Wave Theory of X-ray Phase-Contrast Radiography

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Received May 13, 1997

Abstract—The consistent wave theory of imaging for weakly absorbing noncrystalline objects is suggested within the method of X-ray phase-contrast radiography with the crystal-analyzer being located in the Bragg geometry. The sensitivity and the spatial resolution of the method were studied, both theoretically and experimentally, as functions of the angular position and the asymmetry coefficient of reflection from the analyzer. The validity ranges of the geometric-optics approximation are discussed. The phase-contrast images of a number of model objects (filaments, capillaries) and also of medical and biological objects are obtained and analyzed.

It is well known that the traditional methods of X-ray diffraction based on recording of absorption images are of a limited sensitivity and therefore cannot provide reliable identification of internal organs of living organisms without special substances used to increase their contrast (most often, barium salts and solutions of iodine-containing compounds) [1]. The monochromatization of an X-ray beam allows one to increase the contrast and considerably reduce the dose of the absorbed energy [2], but nevertheless the absorption methods become absolutely inapplicable to the studies of composition-homogeneous biological objects and, in particular, their fine details (blood vessels, lymphatic nodes, etc.). This is explained by the fact that the densities of all the soft tissues of the living organisms are close to the density of water, and therefore absorption coefficients of various tissues (with the exception of the osseous tissue) differ by not more than several percent.

On the other hand, it is well known that along with absorption, there exists the phenomenon of refraction characterized by the refractive index \( n = 1 - \delta \). For X-rays with the wavelength \( \lambda \approx 1 \text{ Å} \) and the medium consisting mainly of light carbon-containing compounds, the value of \( \delta \) is of the order of \( 10^{-6} \). The phase of a wave passing through an object changes by \( \varphi = 2\pi\delta l/\lambda \) determined by the radiation path \( l \) in the object and the relative decrement in refraction, \( \delta \). It is just this phenomenon of the phase shift that underlies several modifications of the method used for obtaining the so-called phase-contrast X-ray images.

The interferometry method. The method is based on the use of multiblock Bone–Flint X-ray interferometers [3] prepared from bulky single crystals. An incident X-ray beam is split into two beams that experience multiple Laue reflection and then are brought together again. An object is placed into one of the shoulders of the interferometer and thus changes the initial interference pattern at the interferometer exit. Since the phase shift by \( 2\pi \) occurs at the path \( l \) \( \approx 100 \text{ μm} \), the unique interpretation of the image is possible only for rather thin objects. This method was first used for obtaining phase-contrast images from a number of biological and mineralogical objects [4]. The use of the synchrotron radiation allowed one to implement the scheme of the phase-contrast computer tomography of sections of rat cerebellum and carcinoma of rabbit liver [5, 6].

The methods of phase-contrast imaging [7–20]. Despite several differences between the methods of obtaining images used by various authors and different names given to these methods, all of them are based on the record of space-inhomogeneous angular distribution of X-ray radiation transmitted by an object with the use of the diffraction reflection from a perfect crystal-analyzer (CA). The phase shifts of the wave front of the transmitted wave change the direction of the wave propagation, i.e., provide its refraction. Despite the fact that the refraction angles are very small (of the order of \( \delta \)) and do not exceed several angular seconds or even fractions of an angular second, the crystal-analyzer transforms these small wave-front perturbations into rather considerable variations of the reflected intensity.

A higher sensitivity to the internal structure of the object is attained due to the record of the interfaces between the media having different refractive indices, because, in the vicinity of such interfaces, the phase gradient in the transverse direction is rather high. To form an almost parallel primary X-ray beam, one uses one or several Bragg reflections from perfect crystals—monochromators. The crystal-analyzer can be placed in both Bragg [7, 8, 12–14] and Laue [9, 10, 15–18] geometries. It is clear that the width of the diffraction-reflection curve of the crystal-analyzer and the choice of the working point on this curve are the decisive factors that increase the method sensitivity, the contrast of the images formed, their adequacy to the real objects, and the rate of their recording. In virtue of the diffraction nature of the dynamical reflection from the crystal-analyzer, the spatial resolution of the phase-contrast imaging is of the order of the extinction length.
Obtaining of phase-contrast images without an analyzer [21–23]. If the spatial coherency of an X-ray beam is rather high, one can obtain the phase-contrast image of an object by using a high-resolution film behind the object [21]. At the stage of recording the primary image, this method is close to the axial (transmission) Gabor holography [24] in which the radiation scattered by an object interferes with the incident (reference) beam at a certain distance \( L \) from the object. The distortion of the wave front is caused by both refraction at the interfaces of the object and the ambient medium and diffraction of waves by an object of finite dimensions. If, e.g., a transverse dimension of the object is \( R \approx 10 \, \mu m \), then \( L \approx (0.01-1)R^2/\lambda - 1-100 \, cm \) [21]. This method is very convenient for studying very small objects (\( \approx 0.1-50.0 \, \mu m \)), and therefore it was used for obtaining X-ray holograms from thin organic fibers with the use of a highly coherent beam from the ESRF synchrotron source [21]. The methods of phase-contrast imaging described above seem to be appropriate for studying macroscopic polymer and biomedical objects and their fragments with the characteristic dimensions from 0.1 mm up to 1.0–10.0 cm [15–17].

