. The distances are the same as seen in the rocket ship frame of reference.
8. The light traveling to the rear of the compartment moves a shorter distance.
9. Three dimensions for space, a forth dimension for time.
10. You'll share the same spacetime if you're at rest relative to each other. If there is relative motion between you, you'll not share the same spacetime.
11. What is special is that the ratio is a constant, the speed of light.
12. A longer path requires a longer time.
13. The stretching of time is called *time dilation*.
14. Gamma = $\sqrt{1\text{-}v^2/c^2}$, which is never less than 1 because v^2/c^2 is always positive.
15. Measurements of time differ by 1.15 at half the speed of light. At 99.5%, time measurements differ by a factor of 10.
16. Evidence for time dilation comes from many experiments performed in labs and on airplane flights, as well as the accuracy of GPS systems that depend on time dilation.
17. Coming toward you, flashes appear more frequently.
18. Frequency increases, while speed remains constant.
19. For the moving-away source the flashes are seen twice as long in duration.
20. While the stay-at-home twin experiences a single frame of reference, the traveling twin experiences two, one going and one returning.
21. Maximum would be when speeds $v = c$, then $c_1c_2/c^2 = 1$. Smallest would be zero when either v_1 or v_2 equal zero.
22. The rule is consistent with light speed being constant, for no values can add to be greater than c.
23. One obstacle is the energy required, another is shielding the radiation encountered.
24. There is no universal standard of time!
25. Thrown at 99.5% the speed of light, the meterstick would be one-tenth its original length.
26. The same stick traveling perpendicular to velocity would not change in length, because length contraction occurs only in the direction of travel.
27. In your own frame of reference, no local length contraction would occur.
28. At the speed of light, momentum would be infinite.
29. High speed particles exhibit a less-bent trajectory due to relativity.
30. The amount of mass conversion in nuclear reactions is millions of times greater than in chemical reactions.
31. Fissioning a single uranium nucleus produces 10 million times as much energy as combustion of a single carbon atom.
32. Yes, $E = mc^2$ applies to chemical as well as nuclear reactions.
33. $E = mc^2$ tells us that mass is congealed energy.
34. The correspondence principle predicts that for low speed the equations of relativity reduce to the classical equations.
35. The relativity equations hold for everyday speeds, and appreciably differ from classical equations only for speeds near the speed of light.

Think and Do

36. Open ended.

Think and Solve

37. Frequency and period are reciprocals of one another (Chapter 19). If the frequency is doubled, the period is halved. For uniform motion, one senses only half as much time between flashes that are doubled in frequency. For accelerated motion, the situation is different. If the source gains speed in approaching, then each successive flash has even less distance to travel and the frequency increases more, and the period decreases more as well with time.

38. We must use the formula

$$V = \frac{v_1 + v_2}{1 + \frac{v_1 v_2}{c^2}}.$$

Putting v_1 = 0.80 c and v_2 = 0.50 c, we get V = (0.80 c + 0.50 c)/[1 + (0.80)(0.50)] = 1.30 c/1.40 = 0.93 c. The drone moves at 93% of the speed of light relative to the Earth.

39. $V = \frac{c + c}{1 + \frac{c^2}{c^2}} = \frac{2c}{1 + 1} = c$

40. The factor γ = 1/√[1 - (v^2/c^2)] = 1/ √[1 - $(0.99)^2$] = 1/√(1 - 0.98) = 1/√0.02 = 7.1. Multiplying 5 min by γ gives 35 min. According to your watch, the nap lasts 35 minutes.

41. In the previous problem we see that for v = 0.99 c, γ is 7.1. The momentum of the bus is more than seven times greater than would be calculated if classical mechanics were valid. The same is true of electrons, or anything traveling at this speed.

42. Gamma, as in the previous two problems, is 7.1. So you measure the length of the bus to be 70 ft /7.1 = 9.9 ft. (Remember, *divide* by γ for length contraction.)

43. Gamma at v = 0.10 c is 1/√[1 - (v^2/c^2)] = 1/√[1 - $(0.10)^2$] = 1/√(1 - 0.01) = 1/√0.99 = 1.005. You would measure the passenger's catnap to last 1.005 (5 m) = 5.03 min.