It was shown [23] that the use of a fine-focus (~20 \( \mu m \)) X-ray tube allows one to record the boundaries of weakly absorbing objects at a rather large distance from these objects (~1 m) even against the background of the polychromatic bremsstrahlung of an X-ray tube.

Below, we suggest a rigorous theory of phase-contrast X-ray imaging that consistently takes into account the wave phenomena both in scattering of an X-ray radiation by a specimen and in the process of diffraction reflection of this radiation characterized by the inhomogeneous distribution of the phase from a crystal-analyzer. The sensitivity and spatial resolution of the phase-contrast imaging with an analyzer being located in the Bragg geometry were studied, both theoretically and experimentally, depending on the angular position and the asymmetry coefficient in reflection from the crystal-analyzer. We also considered the criteria of applicability of the geometric-optics approximation. The X-ray phase-contrast images of a number of model and biomedical objects were obtained and analyzed.

1. FIELD BEHIND THE OBJECT

First, consider the spatial distribution of the field \( E_0(r) \) formed at a certain arbitrary distance \( z = L \) behind the object \( V(r) \) that scatters an incident plane wave \( \exp(i k_0 r) \), where \( k_0 = 2\pi/\lambda \). In the general case, the object has an inhomogeneous polarizability distribution \( \chi(r) = \chi_0 + i\chi_0 \), the related refractive index \( \delta(r) = -\chi_0/\lambda \), and the absorption coefficient \( \mu(r) = k\chi_0 \). We search for the solution of the Helmholtz equation

\[
\nabla^2 E_0(r) + k^2 E_0(r) = -k^2 \chi(r) E_0(r)
\]

in the form \( E_0(r) = A_0(r) \exp(i k_0 r) \), where \( A_0(r) = \exp[i F(r)] \). The phase \( F(r) \) describes the diffraction distortion of the wave front caused by the scattering of the radiation by an inhomogeneous object of finite dimensions. The substitution of \( E_0(r) \) into (1) leads to the parabolic diffusion-type equation

\[
\nabla^2 F(r) + 2i k_0 \nabla F(r) = ik^2 \chi(r).
\]

Since in the X-ray wavelength range, \( \chi \ll 1 \), one can neglect the term \( (\nabla F)^2 \ll |\nabla F| \) in derivation of (2) (the so-called Rylov approximation [25, 26]). The solution of inhomogeneous equation (2) has the form

\[
F(r) = -(i k^2/4\pi) \int \frac{\chi(r')}{r} G_0(r-r') \exp[-i k_0 (r-r')] dr',
\]

where \( G_0(r) = \exp(i k_0 r)/r_1 \) is the Green function for a free space and \( r_1 = |r-r'| \), \( r_1 \gg \lambda \).

For simplicity, we assume that an object has a considerable length and is homogeneous along the \( OY \)-axis, i.e., that \( \chi(r) = \chi(x, z) \) and that the plane wave propagates at an angle \( \alpha \ll 1 \) to the \( YZ \)-plane, \( k_0 = 0 \). Then, one can pass in (3) to the integration with respect to \( y \) within infinite limits and apply the method of a stationary phase in the vicinity of the stationary point \( y' = y \). In the direct vicinity of an object, where its transverse dimension \( R \) is much larger than the first Fresnel zone \( (\lambda z)^{1/2} \), i.e., where the wave parameter obeys the inequality \( D = (\lambda z)^{1/2} / R < 1 \), the repeated application of the stationary-phase method in the vicinity of the point \( x = x - \alpha z \) yields the following simple result

\[
F(x, z) = \Phi(x) = (k/2) \int \chi(x, z') dz' = \Phi(x) + i \mu \chi(x),
\]

where

\[
\Phi(x) = -(1/2) \int \delta(x, z') dz',
\]

\[
\mu \chi(x) = (1/2) \int \mu(x, z') dz'.
\]

The integration in (4) and (5) is performed along the path of radiation propagation within the volume \( V(x, z) \). Relationship (4) can also be obtained directly from (2) if one neglects the term \( \nabla^2 F \) describing the diffraction spread of the beam in the transverse direction [26].