44. Gamma at v = 0.9999 c is 1/√[1 - (v^2/c^2)] = 1/√[1 - $(0.9999)^2$] = 1/√0.0002 = 70.71. So you would measure the length of the bus to be 70 ft/70.71 = 0.99 ft.

45. Gamma at v = 0.5 c is 1/[√1 + (v^2/c^2)] = 1/[√1 - 0.5^2] = 1/[√1 - 0.25] = 1/√0.75 = 1.15. Multiplying 1 h of taxi time by γ gives 1.15 h of Earth time. The drivers' new pay will be (10 hours)(1.15) = 11.5 stellars for this trip.

46. When 1 kg of uranium-235 undergoes fission, the loss of mass is 1 g or 0.001 kg. We note here that c^2 is 9.0 × 10^{16} J/kg (recall that 1 J = 1 kg m^2/s^2). So the energy released by this loss of mass is $E = mc^2$ = (0.001 kg)(9.0 × 10^{16} J/kg) = 9.0 × 10^{13} J, or 9.0 × 10^7 MJ (megajoules). Multiply this energy by $0.03 per MJ and you find that the energy in one gram of matter is worth $2.7 million dollars! (Note: Three cents per MJ corresponds to about 11 cents per kWh.)

Think and Rank

47. C, B, A.

48. C, B, A.

Think and Explain

49. Only accelerated motion, and not uniform motion, can be sensed. You could not detect your motion when traveling uniformly, but accelerated motion could be easily detected by observing that the surface of the water in a bowl is not horizontal.

50. (a) The ball is moving faster relative to the ground when the train is moving (forward).
(b) The ball moves at the same speed relative to the freight car whether the train is moving or not.

51. In the case of a light beam shining from atop a moving freight car, the light beam has the same speed relative to the ground as it has relative to the train. The speed of light is the same in all reference frames.

52. When you drive down the highway you are moving through space and also "through time."

53. The *average* speed of light in a transparent medium is less than c, but in the model of light discussed in Chapter 26, the photons that make up the beam travel at c in the void that lies between the atoms of

the material. Hence the speed of individual photons is always *c*. In any event, Einstein's postulate is that the speed of light in *free* space is invariant.

54. It meets them at *c*.

55. No energy or information is carried perpendicular to the swept beam.

56. Yes. No *thing* can move faster than light, but the spot on the face of the tube is not a "thing." No material thing is moving from one spot to the other. The electrons causing the spot, however, move at less than the speed of light. (Also, no information can travel faster than light, but the spot on the screen is not carrying information from one place to another.)

57. It's all a matter of relative velocity. If two frames of reference are in relative motion, events can occur in the order AB in one frame and in the order BA in the other frame. (See the next exercise.)

58. If by now you expect relativity to be surprising, you might want to say yes. But the answer is no. A time difference between two events at the same place in one frame of reference (such as two readings on a given clock) is multiplied by a time-dilation factor to get the time difference in another frame. Similarly, the distance between two points (or two events) at the same time in one frame of reference (for instance, the distance between the ends of a meterstick) is multiplied by a Lorentz-contraction factor to get the distance in another frame of reference. But if the two events are at the same place *and* the same time in one frame, they are also coincident and simultaneous in all frames of reference, for zero time difference multiplied by any factor is still zero, and zero distance multiplied by any factor is still zero. This much of common sense is preserved by relativity!

59. Yes. If the distance the rocket ship moves forward in the time it takes the light to reach the rear is greater than the distance by which the light bulb was shifted, the outside observer will still see the light getting to the rear first. You can see this, too, by considering such a tiny displacement of the light bulb that it makes hardly any difference to the outside observer, who still sees the light reaching the rear first. But to the inside observer, the light will reach the front first no matter how tiny the displacement.

60. There is an upper limit on speed, but no upper limit on the factor we call gamma, and accordingly no upper limit on either the momentum or kinetic energy of a particle. Since momentum is given by $p = \gamma mv$, p can grow without limit as γ grows without limit, even though m is constant and v is limited. Similarly, kinetic energy can grow without limit. As p gets larger, so does KE.