Expression (4) corresponds to the geometric-optics (GO) approximation. In the region of the geometric shadow, the change in the phase of the wave \( \Phi(x) \) transmitted by the object in comparison with the phase advance in vacuum is determined by the refractive index and the path length in the object. The dependence
of the phase $\varphi(x)$ on the transverse coordinate $x$ results in the refractive deviation of the rays by angles $\beta(x) = \nabla \varphi(x)/k$, where $\nabla \varphi = \partial \varphi/\partial x$ is the phase gradient. It is just this GO approximation that is used in the qualitative analysis of the images with the phase-contrast [7–18, 20, 27]. It is valid for rather large objects far from their boundaries. Let, e.g., $\lambda \sim 1$ Å and $z \sim 10$–100 cm. Then $R \geq 30$–100 μm.

If the observation point lies at a rather large distance from the object, so that $z \gg x'$ and $x \gg |x' - x|$, then expression (3) reduces to the Fresnel-type integral with the weighting factor $\Phi(x)$:

$$F(x, z) = \frac{1}{\sqrt{i\lambda z}} \left[ \Phi(x') \exp \left[ i \frac{k(x - \alpha x - x')^2}{2z} \right] \right] dx'. \quad (6)$$

The dependence of the intensity $I_0(x) = |A_0|^2$ on the transverse coordinate $x$ in the region of the Fresnel diffraction $D \rightarrow 1$ is caused by the change of the imaginary part of the phase $F$ (6). At rather high $\text{Im} F$ values in the region $D \geq 1$, the Rytov approximation (6) becomes invalid, and one has to use the Kirchhoff method. Let us use instead of an object its phase analogue in the “near” zone ($z \approx R$): $A_0 = \exp[i\Phi(x)]$, where the phase $\Phi(x)$ was determined in (4) in the region of the object and $\Phi(x) = 0$ was determined outside it. Then, for an object elongated in the direction of the $OY$-axis, we have

$$A_0(x, z) = 1 + \frac{1}{\sqrt{i\lambda z}} \left[ \exp \left[ i \Phi(x') \right] - 1 \right] \times \exp \left[ i \frac{k(x - \alpha x - x')^2}{2z} \right] dx'. \quad (7)$$

Relationship (7) coincides with the corresponding results obtained in [21] at $\alpha = 0$ and $z_0 \gg z$, where $z_0$ is the distance from the point source to the object.

Figure 1 shows the spatial dependences of the intensities $I_0(x)$, phases $F(x)$, and the effective refraction angles $\beta(x) = \nabla F/k$ calculated for a thin Kapron (nylon-like synthetic fiber) filament of radius $R = 10$ μm at distances $L = 5$ and 50 cm (CuK$_\alpha$ radiation, $\delta = 3.52 \times 10^{-6}$, $\mu = 5.5$ cm$^{-1}$). For a cylindrical homogeneous object, the phase given by (5) is $\varphi(x) = -2k\delta(R^2 - x^2)^{1/2}$. In the near zone ($L = 5$ cm, $D = 0.3$), the calculation of the functions $F(x)$ and $\beta(x)$ within the frameworks of the wave theory (7) and the GO approximation, expression (4) yields rather close results (Figs. 1b, 1c). Nevertheless, with an increase of the distance to the object ($L = 50$ cm, $D = 0.9$, Figs. 1e, 1f) or with a decrease of the filament radius, the GO approximation becomes invalid. At the boundaries of the object image $I_0(x)$, there are obvious oscillations characteristic of the Fresnel diffraction (Figs. 1a, 1d). At the same time, the intensity of the absorption contrast within the framework of the GO approximation, $I_0(x) = \exp[-2\mu_2(x)]$, almost does not differ from unity (dashed lines in Figs. 1a, 1d). The oscillations at the interfaces become negligibly small and the distribution $I_0(x)$ tends to $I_0(x)$ only for objects with $R \gg (\lambda z)^{1/2}$. It is seen from Fig. 1d that in the region $D \geq 1$, the Rytov approximation (6) also becomes invalid.

### 2. DIFFRACTION IMAGING

Consider the process of phase-contrast diffraction imaging for the case were incident a primary X-ray wave is transmitted by an object and is incident onto the surface $z = 0$ of a plane-parallel crystal–analyzer of an arbitrary thickness $t$ in the Bragg geometry. The $OX$-axis is directed along the crystal surface. One has to determine the distribution of the diffracted-radiation field, $E_0(\alpha, z)$.

Represent the fields $E_0$ and $E_h$ on the surface of a crystal–analyzer in the form $E_g(x) = A_g(x) \exp(ik_\alpha x)$, where $g = 0, h$;

$$k_\alpha = k \cos(\theta_\alpha + \psi) - s, \quad k_h = k \cos(\theta_\beta - \psi) - s.$$

Here $s = k_{\gamma_0} \Delta \theta$, where $\Delta \theta = \theta - \theta_0$ is the rotation angle of the crystal–analyzer, $\theta_\beta$ is the Bragg angle, $\gamma_0 = \sin(\theta_\alpha + \psi) + \psi$ is the inclination angle of the reflecting planes with respect to the surface of the crystal–analyzer ($|\psi| < \theta_0$). It should be noted that the coordinate $X$ in Sect.1 is related to the coordinate $x$ in Sect. 2 by a simple relationship $X = \gamma_0 x$.

The complex amplitude $A_g(x)$ is determined by the collimation of an X-ray beam formed by the reflection from the block of monochromators and depends on the material, shape, and dimensions of an object as well as the distance from the crystal–analyzer (see Sect. 1). To determine the amplitude of the reflected wave $A_g(x)$, one has to expand $A_g(x)$ and $A_h(x)$ in plane waves:

$$A_g(x) = \int A_g(q) \exp(iqx) dq, \quad (8)$$

$$A_h(x) = (1/2\pi) \int A_g(q) \exp(-iqx) dq, \quad (9)$$

and to use the result for the Fourier-components of the fields well known from the dynamical theory of diffraction [28]:

$$A_h(q) = R(s - q) A_g(q), \quad (10)$$

where $R(s - q)$ is the amplitude coefficient of the reflection of the plane-wave incident onto the crystal–analyzer at an angle $(s - q)/k_{\gamma_0}$ with respect to the exact Bragg angle.

Substituting (10) and (9) into (8) and changing the variables $x = \xi$ and $s - q \rightarrow q$, we arrive at the following integral relationship for the spatial distribution of the reflected wave $A_g(x)$ at the arbitrary distribution
of the incident wave $A_0(x)$:

$$ A_k(x) = \int G(\xi)A_0(x-\xi)\exp(i\kappa\xi)d\xi, \quad (11) $$

$$ G(\xi) = (1/2\pi)\int R(q)\exp(-iq\xi)dq, \quad (12) $$

where $G(\xi)$ is the Green function [28, 29]. The integration variable $q$ in (12) is a real quantity.

The experimental image of an object is, in fact, an intensity distribution $I_i(x) = |A_k(x)|^2$ recorded either on a film or by a coordinate detector. The spatial resolution of the phase-contrast imaging is determined by both degree of the spread of the image $|G(x)|^2$ of a point $\delta$-like source on the surface of the crystal-analyzer and the resolution of the recording system.

It follows from the dynamical theory of X-ray diffraction for an ideal limited crystal [28] that

$$ R(q) = -q_0\eta(q + q_0 + iQ\cot\omega t)^{-1}, \quad (13) $$
where
\[ q_B = kC(\chi_{\text{abs}} \Delta \gamma_B)/\sin 2\theta_B, \quad \eta = (b\chi_{\text{abs}}/\chi_{\text{abs}})^{1/2}, \]
\[ q_0 = kC(\gamma_B + \gamma_B)/2\sin 2\theta_B, \quad Q = [(q + q_0^2 - q_B^2)^{1/2}], \]
\[ w = (\sin 2\theta_B/2\gamma_B)^{1/2}. \]