61. When we say that light travels a certain distance in 20,000 years we are talking about distance in our frame of reference. From the frame of reference of a traveling astronaut, this distance may well be far shorter, perhaps even short enough that she could cover it in 20 years of her time (traveling, to be sure, at a speed close to the speed of light). In a distant future, astronauts may travel to destinations many light years away in a matter of months in their frame of reference.

62. Yes. If a person travels at relativistic speeds—that is, very close to the speed of light—distances as far as those that light takes thousands of years to travel (in our frame of reference) could be traversed well within an average lifetime. This is because distance depends on the frame of reference in which it is measured. Distances that are quite long in one frame of reference may be quite short in another.

63.A twin who makes a long trip at relativistic speeds returns younger than his stay-at-home twin sister, but both of them are older than when they separated. If they could watch each other during the trip, there would be no time where either would see a reversal of aging, only a slowing or speeding of aging. A hypothetical reversal would result only for speeds greater than the speed of light.

64.Yes, as strange as it sounds, it is possible for a son or daughter to be biologically older than his or her parents. Suppose, for example, that a woman gives birth to a baby and then departs in a high-speed rocket ship. She could theoretically return from a relativistic trip just a few years older than when she left to find her "baby" 80 or so years older.

65. If you were in a high-speed (or no speed) rocket ship, you would note no changes in your pulse or in your volume. This is because the velocity between the observer, that is, yourself, and the observed is zero. No relativistic effect occurs for the observer and the observed when both are in the same reference frame.

66. In contrast to the previous exercise, if you were monitoring a person who is moving away from you at high speed, you would note both a decrease in pulse and in volume. In this case, there is very definitely a velocity of the observed with respect to the observer.

67. Narrower as well.

68. Your friend sees your watch running as slowly as you see hers.

69. Yes, although only high speeds are significant. Changes at low speeds, although there, are imperceptible.

70. Making such an appointment would not be a good idea because if you and your dentist moved about differently between now and next Thursday, you would not agree on what time it is. If your dentist zipped off to a different galaxy for the weekend, you might not even agree on what day it is!

71. The density of a moving body is measured to increase because of a decrease in volume for the same mass.

72. Elliptical, longer in the direction of motion than perpendicular to that direction. The Lorentz contraction shortens the long axis of the elliptical shape to make it no longer than the short axis.

73. For the speed of light equation, v is c. Before relativity, c might have one value in one frame of reference and a different value in another frame. It depended on the motion of the observer. According to relativity, c is a constant, the same for all observers.

74. Both the frequency and the wavelength of the light change (and, of course, its direction of motion changes). Its speed stays the same.

75. The stick must be oriented in a direction perpendicular to its motion, unlike that of a properly-thrown spear. This is because it is traveling at relativistic speed (actually $0.87c$) as evidenced by its increase in momentum. The fact that its length is unaltered means that its long direction is not in the direction of motion. The thickness of the stick, not the length of the stick, will appear shrunken to half size.

76. The stick will appear to be one-half meter long because for it to have a momentum equal to $2mv$, its speed must be $0.87c$.

77. As with the stick in the preceding exercise, the momentum of the rocket ship will be twice the classical value if its measured length is half its normal length.

78. As speed increases, the increase in momentum is not linear, but increases by γ. Near the speed of light the percentage of momentum increase can be much greater than the percentage of speed increase.

79. For the moving electron, length contraction reduces the apparent length of the 2-mile long tube. Because its speed is nearly the speed of light, the contraction is great.

80. If you were traveling with the electrons, no matter how fast they and you were moving, you would see nothing out of the ordinary. In your frame of reference, the electrons would just be “sitting there.” (The v in γ would be zero.) They would have zero momentum and their normal rest energy mc^2. But you would see the target coming toward you at close to the speed of light. (The *relative* speed of two frames of reference is the same for observers in both frames. Someone on the ground measures a certain speed of the electrons moving toward the target. Traveling with the electrons, you measure the same speed of the target moving toward you.)

81. To make the electrons hit the screen with a certain speed, they have to be given more momentum and more energy than if they were nonrelativistic particles. The extra energy is supplied by the power utility. The electric bill is more!