Here \( \chi_{\text{abs}} \) and \( \chi_{\text{abs}} \) are the polarizabilities of the crystal, \( \gamma_B = \sin(\theta_B - \psi) > 0, \quad b = \gamma_B/\chi_{\text{abs}} \) is the asymmetry coefficient of the Bragg reflection, \( C \) is the polarization factor \( (C = 1 \) for the \( \sigma \)-polarization and \( C = \cos 2\theta_B \) for the \( \pi \)-polarization). To increase the reflection intensity and exclude the effect of image splitting (see [11]), one has to use a rather thick crystal–analyzer. If to put \( k\chi_{\text{abs}}/\gamma_B \gg 1 \) in (13), then
\[ R(q) = (\eta/q_B)(-q - q_0 \pm Q), \]
where the sign before \( Q \) coincides with the sign of the imaginary part \( \text{Im} \omega \). Substituting (14) into (12), we arrive at the explicit form of the Green function [29]:
\[ G(\xi) = i(kC\chi_{\text{abs}}/2\sin 2\theta_B), \]
\[ \times \exp(iq_0\xi)(J_0(q_0\xi) + J_2(q_0\xi))H(\xi), \]
where \( J_0 \) and \( J_2 \) are the zeroth- and second-order Bessel functions, respectively, and \( H(\xi) \) is the stepwise Heaviside function equal to zero at \( \xi < 0 \) and to unity at \( \xi > 0 \). With due regard for the behavior of the function \( H(\xi) \) upon the change of the integration limits in (11), we arrive at the image amplitude \( A_h(x) \):
\[ A_h(x) = \int_0^{x_a} G(\xi)A_0(x - \xi)e^{i\phi(\xi - \xi)}d\xi, \]
where \( a \) is the minimum value of the coordinate of the region in which the incident field \( A_0 \) has the nonzero value.

The value of \( q_B \) in (13), which determines the width of the diffraction reflection curve \( P = |R(q)|^2 \) in the arguments of the Bessel functions in (15), is inversely proportional to the extinction length \( \Lambda = \lambda(\gamma_B)^{1/2}/\pi C\chi_{\text{abs}} \). For the analysis of the spatial resolution of the phase-contrast imaging, it is expedient to introduce into consideration a certain characteristic length \( \Lambda_x = 1/|q_B| \) along the surface of a crystal-analyzer, where
\[ \Lambda_x = \lambda\sin 2\theta_B/2\pi C\chi_{\text{abs}}(\gamma_B)^{1/2} = \Lambda \sin 2\theta_B/2\gamma_B. \]

The length \( \Lambda_x \) is minimal in the case of symmetric reflection \( (b = 1) \) and increases with both increase and decrease of the asymmetry coefficient \( b \). Since \( \chi_{\text{abs}}^2 \sim \Lambda^2 \), then \( \Lambda_x / \Lambda \sim 1/\Lambda \). The angular halfwidth of the diffraction-reflection curve is \( \Delta \theta_B = \Delta \theta_B/b^{1/2} \), where \( \Delta \theta_B = C\chi_{\text{abs}}/\sin 2\theta_B \) is the halfwidth of the diffraction-reflection curve of the symmetric reflection. If \( a_b \) is the spatial width of the incident beam, then the width of the reflected beam is \( a_b = a_B/b \). Therefore if \( b > 1 \), the angular width of the diffraction-reflection curve and the spatial width of the reflected beam decrease, and, at \( b < 1 \), they increase. It will be shown later, that such a behavior of \( \Lambda_x, \Delta \theta_B, \) and \( a_b \) plays a decisive role in the sensitivity and spatial resolution of the methods of phase-contrast imaging.

Prior to proceeding to the calculation of images of some model objects, consider a number of the general features of phase-contrast imaging. We represent the incident field \( A_0(x) \) for a crystal–analyzer as
\[ A_0(x) = |A_0(x)|\exp[i\phi(x)], \]
where \( |A_0(x)| = \exp[-\mu(x)] \). The dependence of the phase \( \phi(x) \) on the coordinate results in the refraction of beams by an angle \( \beta(x) = \nabla\phi/k\gamma_B \). Substituting (18) into (16), we obtain
\[ A_h(x) = \int_0^{x_a} G(\xi)A_0(x - \xi)e^{i\phi(x - \xi)}d\xi, \]
where \( x_a \) are the coordinates of the object boundaries. If \( x - x_a \gg \Lambda_x \), then the upper integration limit in (20) can be taken to be infinite. As a result, we obtain that
\[ A_h(x) = A_0(x) \int_0^{x_a} G(\xi)e^{i(x - \xi)}d\xi, \]
where \( x_a \) is the distance between the point \( x \) and the nearest object boundary, and \( \Delta \theta_B = \beta(x) \) is the angular deviation determined by the rotation angle of the analyzer and the local refraction angle. Relationship (21) is a geometric-optics approximation of the dynamical theory of diffraction and can also be obtained by the method of the stationary phase (see [20]) when estimating the double integral obtained upon the substitution of (12) into (11). It follows from (20) that the image is obtained only in the vicinity of the object boundaries \( x = x_a \), the image has the oscillation structure, and therefore the geometric-optics approximation (21) becomes invalid.
In the other limiting case, \( |V^2 \phi| \gg q^2 \), the method of the stationary phase and (19) yield

\[
A_0(x) = \sum_m G(x - x_m)A_0(x_m)C_m(x) \exp[is(x - x_m)],
\]

where

\[
C_m(x) = \int_a^x \exp \left[ \frac{i}{2} V^2 \varphi(x_m)(\xi - x_m)^2 \right] d\xi.
\]

The stationary points \( x = x_m \) are determined from the equation \( V\varphi(x) = s \). This case corresponds to the sources of strongly divergent waves concentrated at points \( x_m \). Usually, this case is observed in the vicinity of the edges of the objects having circular shapes. If the points \( x_m \) are located at considerable distances from \( a \) and \( x \), then \( C_m = \sqrt{\pi (1 \pm j)} V^2 \varphi(x_m)^{-1/2} \), and equation (22) reduces to the so-called refractometric mode described in [20] (the sign in the expression for \( C_m \) in parentheses is the same as the sign of \( V^2 \varphi(x_m) \)). In this case, the image in the vicinity of an isolated stationary point is determined by the spatial distribution of the Green function.

3. SENSITIVITY AND SPACIAL RESOLUTION

Let us estimate the sensitivity of phase-contrast imaging to small variations in the object density or, which is the same, to small variations in the refraction angles \( \beta(x) \) or the phase gradients \( V\varphi(x) \). If the absorption in the object can be ignored, then, in accordance with (21), the image intensity \( I_0(x) = P(\Delta \theta - \beta) \) is determined by the behavior of the diffraction-reflection curve in the vicinity of the working point \( \Delta \theta = s/k\gamma_0 \). It is well known that in Bragg reflection from a thick crystal, the diffraction reflection curve attains a value very close to unity in the vicinity of the maximum \( |\Delta \theta| \leq \Delta \theta_0 \) and dramatically falls down with an increase of the rotation angle of the crystal-analyzer, \( \Delta \theta \). Therefore, in order to increase the method sensitivity, the working point should be chosen at the steep (almost linear) slope of the diffraction-reflection curve with \( |y| = 1 - 1.5 \), where \( y = \Delta \theta / \Delta \theta_0 \) is the angular deviation of the crystal-analyzer normalized to the halfwidth of the reflection-diffraction curve, \( \Delta \theta_0 \). The image contrast \( K = I_0/I_1 \) at small refraction angles \( \beta \) is determined by the derivative of the diffraction-reflection curve:

\[
K(x) = (\beta / \Delta \theta_0) f(y),
\]

where \( f(y) = P^{-1} dP/ dy \). The function \( f(y) \) on the scale \( y \) is a universal function almost independent of the order of reflection and the asymmetry coefficient \( b \) of the crystal-analyzer. It is seen from (23) that in order to increase the contrast, one has to use diffraction-reflection curves with a small width \( \Delta \theta_0 \), i.e., to use the symmetric reflections with \( b \gg 1 \) and the working point at which the \( dP/ dy \) value is maximal.

Now estimate (23) on an example of the (220) AgK\( \alpha \) radiation from a silicon crystal with \( b = 10 \) and \( \Delta \theta_0 = 0.27^\circ \). We use the experimentally measured value \( K = 10\% \) and the working point \( y = -1.1 \) at which \( f = 4.6 \). Then, it is possible to record very small refraction angles \( \beta = 6 \times 10^{-3}^\circ \) corresponding to the relative difference between the refractive indices of two media \( \Delta \theta = 3 \times 10^{-8} \). On the small-angle slope of the diffraction-reflection curve, the value of \( f(y) \) is almost 1.5 times higher than in the region of positive angles, and the experiment confirms much higher sensitivity of the method at the working points lying at the small-angle slope. In real experiments, the sensitivity is limited by the divergence of the incident radiation, \( \Delta \theta_0 \). Therefore, in order to attain such a high sensitivity, one has to use the strongly asymmetric reflections from one (7-10, 17) or several (9, 10, 12-16, 30) monochromators with \( b_m \ll 1 \). If, e.g., \( b_m = 1/54 \), then \( \Delta \theta_0 = 0.12^\circ \).