82. The correspondence principle just makes good sense. If a new idea is valid, then it ought to be in harmony with the areas it overlaps. If it doesn’t, then either the areas themselves are suspect, or the new idea is suspect. If a new theory is valid, it must account for the verified results of the older theory, whether the theory is or isn’t in the field of science.

83. $E = mc^2$ means that energy and mass are two sides of the same coin, mass-energy. The c^2 is the proportionality constant that links the units of energy and mass. In a practical sense, energy and mass are one and the same. When something gains energy, it gains mass. When something loses energy, it loses mass. Mass is simply congealed energy.

84. Both have both the same mass and, hence the same energy.

85. Yes, for it contains more potential energy, which has mass.

86. Just as time is required for knowledge of distant events to reach our eyes, a lesser yet finite time is required for information on nearby things to reach our eyes. So the answer is yes, there is always a finite interval between an event and our perception of that event. If the back of your hand is 30 cm from your eyes, you are seeing it as it was one-billionth of a second ago.

87. Open-ended.

Think and Discuss

88. The effects of relativity become pronounced only at speeds near the speed of light or when energies change by amounts comparable to mc^2. In our "non-relativistic" world, we don't directly perceive such things, whereas we do perceive events governed by classical mechanics. So the mechanics of Newton is consistent with our common sense, based on everyday experience, while the relativity of Einstein is not consistent with common sense. Its effects are outside our everyday experience.

89. Michelson and Morley considered their experiment a failure in the sense that it did not confirm the expected result, namely that differences in the velocity of light would be encountered and measured. No such differences were found. The experiment was successful in that it widened the doors to new insights in physics.

90. Yes, for example, a distant part of a beam sweeping the sky. What it doesn't allow is energy or particles or the transmission of information to exceed c.

91. No, for speed and frequency are entirely different from each other. Frequency, how frequent, is not the same as speed, how fast.

92. The moving points are not material things. No mass or no information can travel faster than c, and the points so described are neither mass nor information. Hence, their faster motion doesn't contradict special relativity.

93. Experimental evidence in accelerators has repeatedly shown that as more and more energy is put into a particle, the particle never reaches the speed of light. Its momentum grows without limit, but not its speed. As the speed of light is approached, the momentum of the particle approaches infinity. There is an infinite resistance to any further increase in momentum, and hence speed. So c is the speed limit for material particles. (Kinetic energy likewise approaches infinity as the speed of light is approached.)

94. The acid bath that dissolved the latched pin will be a little warmer, and a little more massive (in principle). The extra potential energy of the latched pin is transformed into a bit more mass.

95. The mass of the radioactive material decreases, but the mass of the lead container increases. So no net mass change results. There has been a redistribution of energy within the chunk-container system, but no change in total energy and therefore no change in mass.

96. At $0.995c$ the muon has ten times as much time, or twenty-millionths of a second, to live in our frame of reference. From the stationary Earth, the muon's "clock" is running ten times slower than Earth clocks, allowing sufficient time to make the trip. (Interestingly, from the muon's frame of reference, the distance to Earth is contracted by ten times, so it has sufficient time to get there.)

97. There are two enormous obstacles to the practice of "century hopping" at this time. First, although we have the means to easily accelerate atomic particles to speeds approaching the speed of light, we as yet have no means of propelling a body as massive as an inhabited rocket to relativistic speeds. Second, if we did, we currently have no way of shielding the occupants of such a rocket from the radiation that would result from the high-speed collisions with interstellar matter.

98. Kierkegaard's statement, "Life can only be understood backwards; but it must be lived forwards.," is consistent with special relativity. No matter how much time might be dilated as a result of high speeds, a space traveler can only effectively slow the passage of time relative to various frames of reference, but can never reverse it—the theory does not provide for traveling backward in time. Time, at whatever rate, flows only forward.

99. Rather than say matter can neither be created nor destroyed, it is more accurate to say that mass-energy can neither be created nor destroyed.

100. The decrease is given by $\Delta L = L_0/\gamma$, which is enormously tiny. For speeds near the speed of light, however, decrease in length cannot be ignored.