Now consider the optimum conditions for increasing the spatial resolution. It follows from (10) and (14) that with an increase of the ratio \( |V\varphi| / q_0 \), a certain part of the spectrum range \( A_0(q) \) goes outside the region of "strong reflection." This results in the suppression of short-wavelength components of the reflection spectrum \( A_0(q) \) and, as a consequence, also to the distortion of the space distribution \( A_0(x) \). Therefore, in order to improve the quality of the transformation of \( A_0(x) \) into \( A_0(x) \), one has to use the asymmetric reflections from the crystal-analyzer with \( b \ll 1 \), i.e., the reflections with the considerable width of the diffraction-reflection curve. However, with a decrease of the asymmetry coefficient \( b \), the longitudinal extinction length \( \Lambda \approx 1/\gamma_0^{1/2} \) (17) increases. At first glance, it seems that the resolution should decrease. Nevertheless, the image becomes broader by a factor \( a_0 / b \) along the \( OX \) direction. As a result, the ratio of the total dimension of the image to the value of \( \Lambda \) characterizing the spread of an infinitely narrow beam increases as \( 1/b^{1/2} \).

Figure 2 shows the phase-contrast images of the profile of a Kapron capillary at two values of the asymmetry coefficient of the analyzer, \( b = 20 \) and \( b = 1/20 \), calculated by formula (19). For comparison, the dashed lines show the data calculated by the formula (21) free of the extinction distortions. It is seen from Fig. 2 that the best reproduction of the object details is observed just in the case where \( b \ll 1 \).

As has already been indicated, the wave phenomena manifest themselves most clearly for rather thin objects with the characteristic dimensions \( R \sim (\lambda L)^{1/2} \) (during the formation of the field \( A_0 \) on the surface of the crystal-analyzer behind the object) or \( R \ll \Lambda \) (in the diffraction reflection from the crystal-analyzer). For comparison, Fig. 3 shows the calculated data for Kapron filaments of radii \( R = 10 \) and 50 \( \mu \)m in the following three cases: (1) the use of the rigorous theory for the fields \( A_0 \) (7) and \( A_b \) (16) (curves 1 and 2); (2) the use of the geo-
metric-optics approximation for the field $A_0 (4)$ and the wave theory for $A_0$ (16) (curves 2); and (3) the use of geometric-optics approximation for both $A_0$ (4) and $A_h$ (21) (curves 3). It is seen from Fig. 3 that with an increase of the filament dimensions, the results of all the calculations almost coincide with those obtained in the simplest geometric-optics approximation. The computational time in the last instance is minimal, which can be of great importance for solving the inverse problem of the restoration of the internal structure of various objects, i.e., for the computer diffraction tomography.

In order to obtain the stereographic images and the corresponding tomographic sections, an object should be rotated about one or several axes in the X-ray beam. The information thus obtained is more detailed than the information provided by a conventional tomograph: it would reflect not only the distribution of the regions of
the internal structure, which differently absorb the radiation, but also the distribution of the boundaries at which the phase of the incident beam has changed.

The corresponding experiments were performed on a double-crystal X-ray spectrometer in the dispersion-free geometry of the 220 and -220 monochromators and the crystal-analyzer prepared from a perfect 70 x 40 x 3 mm³ large silicon single crystals. We used the AgKα radiation (λ = 0.559 Å, θ₀ = 8.38°). The asymmetry reflection coefficients of the monochromator and the analyzer were b₁ = 1.54 and b = 2, respectively. The objects were a Kapron filament of radius R = 0.22 mm and a polyethylene capillary of radius R = 1.4 mm and the wall thickness d = 0.35 mm filled either with water or alcohol. In the first series of experiments (test A), the objects were studied in air; in the second series of experiments (test B) they were placed into a cell filled with alcohol. The objects–analyzer distance and the distance from the analyzer to the MK-type photoplate with the 50 µm-thick emulsion layer were 28 cm. The preliminary tuning of the circuit and the image control were provided by a area-sensitive TV detector.

The results obtained in photometric measurements of the capillary image at different angular deviations of the crystal-analyzer Δθ are shown in Fig. 4. Solid lines show the corresponding intensities I₀(x) calculated based on (19) with due regard for the convolution with the diffraction reflection curve of a monochromator and the smoothening of the curve due to the finite width (50 µm) of the entrance slit of the microphotometer.

First of all, it should be noted that if the crystal-monochromator was located in the position close to the maximum of the diffraction-reflection curve (Δθ = -0.13°), the filament and the capillary showed only a weak absorption contrast, especially in test B. This is explained by the fact that the refraction angles β(x) for the materials used only slightly exceed the halfwidth of the reflection-diffraction curve Δθ₁ = 0.60° (Fig. 5), and therefore the image is formed due to reflection from almost a flat surface of the Darwin
The spatial distribution of the refraction angle in the capillary is determined by the relationships:

$$\beta(x) = 2\gamma_0 x ((\delta - \delta_0)/l + (\delta_1 - \delta)/l_1), \quad \gamma_0 |x| \leq R - d,$$

$$\beta(x) = 2\gamma_0 x (\delta - \delta_0)/l, \quad R - d < |x| \leq R,$$

where $\delta$, $\delta_0$, and $\delta_1$ are the refractive indices of the capillary material, the substance filling the capillary tube, and the ambient medium, respectively; $l = (R^2 - \gamma_0^2 x^2)^{1/2}, l_1 = [(R - d)^2 - \gamma_0^2 x^2]^{1/2}$. The refractive indices $\delta$ (in the units of $10^{-6}$) for polyethylene, water, and alcohol are equal to 0.415, 0.469, and 0.377 and the absorption coefficients are $\mu = 0.31$, $0.63$, and $0.37$ cm$^{-1}$, respectively [17].

Since the difference in the refractive indices at the capillary–air interface is more pronounced than at the capillary–alcohol interface, then in test A (unlike test B), one obtains a weak contrast of the external capillary boundary even in the case where $\Delta \theta = -0.13''$ (the maximum of the diffraction-reflection curve, curve 3). This is explained by the fact that the radiation refracted at the external edges corresponds to the slopes of the diffraction-reflection curve having lower intensities.

The situation is quite different if the working point is displaced to the slopes of the diffraction-reflection curve. In both tests, one can see the sharp external boundaries of the capillary. Because of a large difference in the refractive indices indicated above, the external boundaries in test A have a higher contrast than in test B. At the same time, the internal boundaries of the capillary tube in test A are almost invisible, whereas in test B they are as sharp as the external boundaries (Fig. 4). This is explained by the fact that the gradient of the refractive index at the internal polyethylene-water interface in the object (test A) is masked by a more powerful refraction gradient at the external capillary–air interface having the same direction (Fig. 5). To the contrary, in the object studied in test B, both gradients have comparable values, but opposite signs. Therefore the contrast values at the internal and external interfaces become rather close and can hardly be distinguished. The fact that the contrast has a characteristic black–white structure in the vicinity of the capillary walls increases the visibility of the fine details of the internal structure (Fig. 4B). This result leads to a very important practical conclusion: to be able to observe the internal boundaries of an object characterized by a small refractive-index gradient, one has to place this object into an immersion medium with the refractive index close to that of the object material.

**CONCLUSION**

It should be noted that the wave theory of the contrast formation in the method of phase-contrast imaging suggested in this work satisfactorily describes the experimental data obtained for the objects studied. The sensitivity of the method to extremely low density gradients at the interfaces increases with an increase of the asymmetry coefficient of the crystal-analyzer, $b > 1$. It also increases when it is located in the positions corresponding to the small-angle slope of the diffraction reflection curve. To increase the spatial resolution of the method, one has to use the reflections with $b < 1$. In addition to the model objects, we also studied the possibilities provided by the phase-contrast imaging for studying complicated rather large medical–biological objects, e.g., 1.5 x 2.0 cm$^2$ large adenocarcinoma of the mammary gland inside a 10 x 10 x 3 cm$^3$ large object. It is shown that the images of the adenocarcinoma are similar to the corresponding images obtained from the histological 1-µm-thick sections of the same tumor, which allows the study of the morphological changes of neoplasms and the ways of metastasis formation. The tumor structure was best revealed when the object in the beam had no contact with air. The more detailed description of these experiments and the discussion of possible methods of construction of stereographic images of the internal structure of an object and obtaining of tomographic sections will be the subject of another article.

**REFERENCES**


Translated by L. Man